Computer Algebra Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

1-Algebraic-functions/1.1-Binomial-products/1.1.1-Linear/17-1.1.1.6-P-x-a+b-x-^m-c+d-x-^n-e+f-x-^p

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [78]. This is test number [17].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (78)	0.00 (0)
Mathematica	100.00 (78)	0.00 (0)
Maple	100.00 (78)	0.00 (0)
Fricas	82.05 (64)	17.95 (14)
Giac	61.54 (48)	38.46 (30)
Mupad	50.00 (39)	50.00 (39)
Maxima	34.62 (27)	65.38 (51)
Sympy	5.13 (4)	94.87 (74)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

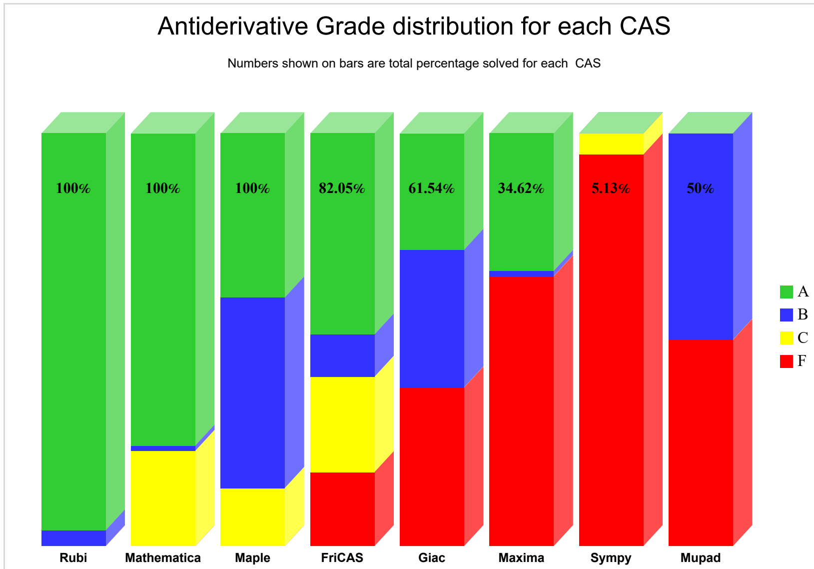
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

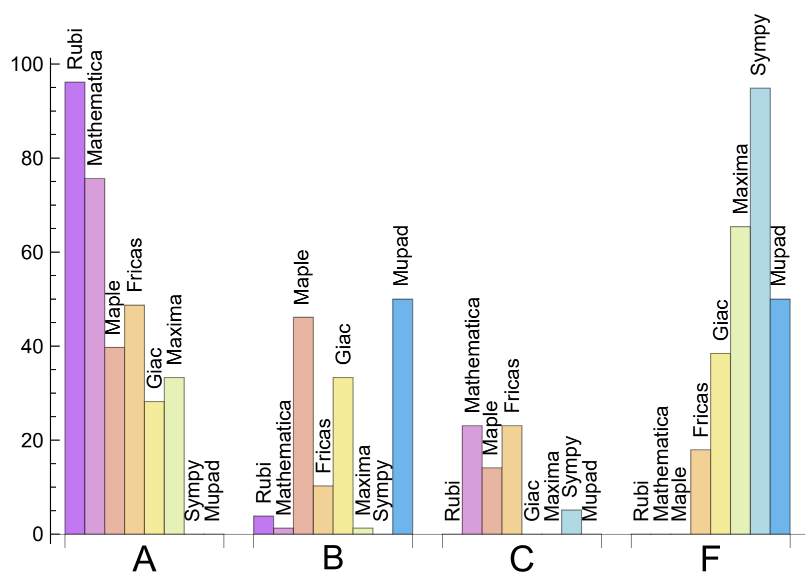
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Mathematica	75.641	1.282	23.077	0.000
Fricas	48.718	10.256	23.077	17.949
Maple	39.744	46.154	14.103	0.000
Maxima	33.333	1.282	0.000	65.385
Giac	28.205	33.333	0.000	38.462
Mupad	0.000	50.000	0.000	50.000
Sympy	0.000	0.000	5.128	94.872

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maple	0	0.00	0.00	0.00
Fricas	14	0.00	100.00	0.00
Giac	30	63.33	0.00	36.67
Mupad	39	0.00	100.00	0.00
Maxima	51	35.29	0.00	64.71
Sympy	74	67.57	32.43	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.28
Rubi	0.91
Maple	2.41
Giac	4.31
Fricas	4.92
Mathematica	8.42
Sympy	28.64
Mupad	34.47

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	187.07	0.99	100.00	1.01
Sympy	230.50	4.50	230.50	4.48
Rubi	426.36	1.02	357.00	1.03
Mathematica	472.68	1.00	260.50	0.95
Fricas	1230.86	2.51	823.50	1.88
Maple	1883.79	3.85	880.00	1.97
Giac	2271.65	4.58	526.50	2.07
Mupad	3248.46	15.07	1732.00	6.00

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

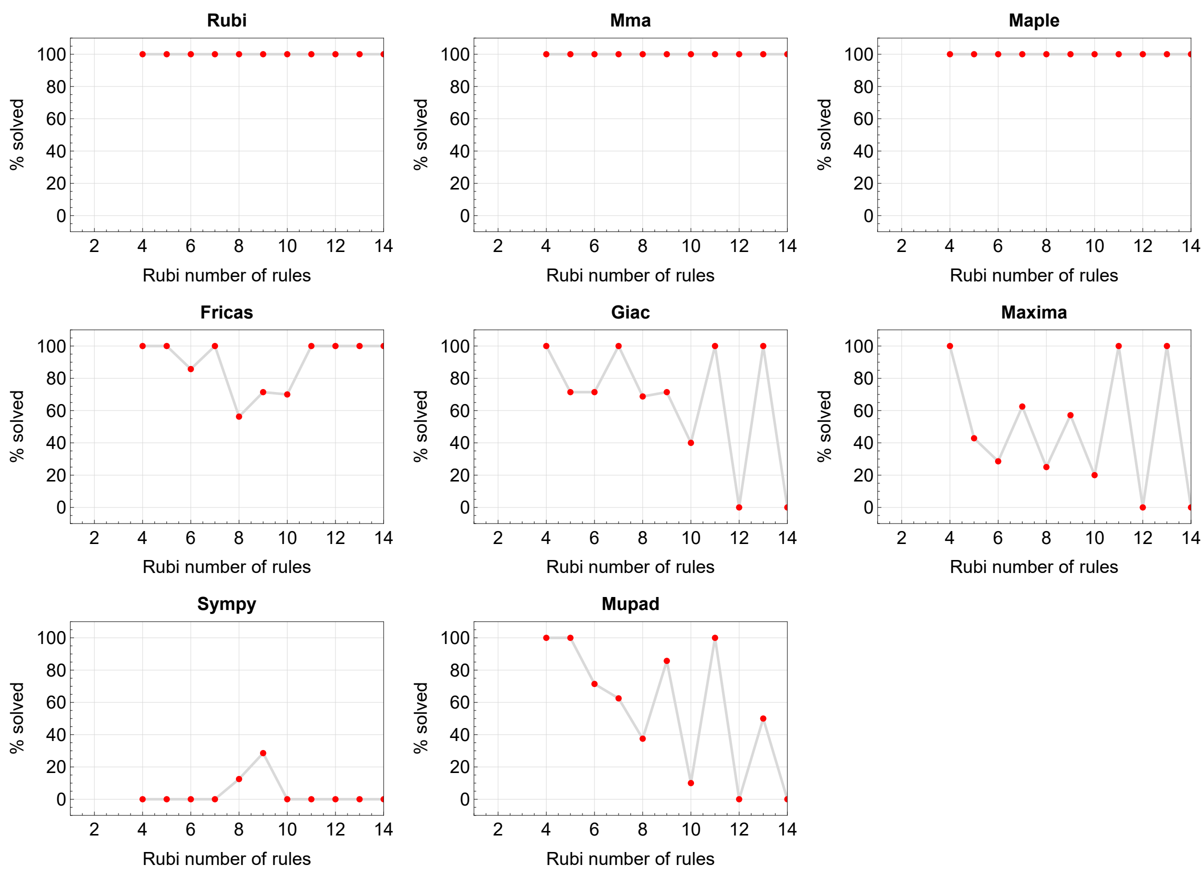


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

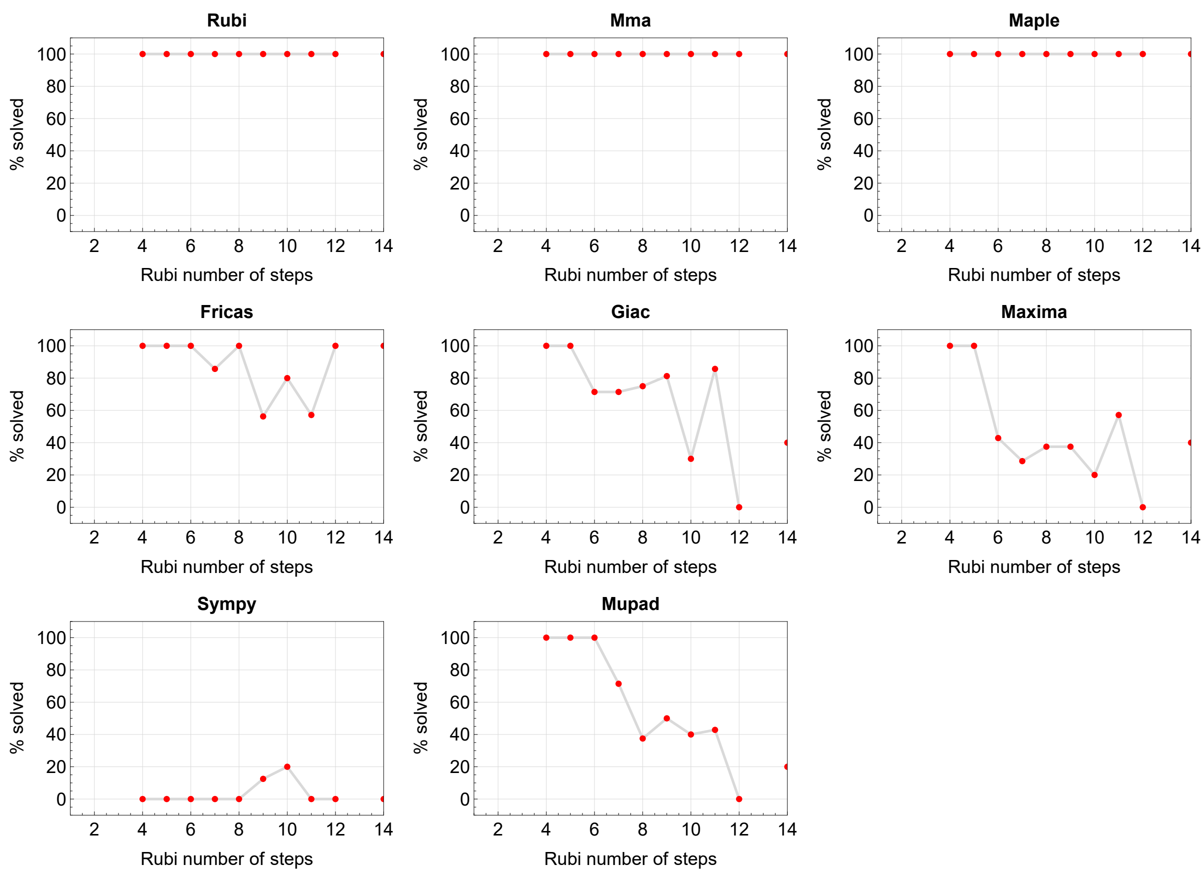


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

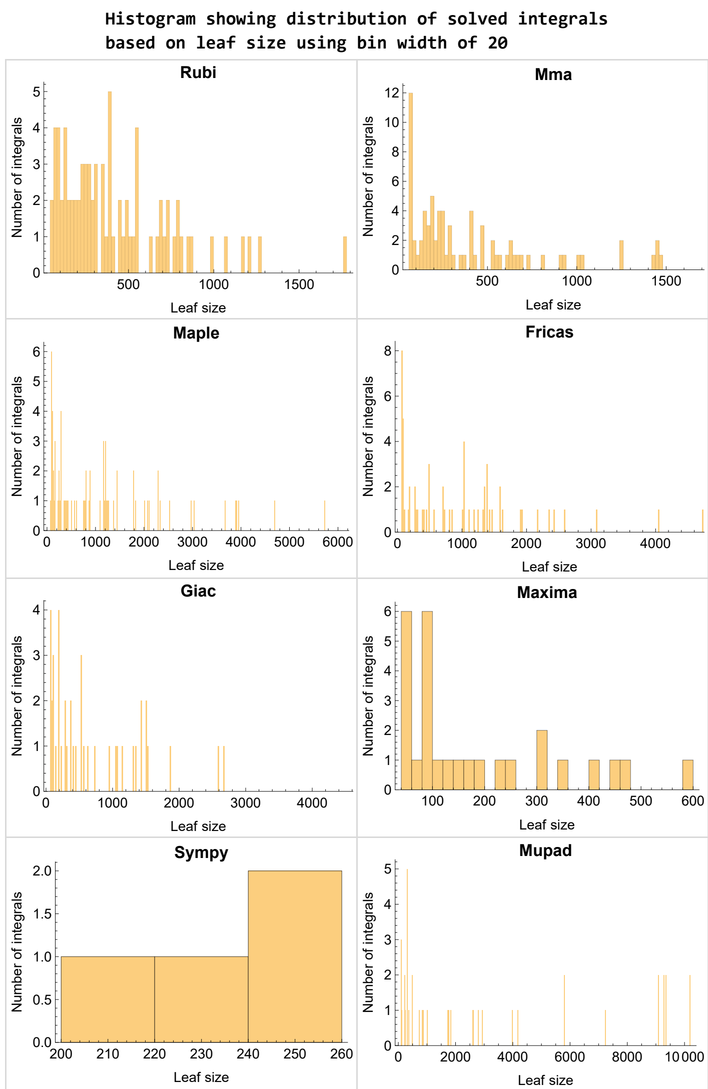


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

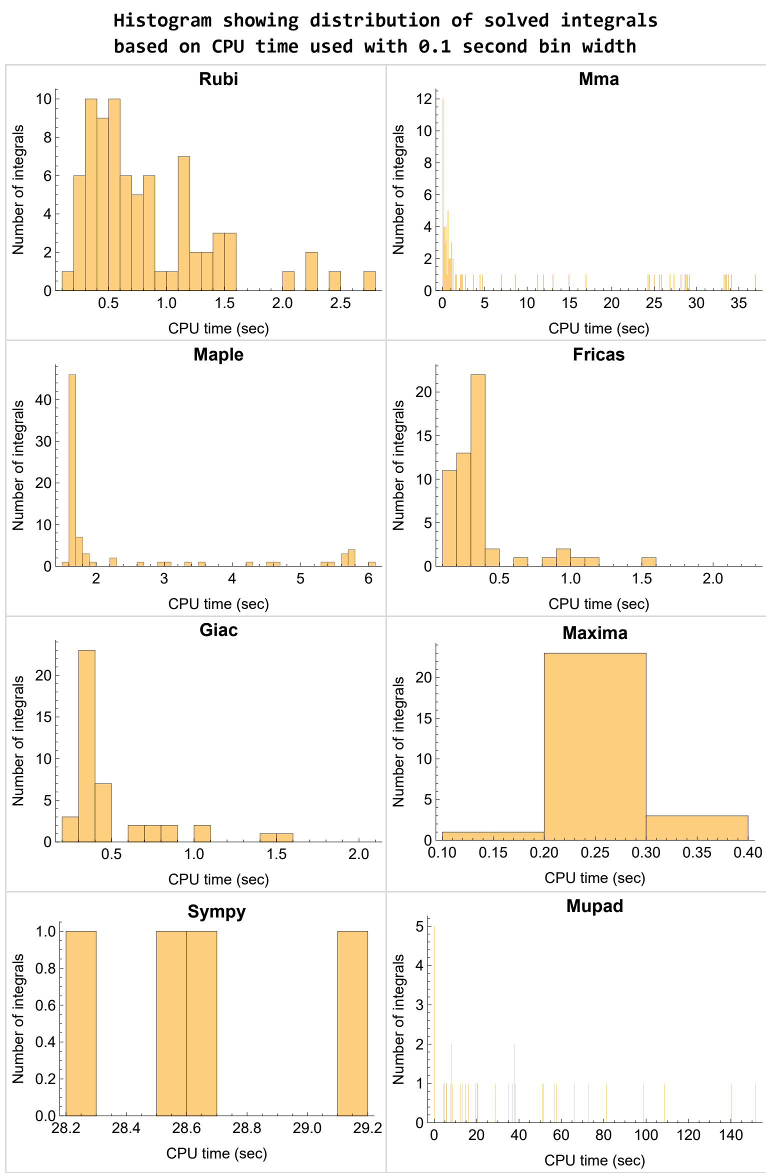


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

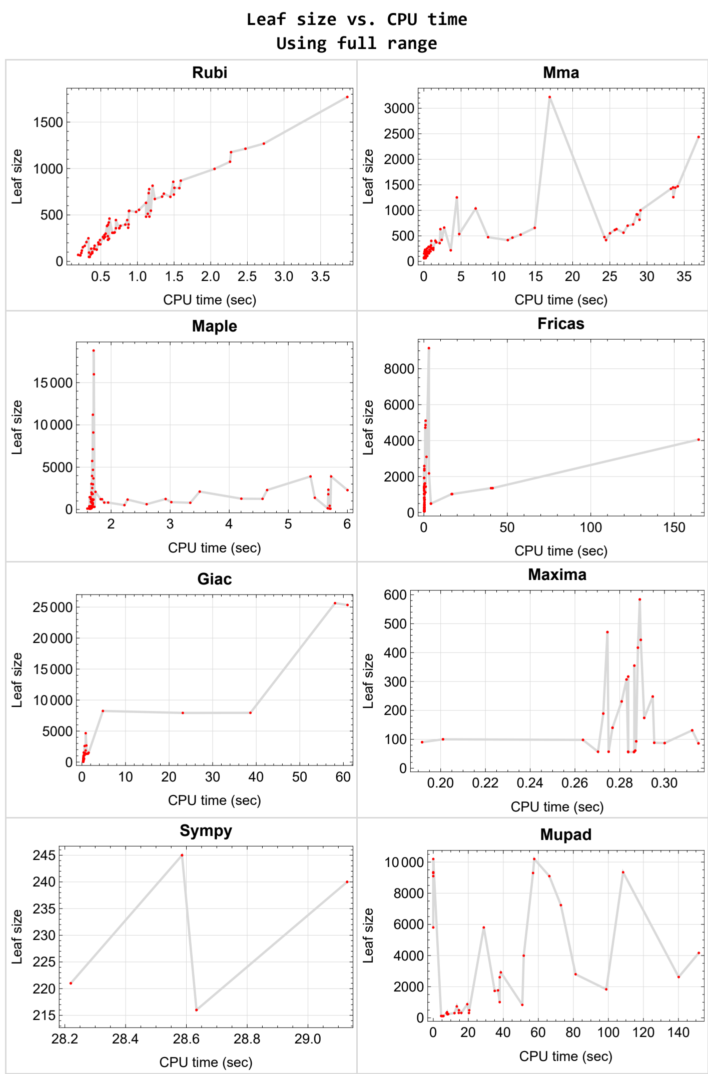


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica $\{35, 36, 37, 38\}$

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

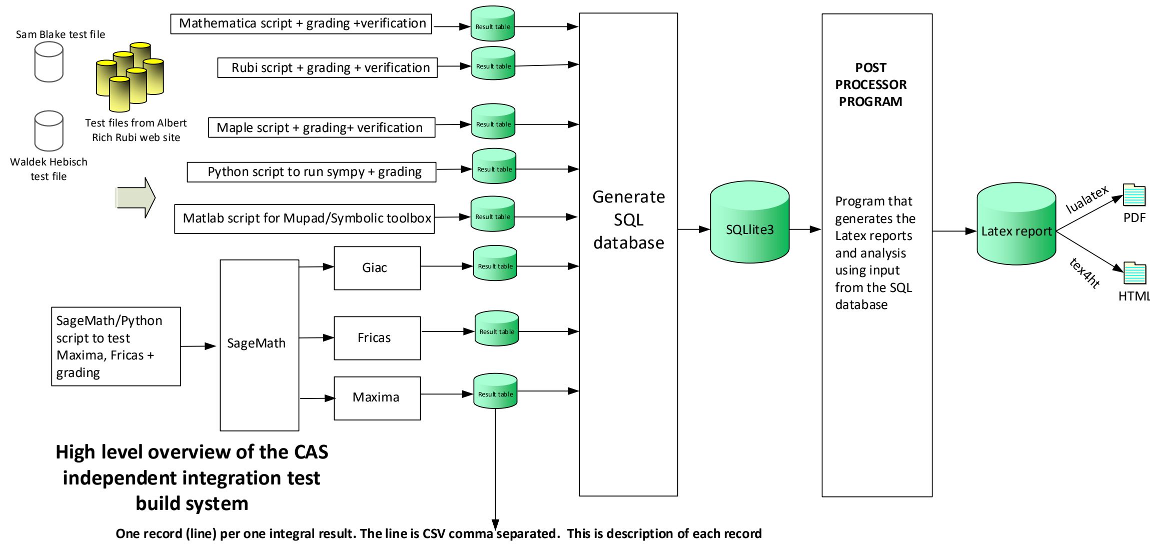
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,..}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60 }

B grade { 47 }

C grade { 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 8, 9, 10, 20, 21, 22, 23, 24, 27, 28, 29, 30, 31, 34, 37, 38, 39, 61, 62, 63, 67, 68, 69, 70, 73, 74, 75 }

B grade { 5, 12, 25, 26, 32, 33, 35, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 64, 65, 66, 71, 72, 76, 77, 78 }

C grade { 6, 7, 11, 13, 14, 15, 16, 17, 18, 19, 36 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 4, 8, 9, 10, 11, 15, 16, 17, 18, 19, 20, 21, 22, 23, 26, 27, 28, 29, 30, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 47, 48, 49, 54, 55, 56 }

B grade { 5, 6, 7, 12, 13, 14, 40, 59 }

C grade { 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78 }

F normal fail { }

F(-1) timedout fail { 24, 25, 31, 32, 44, 45, 46, 50, 51, 52, 53, 57, 58, 60 }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 1, 2, 3, 4, 8, 9, 10, 11, 15, 16, 17, 18, 19, 20, 21, 22, 23, 27, 28, 29, 30, 34, 36, 37, 38, 39 }

B grade { 35 }

C grade { }

F normal fail { 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78 }

F(-1) timedout fail { }

F(-2) exception fail { 5, 6, 7, 12, 13, 14, 24, 25, 26, 31, 32, 33, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60 }

2.1.6 Giac

A grade { 8, 9, 10, 11, 15, 16, 25, 27, 28, 29, 30, 32, 34, 35, 36, 37, 47, 48, 49, 54, 55, 56 }

B grade { 1, 2, 3, 4, 17, 18, 19, 20, 21, 22, 23, 26, 33, 38, 39, 41, 42, 43, 45, 46, 51, 52, 53, 58, 59, 60 }

C grade { }

F normal fail { 40, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78 }

F(-1) timedout fail { }

F(-2) exception fail { 5, 6, 7, 12, 13, 14, 24, 31, 44, 50, 57 }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 22, 23, 24, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 40, 49, 55, 56 }

C grade { }

F normal fail { }

F(-1) timedout fail { 20, 21, 25, 32, 41, 42, 43, 44, 45, 46, 47, 48, 50, 51, 52, 53, 54, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { }

B grade { }

C grade { 17, 18, 36, 37 }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 12, 13, 14, 20, 21, 22, 23, 24, 25, 31, 32, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 57, 58, 61, 62, 63, 64, 65, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77 }

F(-1) timedout fail { 8, 9, 10, 11, 15, 16, 19, 26, 27, 28, 29, 30, 33, 34, 35, 38, 39, 40, 53, 59, 60, 66, 72, 78 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	415	383	373	508	444	406	0	1539	3993
N.S.	1	0.92	0.90	1.22	1.07	0.98	0.00	3.71	9.62
time (sec)	N/A	0.816	1.670	2.226	0.289	0.290	0.000	0.430	51.668

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	271	262	358	307	279	0	1059	2920
N.S.	1	0.95	0.92	1.25	1.07	0.98	0.00	3.70	10.21
time (sec)	N/A	0.643	1.274	1.633	0.283	0.275	0.000	0.412	38.594

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	169	159	231	174	170	0	631	736
N.S.	1	1.01	0.95	1.38	1.04	1.01	0.00	3.76	4.38
time (sec)	N/A	0.439	0.701	1.643	0.291	0.291	0.000	0.337	13.443

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	91	89	155	93	95	0	284	361
N.S.	1	0.96	0.94	1.63	0.98	1.00	0.00	2.99	3.80
time (sec)	N/A	0.250	0.302	1.646	0.287	0.277	0.000	0.316	7.872

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	127	156	289	0	493	0	0	5803
N.S.	1	1.04	1.28	2.37	0.00	4.04	0.00	0.00	47.57
time (sec)	N/A	0.465	0.610	1.676	0.000	4.253	0.000	0.000	28.855

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	184	197	899	0	1025	0	0	10198
N.S.	1	1.13	1.21	5.52	0.00	6.29	0.00	0.00	62.56
time (sec)	N/A	0.516	0.898	1.671	0.000	16.648	0.000	0.000	57.674

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	263	228	1449	0	1580	0	0	9097
N.S.	1	1.06	0.92	5.84	0.00	6.37	0.00	0.00	36.68
time (sec)	N/A	0.583	1.263	1.637	0.000	0.363	0.000	0.000	66.294

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	340	359	259	361	355	286	0	374	2606
N.S.	1	1.06	0.76	1.06	1.04	0.84	0.00	1.10	7.66
time (sec)	N/A	0.802	1.017	1.631	0.287	0.277	0.000	0.337	37.958

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	244	178	256	231	192	0	232	1732
N.S.	1	1.07	0.78	1.12	1.01	0.84	0.00	1.02	7.60
time (sec)	N/A	0.640	0.680	1.644	0.281	0.279	0.000	0.319	35.089

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	140	107	173	131	114	0	112	492
N.S.	1	1.08	0.82	1.33	1.01	0.88	0.00	0.86	3.78
time (sec)	N/A	0.425	0.463	1.655	0.312	0.283	0.000	0.288	14.683

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	65	64	117	57	67	0	60	232
N.S.	1	1.03	1.02	1.86	0.90	1.06	0.00	0.95	3.68
time (sec)	N/A	0.234	0.251	1.634	0.270	0.264	0.000	0.303	8.146

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	127	156	289	0	493	0	0	5803
N.S.	1	1.04	1.28	2.37	0.00	4.04	0.00	0.00	47.57
time (sec)	N/A	0.470	0.042	1.662	0.000	4.215	0.000	0.000	0.005

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	184	197	899	0	1025	0	0	10198
N.S.	1	1.13	1.21	5.52	0.00	6.29	0.00	0.00	62.56
time (sec)	N/A	0.500	0.101	1.657	0.000	16.955	0.000	0.000	0.010

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	263	228	1449	0	1580	0	0	9097
N.S.	1	1.06	0.92	5.84	0.00	6.37	0.00	0.00	36.68
time (sec)	N/A	0.568	0.157	1.652	0.000	0.415	0.000	0.000	0.007

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	95	75	139	87	78	0	76	244
N.S.	1	1.20	0.95	1.76	1.10	0.99	0.00	0.96	3.09
time (sec)	N/A	0.365	0.021	5.667	0.300	0.335	0.000	0.326	8.580

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	65	64	117	57	67	0	60	232
N.S.	1	1.03	1.02	1.86	0.90	1.06	0.00	0.95	3.68
time (sec)	N/A	0.229	0.013	1.635	0.275	0.350	0.000	0.309	8.110

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	73	96	57	81	245	196	122
N.S.	1	1.00	1.52	2.00	1.19	1.69	5.10	4.08	2.54
time (sec)	N/A	0.367	0.017	1.598	0.284	0.300	28.586	0.346	4.514

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	73	97	57	84	221	282	114
N.S.	1	1.00	1.52	2.02	1.19	1.75	4.60	5.88	2.38
time (sec)	N/A	0.361	0.020	1.635	0.287	0.309	28.220	0.376	4.937

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	74	70	108	98	65	0	407	312
N.S.	1	1.04	0.99	1.52	1.38	0.92	0.00	5.73	4.39
time (sec)	N/A	0.387	0.024	1.624	0.264	0.302	0.000	0.365	7.626

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	591	512	402	575	584	1001	0	2671	0
N.S.	1	0.87	0.68	0.97	0.99	1.69	0.00	4.52	0.00
time (sec)	N/A	1.216	0.999	1.691	0.289	0.345	0.000	1.023	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	451	398	286	401	417	703	0	1868	0
N.S.	1	0.88	0.63	0.89	0.92	1.56	0.00	4.14	0.00
time (sec)	N/A	0.927	0.686	1.673	0.288	0.335	0.000	0.849	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	284	183	253	248	441	0	1142	1765
N.S.	1	0.95	0.61	0.84	0.83	1.47	0.00	3.81	5.88
time (sec)	N/A	0.591	0.394	1.669	0.295	0.330	0.000	0.632	37.004

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	153	123	163	140	265	0	527	876
N.S.	1	0.69	0.56	0.74	0.63	1.20	0.00	2.38	3.96
time (sec)	N/A	0.265	0.211	1.658	0.277	0.310	0.000	0.431	19.441

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	233	178	299	0	0	0	0	9298
N.S.	1	0.84	0.64	1.08	0.00	0.00	0.00	0.00	33.45
time (sec)	N/A	0.615	0.430	1.719	0.000	0.000	0.000	0.000	56.987

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	308	229	1166	0	0	0	526	0
N.S.	1	0.96	0.71	3.62	0.00	0.00	0.00	1.63	0.00
time (sec)	N/A	0.698	0.756	1.703	0.000	0.000	0.000	0.457	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	363	357	252	1794	0	1355	0	1425	9344
N.S.	1	0.98	0.69	4.94	0.00	3.73	0.00	3.93	25.74
time (sec)	N/A	0.786	1.027	1.674	0.000	41.191	0.000	0.711	108.411

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	501	482	283	390	471	700	0	571	4167
N.S.	1	0.96	0.56	0.78	0.94	1.40	0.00	1.14	8.32
time (sec)	N/A	1.207	0.629	1.654	0.275	0.335	0.000	0.354	151.648

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	368	362	200	270	317	482	0	363	2799
N.S.	1	0.98	0.54	0.73	0.86	1.31	0.00	0.99	7.61
time (sec)	N/A	0.919	0.431	1.683	0.284	0.318	0.000	0.327	81.282

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	248	128	175	189	302	0	191	1011
N.S.	1	1.01	0.52	0.71	0.77	1.23	0.00	0.78	4.11
time (sec)	N/A	0.562	0.235	1.672	0.273	0.310	0.000	0.330	37.946

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	116	90	126	88	196	0	106	489
N.S.	1	0.66	0.51	0.71	0.50	1.11	0.00	0.60	2.76
time (sec)	N/A	0.245	0.135	1.666	0.295	0.282	0.000	0.315	20.546

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	233	178	299	0	0	0	0	9298
N.S.	1	0.84	0.64	1.08	0.00	0.00	0.00	0.00	33.45
time (sec)	N/A	0.616	0.161	1.700	0.000	0.000	0.000	0.000	0.008

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	308	229	1166	0	0	0	526	0
N.S.	1	0.96	0.71	3.62	0.00	0.00	0.00	1.63	0.00
time (sec)	N/A	0.675	0.267	1.677	0.000	0.000	0.000	0.492	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	363	357	252	1794	0	1355	0	1425	9344
N.S.	1	0.98	0.69	4.94	0.00	3.73	0.00	3.93	25.74
time (sec)	N/A	0.745	0.417	5.677	0.000	40.270	0.000	0.718	0.008

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	137	74	108	100	73	0	105	318
N.S.	1	1.57	0.85	1.24	1.15	0.84	0.00	1.21	3.66
time (sec)	N/A	0.382	0.060	1.644	0.201	0.280	0.000	0.309	16.031

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F(-1)	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	52	70	63	96	90	61	0	80	312
N.S.	1	1.35	1.21	1.85	1.73	1.17	0.00	1.54	6.00
time (sec)	N/A	0.197	0.043	5.700	0.192	0.280	0.000	0.304	20.382

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	55	89	69	95	56	73	240	71	118
N.S.	1	1.62	1.25	1.73	1.02	1.33	4.36	1.29	2.15
time (sec)	N/A	0.398	0.047	1.648	0.286	0.281	29.129	0.320	5.821

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	55	89	69	95	56	82	216	83	118
N.S.	1	1.62	1.25	1.73	1.02	1.49	3.93	1.51	2.15
time (sec)	N/A	0.395	0.063	1.651	0.284	0.312	28.633	0.287	5.503

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	83	102	60	76	61	69	0	145	316
N.S.	1	1.23	0.72	0.92	0.73	0.83	0.00	1.75	3.81
time (sec)	N/A	0.398	0.047	5.713	0.287	0.300	0.000	0.316	14.593

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	133	71	89	86	90	0	197	304
N.S.	1	1.15	0.61	0.77	0.74	0.78	0.00	1.70	2.62
time (sec)	N/A	0.417	0.069	1.650	0.315	0.308	0.000	0.325	12.125

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	230	185	1095	0	1186	0	0	7235
N.S.	1	1.16	0.93	5.50	0.00	5.96	0.00	0.00	36.36
time (sec)	N/A	0.503	0.602	1.694	0.000	0.323	0.000	0.000	72.885

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1348	815	1253	5734	0	3096	0	4656	0
N.S.	1	0.60	0.93	4.25	0.00	2.30	0.00	3.45	0.00
time (sec)	N/A	1.259	4.434	1.680	0.000	1.577	0.000	0.865	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	721	461	662	3025	0	1620	0	2592	0
N.S.	1	0.64	0.92	4.20	0.00	2.25	0.00	3.60	0.00
time (sec)	N/A	0.652	2.731	1.667	0.000	0.651	0.000	0.619	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	330	249	283	1207	0	840	0	1073	0
N.S.	1	0.75	0.86	3.66	0.00	2.55	0.00	3.25	0.00
time (sec)	N/A	0.342	0.874	1.657	0.000	0.321	0.000	0.432	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	450	481	404	3898	0	0	0	0	0
N.S.	1	1.07	0.90	8.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.167	1.544	5.725	0.000	0.000	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	521	546	358	4680	0	0	0	1507	0
N.S.	1	1.05	0.69	8.98	0.00	0.00	0.00	2.89	0.00
time (sec)	N/A	1.257	2.127	1.693	0.000	0.000	0.000	1.539	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	658	695	536	11204	0	0	0	8241	0
N.S.	1	1.06	0.81	17.03	0.00	0.00	0.00	12.52	0.00
time (sec)	N/A	1.508	4.755	1.692	0.000	0.000	0.000	4.811	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1032	778	3220	3958	0	2176	0	1509	0
N.S.	1	0.75	3.12	3.84	0.00	2.11	0.00	1.46	0.00
time (sec)	N/A	1.233	16.915	1.678	0.000	3.084	0.000	0.455	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	540	419	474	2002	0	1114	0	733	0
N.S.	1	0.78	0.88	3.71	0.00	2.06	0.00	1.36	0.00
time (sec)	N/A	0.632	8.633	1.668	0.000	0.921	0.000	0.365	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	208	217	763	0	576	0	313	1832
N.S.	1	0.85	0.88	3.10	0.00	2.34	0.00	1.27	7.45
time (sec)	N/A	0.314	3.616	1.672	0.000	0.372	0.000	0.302	98.801

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	312	465	1822	0	0	0	0	0
N.S.	1	1.08	1.60	6.28	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.741	11.908	1.689	0.000	0.000	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	364	396	417	3670	0	0	0	1354	0
N.S.	1	1.09	1.15	10.08	0.00	0.00	0.00	3.72	0.00
time (sec)	N/A	0.858	11.283	1.701	0.000	0.000	0.000	1.402	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	484	533	523	9100	0	0	0	7922	0
N.S.	1	1.10	1.08	18.80	0.00	0.00	0.00	16.37	0.00
time (sec)	N/A	1.062	13.010	1.699	0.000	0.000	0.000	23.147	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	685	719	657	15990	0	0	0	25338	0
N.S.	1	1.05	0.96	23.34	0.00	0.00	0.00	36.99	0.00
time (sec)	N/A	1.586	14.908	1.709	0.000	0.000	0.000	60.944	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	718	735	632	2528	0	1436	0	946	0
N.S.	1	1.02	0.88	3.52	0.00	2.00	0.00	1.32	0.00
time (sec)	N/A	1.199	2.224	1.681	0.000	1.120	0.000	0.359	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	371	381	314	1199	0	720	0	441	2621
N.S.	1	1.03	0.85	3.23	0.00	1.94	0.00	1.19	7.06
time (sec)	N/A	0.610	0.958	1.681	0.000	0.486	0.000	0.326	140.157

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	167	141	425	0	380	0	190	833
N.S.	1	1.02	0.86	2.59	0.00	2.32	0.00	1.16	5.08
time (sec)	N/A	0.292	0.331	5.701	0.000	0.319	0.000	0.290	50.858

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	204	183	746	0	0	0	0	0
N.S.	1	1.09	0.97	3.97	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.486	0.500	1.692	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	299	249	2973	0	0	0	1319	0
N.S.	1	1.18	0.98	11.70	0.00	0.00	0.00	5.19	0.00
time (sec)	N/A	0.582	1.067	1.693	0.000	0.000	0.000	1.099	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	424	445	420	7119	0	4058	0	7939	0
N.S.	1	1.05	0.99	16.79	0.00	9.57	0.00	18.72	0.00
time (sec)	N/A	0.895	2.400	1.690	0.000	164.611	0.000	38.668	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	826	869	1036	18802	0	0	0	25632	0
N.S.	1	1.05	1.25	22.76	0.00	0.00	0.00	31.03	0.00
time (sec)	N/A	1.645	6.959	1.704	0.000	0.000	0.000	58.083	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1182	1213	1422	2077	0	1916	0	0	0
N.S.	1	1.03	1.20	1.76	0.00	1.62	0.00	0.00	0.00
time (sec)	N/A	2.511	33.212	1.736	0.000	0.167	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	774	789	917	1205	0	1393	0	0	0
N.S.	1	1.02	1.18	1.56	0.00	1.80	0.00	0.00	0.00
time (sec)	N/A	1.706	28.729	1.823	0.000	0.149	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	706	729	633	1163	0	1463	0	0	0
N.S.	1	1.03	0.90	1.65	0.00	2.07	0.00	0.00	0.00
time (sec)	N/A	1.416	25.895	2.279	0.000	0.166	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	687	697	815	1378	0	2588	0	0	0
N.S.	1	1.01	1.19	2.01	0.00	3.77	0.00	0.00	0.00
time (sec)	N/A	1.438	28.992	5.449	0.000	0.267	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	964	996	1444	2292	0	4721	0	0	0
N.S.	1	1.03	1.50	2.38	0.00	4.90	0.00	0.00	0.00
time (sec)	N/A	2.170	33.800	4.641	0.000	0.865	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1716	1770	2437	3900	0	9150	0	0	0
N.S.	1	1.03	1.42	2.27	0.00	5.33	0.00	0.00	0.00
time (sec)	N/A	4.100	36.937	5.376	0.000	3.015	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1235	1268	1470	2108	0	1931	0	0	0
N.S.	1	1.03	1.19	1.71	0.00	1.56	0.00	0.00	0.00
time (sec)	N/A	2.933	34.132	3.500	0.000	0.159	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	766	792	922	1205	0	1392	0	0	0
N.S.	1	1.03	1.20	1.57	0.00	1.82	0.00	0.00	0.00
time (sec)	N/A	1.539	28.613	1.846	0.000	0.178	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	527	545	562	812	0	1036	0	0	0
N.S.	1	1.03	1.07	1.54	0.00	1.97	0.00	0.00	0.00
time (sec)	N/A	0.935	26.835	1.950	0.000	0.135	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	540	555	551	861	0	1336	0	0	0
N.S.	1	1.03	1.02	1.59	0.00	2.47	0.00	0.00	0.00
time (sec)	N/A	1.057	25.022	3.020	0.000	0.146	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	597	630	724	1269	0	2429	0	0	0
N.S.	1	1.06	1.21	2.13	0.00	4.07	0.00	0.00	0.00
time (sec)	N/A	1.175	28.129	4.205	0.000	0.307	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1034	1073	1449	2330	0	4867	0	0	0
N.S.	1	1.04	1.40	2.25	0.00	4.71	0.00	0.00	0.00
time (sec)	N/A	2.304	33.497	5.678	0.000	0.999	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	838	857	1000	1233	0	1388	0	0	0
N.S.	1	1.02	1.19	1.47	0.00	1.66	0.00	0.00	0.00
time (sec)	N/A	1.587	29.122	2.923	0.000	0.157	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	528	542	615	812	0	1036	0	0	0
N.S.	1	1.03	1.16	1.54	0.00	1.96	0.00	0.00	0.00
time (sec)	N/A	0.927	25.664	1.884	0.000	0.139	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	387	394	418	615	0	807	0	0	0
N.S.	1	1.02	1.08	1.59	0.00	2.09	0.00	0.00	0.00
time (sec)	N/A	0.630	24.490	2.602	0.000	0.136	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	422	445	477	784	0	1240	0	0	0
N.S.	1	1.05	1.13	1.86	0.00	2.94	0.00	0.00	0.00
time (sec)	N/A	0.779	24.270	3.341	0.000	0.152	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	642	672	699	1249	0	2344	0	0	0
N.S.	1	1.05	1.09	1.95	0.00	3.65	0.00	0.00	0.00
time (sec)	N/A	1.321	27.394	4.564	0.000	0.302	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1116	1176	1258	2283	0	5108	0	0	0
N.S.	1	1.05	1.13	2.05	0.00	4.58	0.00	0.00	0.00
time (sec)	N/A	2.413	33.536	6.002	0.000	1.049	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [61] had the largest ratio of [.3684209999999999998]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	11	11	0.92	37	0.297
2	A	9	9	0.95	37	0.243
3	A	7	7	1.01	35	0.200
4	A	6	6	0.96	30	0.200
5	A	9	8	1.04	37	0.216
6	A	7	6	1.13	37	0.162
7	A	6	5	1.06	37	0.135
8	A	11	11	1.06	37	0.297
9	A	9	9	1.07	37	0.243
10	A	6	6	1.08	35	0.171
11	A	5	5	1.03	30	0.167
12	A	9	8	1.04	37	0.216
13	A	7	6	1.13	37	0.162
14	A	6	5	1.06	37	0.135
15	A	7	7	1.20	31	0.226
16	A	5	5	1.03	30	0.167
17	A	10	9	1.00	33	0.273
18	A	9	8	1.00	33	0.242
19	A	8	7	1.04	33	0.212
20	A	14	13	0.87	40	0.325
21	A	11	10	0.88	40	0.250
22	A	9	8	0.95	38	0.211

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	7	6	0.69	33	0.182
24	A	10	9	0.84	40	0.225
25	A	9	8	0.96	40	0.200
26	A	7	6	0.98	40	0.150
27	A	14	13	0.96	40	0.325
28	A	11	10	0.98	40	0.250
29	A	8	7	1.01	38	0.184
30	A	6	5	0.66	33	0.152
31	A	10	9	0.84	40	0.225
32	A	9	8	0.96	40	0.200
33	A	7	6	0.98	40	0.150
34	A	9	8	1.57	30	0.267
35	A	4	4	1.35	29	0.138
36	A	10	9	1.62	32	0.281
37	A	9	8	1.62	32	0.250
38	A	7	6	1.23	32	0.188
39	A	8	7	1.15	32	0.219
40	A	7	6	1.16	32	0.188
41	A	10	9	0.60	36	0.250
42	A	8	7	0.64	34	0.206
43	A	8	7	0.75	29	0.241
44	A	11	10	1.07	36	0.278
45	A	11	10	1.05	36	0.278
46	A	11	10	1.06	36	0.278
47	A	9	8	0.75	36	0.222
48	A	7	6	0.78	34	0.176
49	A	7	6	0.85	29	0.207
50	A	9	8	1.08	36	0.222
51	A	9	8	1.09	36	0.222
52	A	9	8	1.10	36	0.222
53	A	9	8	1.05	36	0.222
54	A	8	7	1.02	36	0.194
55	A	6	5	1.03	34	0.147
56	A	6	5	1.02	29	0.172

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	7	6	1.09	36	0.167
58	A	7	6	1.18	36	0.167
59	A	7	6	1.05	36	0.167
60	A	9	8	1.05	36	0.222
61	A	14	14	1.03	38	0.368
62	A	12	12	1.02	38	0.316
63	A	12	12	1.03	38	0.316
64	A	12	12	1.01	38	0.316
65	A	12	12	1.03	38	0.316
66	A	14	14	1.03	38	0.368
67	A	14	14	1.03	38	0.368
68	A	12	12	1.03	38	0.316
69	A	10	10	1.03	38	0.263
70	A	10	10	1.03	38	0.263
71	A	10	10	1.06	38	0.263
72	A	12	12	1.04	38	0.316
73	A	12	12	1.02	38	0.316
74	A	10	10	1.03	38	0.263
75	A	8	8	1.02	38	0.211
76	A	8	8	1.05	38	0.211
77	A	10	10	1.05	38	0.263
78	A	12	12	1.05	38	0.316

CHAPTER 3

Listing of integrals

3.1 $\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^3 \left(A+Bx+Cx^2\right) dx$

3.1.1 Optimal result

Integrand size = 37, antiderivative size = 415

$$\begin{aligned}
&\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^3 \left(A+Bx+Cx^2\right) dx \\
&= \frac{(2Cd^2e^3+8Ad^4e^3+6Bd^2e^2f+3Cef^2+6Ad^2ef^2+Bf^3)\,x\sqrt{1-d^2x^2}}{16d^4} \\
&\quad - \frac{(7d^2f(Be+2Af)-C(3d^2e^2-8f^2))\,(e+fx)^2\left(1-d^2x^2\right)^{3/2}}{70d^4f} \\
&\quad + \frac{(3Ce-7Bf)(e+fx)^3\left(1-d^2x^2\right)^{3/2}}{42d^2f} - \frac{C(e+fx)^4\left(1-d^2x^2\right)^{3/2}}{7d^2f} \\
&\quad + \frac{(8(C(3d^4e^4-30d^2e^2f^2-8f^4)-7d^2f(2Af(6d^2e^2+f^2)+B(d^2e^3+6ef^2)))+3d^2f(6Cd^2e^3-14Bd^2e}{840d^6f} \\
&\quad + \frac{(2Cd^2e^3+8Ad^4e^3+6Bd^2e^2f+3Cef^2+6Ad^2ef^2+Bf^3)\arcsin(dx)}{16d^5}
\end{aligned}$$

output
```
-1/70*(7*d^2*f*(2*A*f+B*e)-C*(3*d^2*e^2-8*f^2))*(f*x+e)^2*(-d^2*x^2+1)^(3/
2)/d^4/f+1/42*(-7*B*f+3*C*e)*(f*x+e)^3*(-d^2*x^2+1)^(3/2)/d^2/f-1/7*C*(f*x
+e)^4*(-d^2*x^2+1)^(3/2)/d^2/f+1/840*(8*C*(3*d^4*e^4-30*d^2*e^2*f^2-8*f^4)
-56*d^2*f*(2*A*f*(6*d^2*e^2+f^2)+B*(d^2*e^3+6*e*f^2))+3*d^2*f*(-98*A*d^2*e
*f^2-14*B*d^2*e^2*f+6*C*d^2*e^3-35*B*f^3-41*C*e*f^2)*x)*(-d^2*x^2+1)^(3/2)
/d^6/f+1/16*(8*A*d^4*e^3+6*A*d^2*e*f^2+6*B*d^2*e^2*f+2*C*d^2*e^3+B*f^3+3*C
*e*f^2)*arcsin(d*x)/d^5+1/16*(8*A*d^4*e^3+6*A*d^2*e*f^2+6*B*d^2*e^2*f+2*C*
d^2*e^3+B*f^3+3*C*e*f^2)*x*(-d^2*x^2+1)^(1/2)/d^4
```

3.1.2 Mathematica [A] (verified)

Time = 1.67 (sec) , antiderivative size = 373, normalized size of antiderivative = 0.90

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx$$
$$=\frac{\sqrt{1-d^2x^2}(14Ad^2(-16f^3-d^2f(120e^2+45efx+8f^2x^2)+6d^4x(10e^3+20e^2fx+15ef^2x^2+4f^3x^3))+}{}$$

input
```
Integrate[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^3*(A + B*x + C*x^2),x]
```

output
```
(Sqrt[1 - d^2*x^2]*(14*A*d^2*(-16*f^3 - d^2*f*(120*e^2 + 45*e*f*x + 8*f^2*
x^2) + 6*d^4*x*(10*e^3 + 20*e^2*f*x + 15*e*f^2*x^2 + 4*f^3*x^3)) + 7*B*(-3
*d^2*f^2*(32*e + 5*f*x) - 2*d^4*(40*e^3 + 45*e^2*f*x + 24*e*f^2*x^2 + 5*f^
3*x^3) + 4*d^6*x^2*(20*e^3 + 45*e^2*f*x + 36*e*f^2*x^2 + 10*f^3*x^3)) + C*
(-128*f^3 - d^2*f*(672*e^2 + 315*e*f*x + 64*f^2*x^2) - 6*d^4*x*(35*e^3 + 5
6*e^2*f*x + 35*e*f^2*x^2 + 8*f^3*x^3) + 12*d^6*x^3*(35*e^3 + 84*e^2*f*x +
70*e*f^2*x^2 + 20*f^3*x^3))) + 210*d*(2*C*d^2*e^3 + 8*A*d^4*e^3 + 6*B*d^2*
e^2*f + 3*C*e*f^2 + 6*A*d^2*e*f^2 + B*f^3)*ArcTan[(d*x)/(-1 + Sqrt[1 - d^2
*x^2])])/(1680*d^6)
```

3.1.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 383, normalized size of antiderivative = 0.92, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.297, Rules used = {2112, 2185, 25, 27, 687, 27, 687, 25, 676, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-dx}\sqrt{dx+1}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{ 2112}$$

$$\int \sqrt{1-d^2x^2}(e+fx)^3\left(A+Bx+Cx^2\right)dx$$

$$\Big\downarrow \text{ 2185}$$

$$-\frac{\int -f(e+fx)^3\left(\left(7Ad^2+4C\right)f-d^2(3Ce-7Bf)x\right)\sqrt{1-d^2x^2}dx}{7d^2f^2}-\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^4}{7d^2f}$$

$$\Big\downarrow \; 25$$

$$\frac{\int f(e+fx)^3\left(\left(7Ad^2+4C\right)f-d^2(3Ce-7Bf)x\right)\sqrt{1-d^2x^2}dx}{7d^2f^2}-\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^4}{7d^2f}$$

$$\Big\downarrow \; 27$$

$$\frac{\int (e+fx)^3\left(\left(7Ad^2+4C\right)f-d^2(3Ce-7Bf)x\right)\sqrt{1-d^2x^2}dx}{7d^2f}-\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^4}{7d^2f}$$

$$\Big\downarrow \; 687$$

$$\frac{\frac{1}{6}\left(1-d^2x^2\right)^{3/2}(e+fx)^3(3Ce-7Bf)-\frac{\int -3d^2(e+fx)^2\left(f\left(14Aed^2+5Ce+7Bf\right)+\left(7d^2f(Be+2Af)-C\left(3d^2e^2-8f^2\right)\right)x\right)\sqrt{1-d^2x^2}dx}{6d^2}}{7d^2f}$$

$$\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^4}{7d^2f}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{2}\int (e+fx)^2\left(f\left(14Aed^2+5Ce+7Bf\right)+\left(7d^2f(Be+2Af)-C\left(3d^2e^2-8f^2\right)\right)x\right)\sqrt{1-d^2x^2}dx+\frac{1}{6}\left(1-d^2x^2\right)^{3/}}{7d^2f}$$

$$\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^4}{7d^2f}$$

$$\Big\downarrow \; 687$$

$$\frac{\frac{1}{2}\left(-\frac{\int -\left((e+fx)\left(f\left(70Ae^2d^4+19Ce^2d^2+28Af^2d^2+49Befd^2+16Cf^2\right)-d^2\left(6Cd^2e^3-14Bd^2fe^2-98Ad^2f^2e-41Cf^2e-35Bf^3\right)x\right)\sqrt{1-d^2x^2}\right)dx}{5d^2}\right.}{7d^2f}$$

$$\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^4}{7d^2f}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{1}{2}\left(\frac{\int (e+fx)\left(f\left(70Ae^2d^4+19Ce^2d^2+28Af^2d^2+49Befd^2+16Cf^2\right)-d^2\left(6Cd^2e^3-14Bd^2fe^2-98Ad^2f^2e-41Cf^2e-35Bf^3\right)x\right)\sqrt{1-d^2x^2}dx}{5d^2}-\frac{1}{5}\left(\right.\right.}{7d^2f}$$

$$\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^4}{7d^2f}$$

$$\Big\downarrow \; 676$$

3.1. $\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx$

$$\frac{\frac{1}{2}\left(\frac{\frac{35}{4}f\left(8Ad^4e^3+6Ad^2ef^2+6Bd^2e^2f+Bf^3+2Cd^2e^3+3Cef^2\right)\int\sqrt{1-d^2x^2}dx+\frac{1}{4}fx\left(1-d^2x^2\right)^{3/2}\left(-98Ad^2ef^2-14Bd^2e^2f-35Bf^3+6Cd^2e^3-41\right.}{5d^2}\right.}{}$$

$$\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^4}{7d^2f}$$

$$\Big\downarrow \; 211$$

$$\frac{\frac{1}{2}\left(\frac{\frac{35}{4}f\left(8Ad^4e^3+6Ad^2ef^2+6Bd^2e^2f+Bf^3+2Cd^2e^3+3Cef^2\right)\left(\frac{1}{2}\int\frac{1}{\sqrt{1-d^2x^2}}dx+\frac{1}{2}x\sqrt{1-d^2x^2}\right)+\frac{1}{4}fx\left(1-d^2x^2\right)^{3/2}\left(-98Ad^2ef^2-14Bd^2e^2f-3\right.}{5d^2}\right.}{}$$

$$\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^4}{7d^2f}$$

$$\Big\downarrow \; 223$$

$$\frac{\frac{1}{2}\left(\frac{\frac{35}{4}f\left(\frac{\arcsin(dx)}{2d}+\frac{1}{2}x\sqrt{1-d^2x^2}\right)\left(8Ad^4e^3+6Ad^2ef^2+6Bd^2e^2f+Bf^3+2Cd^2e^3+3Cef^2\right)+\frac{1}{4}fx\left(1-d^2x^2\right)^{3/2}\left(-98Ad^2ef^2-14Bd^2e^2f-35Bf^3+\right.}{5d^2}\right.}{}$$

$$\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^4}{7d^2f}$$

input
```
Int[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^3*(A + B*x + C*x^2),x]
```

output
```
-1/7*(C*(e + f*x)^4*(1 - d^2*x^2)^(3/2))/(d^2*f) + (((3*C*e - 7*B*f)*(e +
f*x)^3*(1 - d^2*x^2)^(3/2))/6 + (-1/5*((7*f*(B*e + 2*A*f) - C*(3*e^2 - (8*
f^2)/d^2))*(e + f*x)^2*(1 - d^2*x^2)^(3/2)) + ((2*(C*(3*d^4*e^4 - 30*d^2*e
^2*f^2 - 8*f^4) - 7*d^2*f*(2*A*f*(6*d^2*e^2 + f^2) + B*(d^2*e^3 + 6*e*f^2)
))*(1 - d^2*x^2)^(3/2))/(3*d^2) + (f*(6*C*d^2*e^3 - 14*B*d^2*e^2*f - 41*C*
e*f^2 - 98*A*d^2*e*f^2 - 35*B*f^3)*x*(1 - d^2*x^2)^(3/2))/4 + (35*f*(2*C*d
^2*e^3 + 8*A*d^4*e^3 + 6*B*d^2*e^2*f + 3*C*e*f^2 + 6*A*d^2*e*f^2 + B*f^3)*
((x*Sqrt[1 - d^2*x^2])/2 + ArcSin[d*x]/(2*d)))/4)/(5*d^2))/2)/(7*d^2*f)
```

3.1. $\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx$

3.1.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2112
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.1.4 Maple [A] (verified)

Time = 2.23 (sec) , antiderivative size = 508, normalized size of antiderivative = 1.22

method	result
risch	$-\frac{(240C\,f^3x^6d^6+280B\,d^6f^3x^5+840C\,d^6e\,f^2x^5+336A\,d^6f^3x^4+1008B\,d^6e\,f^2x^4+1008C\,d^6e^2f\,x^4+1260A\,d^6e\,f^2x^3+1260B\,d^6e^2f\,x}{}$
default	Expression too large to display

input
```
int((f*x+e)^3*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
-1/1680*(240*C*d^6*f^3*x^6+280*B*d^6*f^3*x^5+840*C*d^6*e*f^2*x^5+336*A*d^6
*f^3*x^4+1008*B*d^6*e*f^2*x^4+1008*C*d^6*e^2*f*x^4+1260*A*d^6*e*f^2*x^3+12
60*B*d^6*e^2*f*x^3+420*C*d^6*e^3*x^3+1680*A*d^6*e^2*f*x^2+560*B*d^6*e^3*x^
2-48*C*d^4*f^3*x^4+840*A*d^6*e^3*x-70*B*d^4*f^3*x^3-210*C*d^4*e*f^2*x^3-11
2*A*d^4*f^3*x^2-336*B*d^4*e*f^2*x^2-336*C*d^4*e^2*f*x^2-630*A*d^4*e*f^2*x-
630*B*d^4*e^2*f*x-210*C*d^4*e^3*x-1680*A*d^4*e^2*f-560*B*d^4*e^3-64*C*d^2*
f^3*x^2-105*B*d^2*f^3*x-315*C*d^2*e*f^2*x-224*A*d^2*f^3-672*B*d^2*e*f^2-67
2*C*d^2*e^2*f-128*C*f^3)*(d*x+1)^(1/2)*(d*x-1)/d^6/(-(d*x+1)*(d*x-1))^(1/2
)*((-d*x+1)*(d*x+1))^(1/2)/(-d*x+1)^(1/2)+1/16/d^4*(8*A*d^4*e^3+6*A*d^2*e*
f^2+6*B*d^2*e^2*f+2*C*d^2*e^3+B*f^3+3*C*e*f^2)/(d^2)^(1/2)*arctan((d^2)^(1
/2)*x/(-d^2*x^2+1)^(1/2))*((-d*x+1)*(d*x+1))^(1/2)/(-d*x+1)^(1/2)/(d*x+1)^
(1/2)
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 406, normalized size of antiderivative = 0.98

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^3 \left(A+Bx+Cx^2\right) \,dx$$
$$= \frac{(240\,Cd^6f^3x^6 - 560\,Bd^4e^3 - 672\,Bd^2ef^2 + 280\,(3\,Cd^6ef^2 + Bd^6f^3)x^5 + 48\,(21\,Cd^6e^2f + 21\,Bd^6ef^2 + (7}{}$$

input
```
integrate((f*x+e)^3*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorith
m="fricas")
```

output
```
1/1680*((240*C*d^6*f^3*x^6 - 560*B*d^4*e^3 - 672*B*d^2*e*f^2 + 280*(3*C*d^
6*e*f^2 + B*d^6*f^3)*x^5 + 48*(21*C*d^6*e^2*f + 21*B*d^6*e*f^2 + (7*A*d^6
- C*d^4)*f^3)*x^4 - 336*(5*A*d^4 + 2*C*d^2)*e^2*f - 32*(7*A*d^2 + 4*C)*f^3
 + 70*(6*C*d^6*e^3 + 18*B*d^6*e^2*f - B*d^4*f^3 + 3*(6*A*d^6 - C*d^4)*e*f^
2)*x^3 + 16*(35*B*d^6*e^3 - 21*B*d^4*e*f^2 + 21*(5*A*d^6 - C*d^4)*e^2*f -
(7*A*d^4 + 4*C*d^2)*f^3)*x^2 - 105*(6*B*d^4*e^2*f + B*d^2*f^3 - 2*(4*A*d^6
 - C*d^4)*e^3 + 3*(2*A*d^4 + C*d^2)*e*f^2)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1)
 - 210*(6*B*d^3*e^2*f + B*d*f^3 + 2*(4*A*d^5 + C*d^3)*e^3 + 3*(2*A*d^3 + C
*d)*e*f^2)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/d^6
```

3.1.6 Sympy [F]

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^3 \left(A+Bx+Cx^2\right) \,dx$$
$$= \int (e+fx)^3 \sqrt{-dx+1}\sqrt{dx+1}\left(A+Bx+Cx^2\right) \,dx$$

input
```
integrate((f*x+e)**3*(C*x**2+B*x+A)*(-d*x+1)**(1/2)*(d*x+1)**(1/2),x)
```

output
```
Integral((e + f*x)**3*sqrt(-d*x + 1)*sqrt(d*x + 1)*(A + B*x + C*x**2), x)
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.07

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx$$
$$= -\frac{(-d^2x^2+1)^{\frac{3}{2}}Cf^3x^4}{7\,d^2} + \frac{1}{2}\sqrt{-d^2x^2+1}Ae^3x + \frac{Ae^3\arcsin(dx)}{2\,d} - \frac{(-d^2x^2+1)^{\frac{3}{2}}Be^3}{3\,d^2} - \frac{(-d^2x^2+1)^{\frac{3}{2}}Ae^2f}{d^2} - \frac{4\,(-d^2x^2+1)^{\frac{3}{2}}Cf^3x^2}{35\,d^4} - \frac{(3\,Cef^2+Bf^3)(-d^2x^2+1)^{\frac{3}{2}}x^3}{6\,d^2} - \frac{(3\,Ce^2f+3\,Bef^2+Af^3)(-d^2x^2+1)^{\frac{3}{2}}x^2}{5\,d^2} - \frac{(Ce^3+3\,Be^2f+3\,Aef^2)(-d^2x^2+1)^{\frac{3}{2}}x}{4\,d^2} + \frac{(Ce^3+3\,Be^2f+3\,Aef^2)\sqrt{-d^2x^2+1}x}{8\,d^2} - \frac{8\,(-d^2x^2+1)^{\frac{3}{2}}Cf^3}{105\,d^6} - \frac{(3\,Cef^2+Bf^3)(-d^2x^2+1)^{\frac{3}{2}}x}{8\,d^4} + \frac{(Ce^3+3\,Be^2f+3\,Aef^2)\arcsin(dx)}{8\,d^3} - \frac{2\,(3\,Ce^2f+3\,Bef^2+Af^3)(-d^2x^2+1)^{\frac{3}{2}}}{15\,d^4} + \frac{(3\,Cef^2+Bf^3)\sqrt{-d^2x^2+1}x}{16\,d^4} + \frac{(3\,Cef^2+Bf^3)\arcsin(dx)}{16\,d^5}$$

input
```
integrate((f*x+e)^3*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorith
m="maxima")
```

output
```
-1/7*(-d^2*x^2 + 1)^(3/2)*C*f^3*x^4/d^2 + 1/2*sqrt(-d^2*x^2 + 1)*A*e^3*x +
 1/2*A*e^3*arcsin(d*x)/d - 1/3*(-d^2*x^2 + 1)^(3/2)*B*e^3/d^2 - (-d^2*x^2
+ 1)^(3/2)*A*e^2*f/d^2 - 4/35*(-d^2*x^2 + 1)^(3/2)*C*f^3*x^2/d^4 - 1/6*(3*
C*e*f^2 + B*f^3)*(-d^2*x^2 + 1)^(3/2)*x^3/d^2 - 1/5*(3*C*e^2*f + 3*B*e*f^2
 + A*f^3)*(-d^2*x^2 + 1)^(3/2)*x^2/d^2 - 1/4*(C*e^3 + 3*B*e^2*f + 3*A*e*f^
2)*(-d^2*x^2 + 1)^(3/2)*x/d^2 + 1/8*(C*e^3 + 3*B*e^2*f + 3*A*e*f^2)*sqrt(-
d^2*x^2 + 1)*x/d^2 - 8/105*(-d^2*x^2 + 1)^(3/2)*C*f^3/d^6 - 1/8*(3*C*e*f^2
 + B*f^3)*(-d^2*x^2 + 1)^(3/2)*x/d^4 + 1/8*(C*e^3 + 3*B*e^2*f + 3*A*e*f^2)
*arcsin(d*x)/d^3 - 2/15*(3*C*e^2*f + 3*B*e*f^2 + A*f^3)*(-d^2*x^2 + 1)^(3/
2)/d^4 + 1/16*(3*C*e*f^2 + B*f^3)*sqrt(-d^2*x^2 + 1)*x/d^4 + 1/16*(3*C*e*f
^2 + B*f^3)*arcsin(d*x)/d^5
```

3.1.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1539 vs. 2(391) = 782.

Time = 0.43 (sec) , antiderivative size = 1539, normalized size of antiderivative = 3.71

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorith
m="giac")
```

output
```
1/1680*(840*(sqrt(d*x + 1)*(d*x - 2)*sqrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)
*sqrt(d*x + 1)))*A*d^5*e^3 + 1680*(sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*arcsin
(1/2*sqrt(2)*sqrt(d*x + 1)))*A*d^5*e^3 + 280*(((2*d*x - 5)*(d*x + 1) + 9)*
sqrt(d*x + 1)*sqrt(-d*x + 1) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*B*d^4*
e^3 + 840*(sqrt(d*x + 1)*(d*x - 2)*sqrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*s
qrt(d*x + 1)))*B*d^4*e^3 + 840*(((2*d*x - 5)*(d*x + 1) + 9)*sqrt(d*x + 1)*
sqrt(-d*x + 1) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*A*d^4*e^2*f + 2520*(
sqrt(d*x + 1)*(d*x - 2)*sqrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*sqrt(d*x + 1
)))*A*d^4*e^2*f + 70*(((2*(3*d*x - 10)*(d*x + 1) + 43)*(d*x + 1) - 39)*sqr
t(d*x + 1)*sqrt(-d*x + 1) - 18*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*C*d^3*e^
3 + 280*(((2*d*x - 5)*(d*x + 1) + 9)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 6*arcs
in(1/2*sqrt(2)*sqrt(d*x + 1)))*C*d^3*e^3 + 210*(((2*(3*d*x - 10)*(d*x + 1)
 + 43)*(d*x + 1) - 39)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 18*arcsin(1/2*sqrt(2
)*sqrt(d*x + 1)))*B*d^3*e^2*f + 840*(((2*d*x - 5)*(d*x + 1) + 9)*sqrt(d*x
+ 1)*sqrt(-d*x + 1) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*B*d^3*e^2*f + 2
10*(((2*(3*d*x - 10)*(d*x + 1) + 43)*(d*x + 1) - 39)*sqrt(d*x + 1)*sqrt(-d
*x + 1) - 18*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*A*d^3*e*f^2 + 840*(((2*d*x
 - 5)*(d*x + 1) + 9)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 6*arcsin(1/2*sqrt(2)*s
qrt(d*x + 1)))*A*d^3*e*f^2 + 42*(((2*(3*(4*d*x - 17)*(d*x + 1) + 133)*(d*x
 + 1) - 295)*(d*x + 1) + 195)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 90*arcsin(...
```

3.1.9 Mupad [B] (verification not implemented)

Time = 51.67 (sec) , antiderivative size = 3993, normalized size of antiderivative = 9.62

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx = \text{Too large to display}$$

input
```
int((e + f*x)^3*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)*(A + B*x + C*x^2),x)
```

output
```
- ((((2048*C*f^3)/3 - 640*C*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^6)/((d*x + 1)
^(1/2) - 1)^6 + (((2048*C*f^3)/3 - 640*C*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^
22)/((d*x + 1)^(1/2) - 1)^22 - (((20480*C*f^3)/3 - 448*C*d^2*e^2*f)*((1 -
d*x)^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8 - (((20480*C*f^3)/3 - 448*C*d^2
*e^2*f)*((1 - d*x)^(1/2) - 1)^20)/((d*x + 1)^(1/2) - 1)^20 + (((458752*C*f
^3)/15 + (27136*C*d^2*e^2*f)/5)*((1 - d*x)^(1/2) - 1)^10)/((d*x + 1)^(1/2)
 - 1)^10 + (((458752*C*f^3)/15 + (27136*C*d^2*e^2*f)/5)*((1 - d*x)^(1/2) -
 1)^18)/((d*x + 1)^(1/2) - 1)^18 - (((1011712*C*f^3)/15 - (13184*C*d^2*e^2
*f)/5)*((1 - d*x)^(1/2) - 1)^12)/((d*x + 1)^(1/2) - 1)^12 - (((1011712*C*f
^3)/15 - (13184*C*d^2*e^2*f)/5)*((1 - d*x)^(1/2) - 1)^16)/((d*x + 1)^(1/2)
 - 1)^16 + (((9293824*C*f^3)/105 - (15104*C*d^2*e^2*f)/5)*((1 - d*x)^(1/2)
 - 1)^14)/((d*x + 1)^(1/2) - 1)^14 + (((1 - d*x)^(1/2) - 1)^3*((29*C*d^3*e
^3)/2 - (41*C*d*e*f^2)/4))/((d*x + 1)^(1/2) - 1)^3 - (((1 - d*x)^(1/2) - 1
)^25*((29*C*d^3*e^3)/2 - (41*C*d*e*f^2)/4))/((d*x + 1)^(1/2) - 1)^25 - (((
1 - d*x)^(1/2) - 1)^5*(39*C*d^3*e^3 - (1099*C*d*e*f^2)/2))/((d*x + 1)^(1/2
) - 1)^5 + (((1 - d*x)^(1/2) - 1)^23*(39*C*d^3*e^3 - (1099*C*d*e*f^2)/2))/
((d*x + 1)^(1/2) - 1)^23 - (((1 - d*x)^(1/2) - 1)^7*(209*C*d^3*e^3 + (8755
*C*d*e*f^2)/2))/((d*x + 1)^(1/2) - 1)^7 + (((1 - d*x)^(1/2) - 1)^21*(209*C
*d^3*e^3 + (8755*C*d*e*f^2)/2))/((d*x + 1)^(1/2) - 1)^21 + (((1 - d*x)^(1/
2) - 1)^11*((1767*C*d^3*e^3)/2 - (8267*C*d*e*f^2)/4))/((d*x + 1)^(1/2) ...
```

3.2 $\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx$

3.2.1 Optimal result

Integrand size = 37, antiderivative size = 286

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx$$
$$=\frac{(C(2d^2e^2+f^2)+2d^2(2Bef+A(4d^2e^2+f^2)))\,x\sqrt{1-d^2x^2}}{16d^4}$$
$$+\frac{(Ce-2Bf)(e+fx)^2\left(1-d^2x^2\right)^{3/2}}{10d^2f}-\frac{C(e+fx)^3\left(1-d^2x^2\right)^{3/2}}{6d^2f}$$
$$+\frac{(8(C(d^2e^3-4ef^2)-2f(5Ad^2ef+B(d^2e^2+f^2)))-3f(5(C+2Ad^2)\,f^2-2d^2e(Ce-2Bf))\,x)\,(1-d}{120d^4f}$$
$$+\frac{(C(2d^2e^2+f^2)+2d^2(2Bef+A(4d^2e^2+f^2)))\arcsin(dx)}{16d^5}$$

output
```
1/10*(-2*B*f+C*e)*(f*x+e)^2*(-d^2*x^2+1)^(3/2)/d^2/f-1/6*C*(f*x+e)^3*(-d^2
*x^2+1)^(3/2)/d^2/f+1/120*(8*C*(d^2*e^3-4*e*f^2)-16*f*(5*A*d^2*e*f+B*(d^2*
e^2+f^2))-3*f*(5*(2*A*d^2+C)*f^2-2*d^2*e*(-2*B*f+C*e))*x)*(-d^2*x^2+1)^(3/
2)/d^4/f+1/16*(C*(2*d^2*e^2+f^2)+2*d^2*(2*B*e*f+A*(4*d^2*e^2+f^2)))*arcsin
(d*x)/d^5+1/16*(C*(2*d^2*e^2+f^2)+2*d^2*(2*B*e*f+A*(4*d^2*e^2+f^2)))*x*(-d
^2*x^2+1)^(1/2)/d^4
```

3.2.2 Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.92

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx$$
$$=\frac{d\sqrt{1-d^2x^2}(10Ad^2(12d^2e^2x+16ef(-1+d^2x^2)+3f^2x(-1+2d^2x^2))+4B(-8f^2-d^2(20e^2+15efx+\cdots}{}$$

input
```
Integrate[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^2*(A + B*x + C*x^2),x]
```

output
```
(d*Sqrt[1 - d^2*x^2]*(10*A*d^2*(12*d^2*e^2*x + 16*e*f*(-1 + d^2*x^2) + 3*f
^2*x*(-1 + 2*d^2*x^2)) + 4*B*(-8*f^2 - d^2*(20*e^2 + 15*e*f*x + 4*f^2*x^2)
 + 2*d^4*x^2*(10*e^2 + 15*e*f*x + 6*f^2*x^2)) + C*(30*d^2*e^2*x*(-1 + 2*d^
2*x^2) + 32*e*f*(-2 - d^2*x^2 + 3*d^4*x^4) + 5*f^2*x*(-3 - 2*d^2*x^2 + 8*d
^4*x^4))) + 30*(C*(2*d^2*e^2 + f^2) + 2*d^2*(2*B*e*f + A*(4*d^2*e^2 + f^2)
))*ArcTan[(d*x)/(-1 + Sqrt[1 - d^2*x^2])])/(240*d^5)
```

3.2.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {2112, 2185, 27, 687, 25, 27, 676, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-dx}\sqrt{dx+1}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx$$
$$\Big\downarrow \text{2112}$$
$$\int \sqrt{1-d^2x^2}(e+fx)^2\left(A+Bx+Cx^2\right)dx$$
$$\Big\downarrow \text{2185}$$
$$-\frac{\int -3f(e+fx)^2\left(\left(2Ad^2+C\right)f-d^2(Ce-2Bf)x\right)\sqrt{1-d^2x^2}dx}{6d^2f^2}-\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^3}{6d^2f}$$
$$\Big\downarrow \text{27}$$
$$\frac{\int (e+fx)^2\left(\left(2Ad^2+C\right)f-d^2(Ce-2Bf)x\right)\sqrt{1-d^2x^2}dx}{2d^2f}-\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^3}{6d^2f}$$

$$\Big\downarrow \ 687$$

$$\frac{\frac{1}{5}\left(1-d^2x^2\right)^{3/2}(e+fx)^2(Ce-2Bf)-\frac{\int -d^2(e+fx)\left(f\left(10Aed^2+3Ce+4Bf\right)+\left(5\left(2Ad^2+C\right)f^2-2d^2e(Ce-2Bf)\right)x\right)\sqrt{1-d^2x^2}dx}{5d^2}}{2d^2f}-\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^3}{6d^2f}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int d^2(e+fx)\left(f\left(10Aed^2+3Ce+4Bf\right)+\left(5\left(2Ad^2+C\right)f^2-2d^2e(Ce-2Bf)\right)x\right)\sqrt{1-d^2x^2}dx}{5d^2}+\frac{1}{5}\left(1-d^2x^2\right)^{3/2}(e+fx)^2(Ce-2Bf)}{2d^2f}-\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^3}{6d^2f}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{1}{5}\int(e+fx)\left(f\left(10Aed^2+3Ce+4Bf\right)+\left(5\left(2Ad^2+C\right)f^2-2d^2e(Ce-2Bf)\right)x\right)\sqrt{1-d^2x^2}dx+\frac{1}{5}\left(1-d^2x^2\right)^{3/2}}{2d^2f}-\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^3}{6d^2f}$$

$$\Big\downarrow \ 676$$

$$\frac{\frac{1}{5}\left(\frac{5f\left(2d^2\left(A\left(4d^2e^2+f^2\right)+2Bef\right)+C\left(2d^2e^2+f^2\right)\right)\int\sqrt{1-d^2x^2}dx}{4d^2}+\frac{1}{4}fx\left(1-d^2x^2\right)^{3/2}\left(-10Af^2-4Bef-\frac{5Cf^2}{d^2}+2Ce^2\right)+\frac{2(}{}\right.}{2d^2f}-\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^3}{6d^2f}$$

$$\Big\downarrow \ 211$$

$$\frac{\frac{1}{5}\left(\frac{5f\left(2d^2\left(A\left(4d^2e^2+f^2\right)+2Bef\right)+C\left(2d^2e^2+f^2\right)\right)\left(\frac{1}{2}\int\frac{1}{\sqrt{1-d^2x^2}}dx+\frac{1}{2}x\sqrt{1-d^2x^2}\right)}{4d^2}+\frac{1}{4}fx\left(1-d^2x^2\right)^{3/2}\left(-10Af^2-4Bef-\frac{5Cf}{d^2}\right.\right.}{2d^2f}-\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^3}{6d^2f}$$

$$\Big\downarrow \ 223$$

3.2. $\int\sqrt{1-dx}\sqrt{1+dx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx$

$$\frac{\frac{1}{5}\left(\frac{5f\left(\frac{\arcsin(dx)}{2d}+\frac{1}{2}x\sqrt{1-d^2x^2}\right)\left(2d^2\left(A\left(4d^2e^2+f^2\right)+2Bef\right)+C\left(2d^2e^2+f^2\right)\right)}{4d^2}+\frac{1}{4}fx\left(1-d^2x^2\right)^{3/2}\left(-10Af^2-4Bef-\frac{5Cf^2}{d^2}+2\cdots\right.\right.}{2d^2f}$$

$$\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^3}{6d^2f}$$

input
```
Int[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^2*(A + B*x + C*x^2),x]
```

output
```
-1/6*(C*(e + f*x)^3*(1 - d^2*x^2)^(3/2))/(d^2*f) + (((C*e - 2*B*f)*(e + f*
x)^2*(1 - d^2*x^2)^(3/2))/5 + ((2*(C*d^2*e^3 - 2*B*d^2*e^2*f - 4*C*e*f^2 -
 10*A*d^2*e*f^2 - 2*B*f^3)*(1 - d^2*x^2)^(3/2))/(3*d^2) + (f*(2*C*e^2 - 4*
B*e*f - 10*A*f^2 - (5*C*f^2)/d^2)*x*(1 - d^2*x^2)^(3/2))/4 + (5*f*(C*(2*d^
2*e^2 + f^2) + 2*d^2*(2*B*e*f + A*(4*d^2*e^2 + f^2)))*((x*Sqrt[1 - d^2*x^2
])/2 + ArcSin[d*x]/(2*d)))/(4*d^2))/5)/(2*d^2*f)
```

3.2.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2112
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.2.4 Maple [A] (verified)

Time = 1.63 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.25

method	result
risch	$-\frac{\left(40C f^{2} x^{5} d^{4}+48B d^{4} f^{2} x^{4}+96C d^{4} e f x^{4}+60A d^{4} f^{2} x^{3}+120B d^{4} e f x^{3}+60C d^{4} e^{2} x^{3}+160A d^{4} e f x^{2}+80B d^{4} e^{2} x^{2}+120A d^{4} e^{2} x-10\right.}{2}$
default	$\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(-30C\sqrt{-d^{2}x^{2}+1}\,\operatorname{csgn}(d) d^{3} e^{2} x+30C\arctan\left(\frac{\operatorname{csgn}(d) d x}{\sqrt{-d^{2}x^{2}+1}}\right) d^{2} e^{2}+120A\arctan\left(\frac{\operatorname{csgn}(d) d x}{\sqrt{-d^{2}x^{2}+1}}\right) d^{4} e^{2}+30A\arctan\left(\right.\right.}{}$

input
```
int((f*x+e)^2*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
-1/240/d^4*(40*C*d^4*f^2*x^5+48*B*d^4*f^2*x^4+96*C*d^4*e*f*x^4+60*A*d^4*f^
2*x^3+120*B*d^4*e*f*x^3+60*C*d^4*e^2*x^3+160*A*d^4*e*f*x^2+80*B*d^4*e^2*x^
2+120*A*d^4*e^2*x-10*C*d^2*f^2*x^3-16*B*d^2*f^2*x^2-32*C*d^2*e*f*x^2-30*A*
d^2*f^2*x-60*B*d^2*e*f*x-30*C*d^2*e^2*x-160*A*d^2*e*f-80*B*d^2*e^2-15*C*f^
2*x-32*B*f^2-64*C*e*f)*(d*x+1)^(1/2)*(d*x-1)/(-(d*x+1)*(d*x-1))^(1/2)*((-d
*x+1)*(d*x+1))^(1/2)/(-d*x+1)^(1/2)+1/16*(8*A*d^4*e^2+2*A*d^2*f^2+4*B*d^2*
e*f+2*C*d^2*e^2+C*f^2)/d^4/(d^2)^(1/2)*arctan((d^2)^(1/2)*x/(-d^2*x^2+1)^(
1/2))*((-d*x+1)*(d*x+1))^(1/2)/(-d*x+1)^(1/2)/(d*x+1)^(1/2)
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.98

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx$$
$$= \frac{(40\,Cd^5f^2x^5 - 80\,Bd^3e^2 + 48\,(2\,Cd^5ef + Bd^5f^2)x^4 - 32\,Bdf^2 + 10\,(6\,Cd^5e^2 + 12\,Bd^5ef + (6\,Ad^5 - Cd^3}{}$$

input
```
integrate((f*x+e)^2*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorith
m="fricas")
```

output
```
1/240*((40*C*d^5*f^2*x^5 - 80*B*d^3*e^2 + 48*(2*C*d^5*e*f + B*d^5*f^2)*x^4
 - 32*B*d*f^2 + 10*(6*C*d^5*e^2 + 12*B*d^5*e*f + (6*A*d^5 - C*d^3)*f^2)*x^
3 - 32*(5*A*d^3 + 2*C*d)*e*f + 16*(5*B*d^5*e^2 - B*d^3*f^2 + 2*(5*A*d^5 -
C*d^3)*e*f)*x^2 - 15*(4*B*d^3*e*f - 2*(4*A*d^5 - C*d^3)*e^2 + (2*A*d^3 + C
*d)*f^2)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 30*(4*B*d^2*e*f + 2*(4*A*d^4 +
C*d^2)*e^2 + (2*A*d^2 + C)*f^2)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/
(d*x)))/d^5
```

3.2.6 Sympy [F]

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx$$
$$= \int (e+fx)^2\,\sqrt{-dx+1}\sqrt{dx+1}\left(A+Bx+Cx^2\right)\,dx$$

input
```
integrate((f*x+e)**2*(C*x**2+B*x+A)*(-d*x+1)**(1/2)*(d*x+1)**(1/2),x)
```

output
```
Integral((e + f*x)**2*sqrt(-d*x + 1)*sqrt(d*x + 1)*(A + B*x + C*x**2), x)
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.07

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx$$
$$= -\frac{(-d^2x^2+1)^{\frac{3}{2}}Cf^2x^3}{6\,d^2} + \frac{1}{2}\sqrt{-d^2x^2+1}Ae^2x + \frac{Ae^2\arcsin{(dx)}}{2\,d} - \frac{(-d^2x^2+1)^{\frac{3}{2}}Be^2}{3\,d^2} - \frac{2\,(-d^2x^2+1)^{\frac{3}{2}}Aef}{3\,d^2} - \frac{(-d^2x^2+1)^{\frac{3}{2}}(2\,Cef+Bf^2)x^2}{5\,d^2} - \frac{(-d^2x^2+1)^{\frac{3}{2}}(Ce^2+2\,Bef+Af^2)x}{4\,d^2} - \frac{(-d^2x^2+1)^{\frac{3}{2}}Cf^2x}{8\,d^4} + \frac{\sqrt{-d^2x^2+1}(Ce^2+2\,Bef+Af^2)x}{8\,d^2} + \frac{\sqrt{-d^2x^2+1}Cf^2x}{16\,d^4} + \frac{(Ce^2+2\,Bef+Af^2)\arcsin{(dx)}}{8\,d^3} + \frac{Cf^2\arcsin{(dx)}}{16\,d^5} - \frac{2\,(-d^2x^2+1)^{\frac{3}{2}}(2\,Cef+Bf^2)}{15\,d^4}$$

input
```
integrate((f*x+e)^2*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorith
m="maxima")
```

output
```
-1/6*(-d^2*x^2 + 1)^(3/2)*C*f^2*x^3/d^2 + 1/2*sqrt(-d^2*x^2 + 1)*A*e^2*x +
 1/2*A*e^2*arcsin(d*x)/d - 1/3*(-d^2*x^2 + 1)^(3/2)*B*e^2/d^2 - 2/3*(-d^2*
x^2 + 1)^(3/2)*A*e*f/d^2 - 1/5*(-d^2*x^2 + 1)^(3/2)*(2*C*e*f + B*f^2)*x^2/
d^2 - 1/4*(-d^2*x^2 + 1)^(3/2)*(C*e^2 + 2*B*e*f + A*f^2)*x/d^2 - 1/8*(-d^2
*x^2 + 1)^(3/2)*C*f^2*x/d^4 + 1/8*sqrt(-d^2*x^2 + 1)*(C*e^2 + 2*B*e*f + A*
f^2)*x/d^2 + 1/16*sqrt(-d^2*x^2 + 1)*C*f^2*x/d^4 + 1/8*(C*e^2 + 2*B*e*f +
A*f^2)*arcsin(d*x)/d^3 + 1/16*C*f^2*arcsin(d*x)/d^5 - 2/15*(-d^2*x^2 + 1)^
(3/2)*(2*C*e*f + B*f^2)/d^4
```

3.2.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1059 vs. $2(266) = 532$.

Time = 0.41 (sec) , antiderivative size = 1059, normalized size of antiderivative = 3.70

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorith
m="giac")
```

output
```
1/240*(120*(sqrt(d*x + 1)*(d*x - 2)*sqrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*
sqrt(d*x + 1)))*A*d^4*e^2 + 240*(sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*arcsin(1
/2*sqrt(2)*sqrt(d*x + 1)))*A*d^4*e^2 + 40*(((2*d*x - 5)*(d*x + 1) + 9)*sqr
t(d*x + 1)*sqrt(-d*x + 1) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*B*d^3*e^2
 + 120*(sqrt(d*x + 1)*(d*x - 2)*sqrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*sqrt
(d*x + 1)))*B*d^3*e^2 + 80*(((2*d*x - 5)*(d*x + 1) + 9)*sqrt(d*x + 1)*sqrt
(-d*x + 1) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*A*d^3*e*f + 240*(sqrt(d*
x + 1)*(d*x - 2)*sqrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*A*d
^3*e*f + 10*(((2*(3*d*x - 10)*(d*x + 1) + 43)*(d*x + 1) - 39)*sqrt(d*x + 1
)*sqrt(-d*x + 1) - 18*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*C*d^2*e^2 + 40*((
(2*d*x - 5)*(d*x + 1) + 9)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 6*arcsin(1/2*sqr
t(2)*sqrt(d*x + 1)))*C*d^2*e^2 + 20*(((2*(3*d*x - 10)*(d*x + 1) + 43)*(d*x
 + 1) - 39)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 18*arcsin(1/2*sqrt(2)*sqrt(d*x
+ 1)))*B*d^2*e*f + 80*(((2*d*x - 5)*(d*x + 1) + 9)*sqrt(d*x + 1)*sqrt(-d*x
 + 1) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*B*d^2*e*f + 10*(((2*(3*d*x -
10)*(d*x + 1) + 43)*(d*x + 1) - 39)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 18*arcs
in(1/2*sqrt(2)*sqrt(d*x + 1)))*A*d^2*f^2 + 40*(((2*d*x - 5)*(d*x + 1) + 9)
*sqrt(d*x + 1)*sqrt(-d*x + 1) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*A*d^2
*f^2 + 4*(((2*(3*(4*d*x - 17)*(d*x + 1) + 133)*(d*x + 1) - 295)*(d*x + 1)
+ 195)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 90*arcsin(1/2*sqrt(2)*sqrt(d*x + ...
```

3.2.9 Mupad [B] (verification not implemented)

Time = 38.59 (sec) , antiderivative size = 2920, normalized size of antiderivative = 10.21

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx = \text{Too large to display}$$

input
```
int((e + f*x)^2*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)*(A + B*x + C*x^2),x)
```

output
```
- ((((1 - d*x)^(1/2) - 1)^8*((4928*B*f^2)/3 + (512*B*d^2*e^2)/3))/((d*x +
1)^(1/2) - 1)^8 - (((1 - d*x)^(1/2) - 1)^14*((1408*B*f^2)/3 - (32*B*d^2*e^
2)/3))/((d*x + 1)^(1/2) - 1)^14 - (((1 - d*x)^(1/2) - 1)^6*((1408*B*f^2)/3
 - (32*B*d^2*e^2)/3))/((d*x + 1)^(1/2) - 1)^6 + (((1 - d*x)^(1/2) - 1)^12*
((4928*B*f^2)/3 + (512*B*d^2*e^2)/3))/((d*x + 1)^(1/2) - 1)^12 - (((1 - d*
x)^(1/2) - 1)^10*((11008*B*f^2)/5 - 304*B*d^2*e^2))/((d*x + 1)^(1/2) - 1)^
10 + (64*B*f^2*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (64*B*f^
2*((1 - d*x)^(1/2) - 1)^16)/((d*x + 1)^(1/2) - 1)^16 + (8*B*d^2*e^2*((1 -
d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (8*B*d^2*e^2*((1 - d*x)^(1/2)
 - 1)^18)/((d*x + 1)^(1/2) - 1)^18 + (33*B*d*e*f*((1 - d*x)^(1/2) - 1)^3)/
((d*x + 1)^(1/2) - 1)^3 - (204*B*d*e*f*((1 - d*x)^(1/2) - 1)^5)/((d*x + 1)
^(1/2) - 1)^5 + (204*B*d*e*f*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1
)^7 + (442*B*d*e*f*((1 - d*x)^(1/2) - 1)^9)/((d*x + 1)^(1/2) - 1)^9 - (442
*B*d*e*f*((1 - d*x)^(1/2) - 1)^11)/((d*x + 1)^(1/2) - 1)^11 - (204*B*d*e*f
*((1 - d*x)^(1/2) - 1)^13)/((d*x + 1)^(1/2) - 1)^13 + (204*B*d*e*f*((1 - d
*x)^(1/2) - 1)^15)/((d*x + 1)^(1/2) - 1)^15 - (33*B*d*e*f*((1 - d*x)^(1/2)
 - 1)^17)/((d*x + 1)^(1/2) - 1)^17 + (B*d*e*f*((1 - d*x)^(1/2) - 1)^19)/((
d*x + 1)^(1/2) - 1)^19 - (B*d*e*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2)
- 1))/(d^4 + (10*d^4*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (4
5*d^4*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (120*d^4*((1 -...
```

3.2. $\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx$

3.3 $\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)\left(A+Bx+Cx^2\right)\,dx$

3.3.1 Optimal result

Integrand size = 35, antiderivative size = 168

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)\left(A+Bx+Cx^2\right)\,dx = \frac{(Ce+4Ad^2e+Bf)\,x\sqrt{1-d^2x^2}}{8d^2} - \frac{C(e+fx)^2\left(1-d^2x^2\right)^{3/2}}{5d^2f} - \frac{(4(5d^2f(Be+Af)-C(3d^2e^2-2f^2))-3d^2f(3Ce-5Bf)x)\left(1-d^2x^2\right)^{3/2}}{60d^4f} + \frac{(Ce+4Ad^2e+Bf)\arcsin(dx)}{8d^3}$$

output
```
-1/5*C*(f*x+e)^2*(-d^2*x^2+1)^(3/2)/d^2/f-1/60*(20*d^2*f*(A*f+B*e)-4*C*(3*
d^2*e^2-2*f^2)-3*d^2*f*(-5*B*f+3*C*e)*x)*(-d^2*x^2+1)^(3/2)/d^4/f+1/8*(4*A
*d^2*e+B*f+C*e)*arcsin(d*x)/d^3+1/8*(4*A*d^2*e+B*f+C*e)*x*(-d^2*x^2+1)^(1/
2)/d^2
```

3.3.2 Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.95

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)\left(A+Bx+Cx^2\right)\,dx = \frac{\sqrt{1-d^2x^2}(60Ad^4ex+40Ad^2f(-1+d^2x^2)+15Cd^2ex(-1+2d^2x^2)+5Bd^2(-8e-3fx+8d^2ex^2+6d^2}{120d^4}$$

input
```
Integrate[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)*(A + B*x + C*x^2),x]
```

output
```
(Sqrt[1 - d^2*x^2]*(60*A*d^4*e*x + 40*A*d^2*f*(-1 + d^2*x^2) + 15*C*d^2*e*
x*(-1 + 2*d^2*x^2) + 5*B*d^2*(-8*e - 3*f*x + 8*d^2*e*x^2 + 6*d^2*f*x^3) +
8*C*f*(-2 - d^2*x^2 + 3*d^4*x^4)) + 30*d*(C*e + 4*A*d^2*e + B*f)*ArcTan[(d
*x)/(-1 + Sqrt[1 - d^2*x^2])])/(120*d^4)
```

3.3.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2112, 2185, 25, 27, 676, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-dx}\sqrt{dx+1}(e+fx)\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{ 2112}$$

$$\int \sqrt{1-d^2x^2}(e+fx)\left(A+Bx+Cx^2\right)dx$$

$$\Big\downarrow \text{ 2185}$$

$$-\frac{\int -f(e+fx)\left(\left(5Ad^2+2C\right)f-d^2(3Ce-5Bf)x\right)\sqrt{1-d^2x^2}dx}{5d^2f^2}-\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^2}{5d^2f}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int f(e+fx)\left(\left(5Ad^2+2C\right)f-d^2(3Ce-5Bf)x\right)\sqrt{1-d^2x^2}dx}{5d^2f^2}-\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^2}{5d^2f}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int (e+fx)\left(\left(5Ad^2+2C\right)f-d^2(3Ce-5Bf)x\right)\sqrt{1-d^2x^2}dx}{5d^2f}-\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^2}{5d^2f}$$

$$\Big\downarrow \text{ 676}$$

$$\frac{\frac{5}{4}f\left(4Ad^2e+Bf+Ce\right)\int\sqrt{1-d^2x^2}dx-\frac{1}{3}\left(1-d^2x^2\right)^{3/2}\left(5f(Af+Be)-C\left(3e^2-\frac{2f^2}{d^2}\right)\right)+\frac{1}{4}fx\left(1-d^2x^2\right)^{3/2}(3}{5d^2f}$$
$$\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^2}{5d^2f}$$

$\downarrow$ 211

$$\frac{\frac{5}{4}f\left(4Ad^2e+Bf+Ce\right)\left(\frac{1}{2}\int\frac{1}{\sqrt{1-d^2x^2}}dx+\frac{1}{2}x\sqrt{1-d^2x^2}\right)-\frac{1}{3}\left(1-d^2x^2\right)^{3/2}\left(5f(Af+Be)-C\left(3e^2-\frac{2f^2}{d^2}\right)\right)+\frac{1}{4}}{5d^2f}$$

$$\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^2}{5d^2f}$$

$\downarrow$ 223

$$\frac{\frac{5}{4}f\left(\frac{\arcsin(dx)}{2d}+\frac{1}{2}x\sqrt{1-d^2x^2}\right)\left(4Ad^2e+Bf+Ce\right)-\frac{1}{3}\left(1-d^2x^2\right)^{3/2}\left(5f(Af+Be)-C\left(3e^2-\frac{2f^2}{d^2}\right)\right)+\frac{1}{4}fx(1}{5d^2f}$$

$$\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^2}{5d^2f}$$

input
```
Int[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)*(A + B*x + C*x^2),x]
```

output
```
-1/5*(C*(e + f*x)^2*(1 - d^2*x^2)^(3/2))/(d^2*f) + (-1/3*((5*f*(B*e + A*f)
 - C*(3*e^2 - (2*f^2)/d^2))*(1 - d^2*x^2)^(3/2)) + (f*(3*C*e - 5*B*f)*x*(1
 - d^2*x^2)^(3/2))/4 + (5*f*(C*e + 4*A*d^2*e + B*f)*((x*Sqrt[1 - d^2*x^2])
/2 + ArcSin[d*x]/(2*d)))/4)/(5*d^2*f)
```

3.3.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 2112
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.3.4 Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.38

method	result
risch	$-\frac{\left(24fC\,x^4d^4+30B\,d^4f\,x^3+30C\,d^4e\,x^3+40A\,d^4f\,x^2+40B\,d^4e\,x^2+60A\,d^4ex-8C\,d^2f\,x^2-15B\,d^2fx-15C\,d^2ex-40A\,d^2f-40B\,d^2e}{120d^4\sqrt{-(dx+1)(dx-1)}\,\sqrt{-dx+1}}$
default	$\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(24C\ \mathrm{csgn}(d)d^4f\,x^4\sqrt{-d^2x^2+1}+30B\ \mathrm{csgn}(d)d^4f\,x^3\sqrt{-d^2x^2+1}+30C\ \mathrm{csgn}(d)d^4e\,x^3\sqrt{-d^2x^2+1}+40A\ \mathrm{csgn}(d)d^4\right.}{}$

input
```
int((f*x+e)*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
-1/120*(24*C*d^4*f*x^4+30*B*d^4*f*x^3+30*C*d^4*e*x^3+40*A*d^4*f*x^2+40*B*d
^4*e*x^2+60*A*d^4*e*x-8*C*d^2*f*x^2-15*B*d^2*f*x-15*C*d^2*e*x-40*A*d^2*f-4
0*B*d^2*e-16*C*f)*(d*x+1)^(1/2)*(d*x-1)/d^4/(-(d*x+1)*(d*x-1))^(1/2)*((-d*
x+1)*(d*x+1))^(1/2)/(-d*x+1)^(1/2)+1/8/d^2*(4*A*d^2*e+B*f+C*e)/(d^2)^(1/2)
*arctan((d^2)^(1/2)*x/(-d^2*x^2+1)^(1/2))*((-d*x+1)*(d*x+1))^(1/2)/(-d*x+1
)^(1/2)/(d*x+1)^(1/2)
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.01

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)\left(A+Bx+Cx^2\right)\,dx$$
$$= \frac{(24\,Cd^4fx^4 - 40\,Bd^2e + 30\,(Cd^4e + Bd^4f)x^3 + 8\,(5\,Bd^4e + (5\,Ad^4 - Cd^2)f)x^2 - 8\,(5\,Ad^2 + 2\,C)f - 15}{120\,d}$$

input
```
integrate((f*x+e)*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm=
"fricas")
```

output
```
1/120*((24*C*d^4*f*x^4 - 40*B*d^2*e + 30*(C*d^4*e + B*d^4*f)*x^3 + 8*(5*B*
d^4*e + (5*A*d^4 - C*d^2)*f)*x^2 - 8*(5*A*d^2 + 2*C)*f - 15*(B*d^2*f - (4*
A*d^4 - C*d^2)*e)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 30*(B*d*f + (4*A*d^3 +
 C*d)*e)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/d^4
```

3.3.6 Sympy [F]

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)\left(A+Bx+Cx^2\right)\,dx$$
$$= \int (e+fx)\sqrt{-dx+1}\sqrt{dx+1}\left(A+Bx+Cx^2\right)\,dx$$

input
```
integrate((f*x+e)*(C*x**2+B*x+A)*(-d*x+1)**(1/2)*(d*x+1)**(1/2),x)
```

output
```
Integral((e + f*x)*sqrt(-d*x + 1)*sqrt(d*x + 1)*(A + B*x + C*x**2), x)
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.04

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)\left(A+Bx+Cx^2\right)\,dx$$
$$=\frac{1}{2}\sqrt{-d^2x^2+1}Aex-\frac{\left(-d^2x^2+1\right)^{\frac{3}{2}}Cfx^2}{5\,d^2}+\frac{Ae\arcsin\left(dx\right)}{2\,d}-\frac{\left(-d^2x^2+1\right)^{\frac{3}{2}}Be}{3\,d^2}-\frac{\left(-d^2x^2+1\right)^{\frac{3}{2}}Af}{3\,d^2}-\frac{\left(-d^2x^2+1\right)^{\frac{3}{2}}(Ce+Bf)x}{4\,d^2}+\frac{\sqrt{-d^2x^2+1}(Ce+Bf)x}{8\,d^2}-\frac{2\left(-d^2x^2+1\right)^{\frac{3}{2}}Cf}{15\,d^4}+\frac{(Ce+Bf)\arcsin\left(dx\right)}{8\,d^3}$$

input
```
integrate((f*x+e)*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm=
"maxima")
```

output
```
1/2*sqrt(-d^2*x^2 + 1)*A*e*x - 1/5*(-d^2*x^2 + 1)^(3/2)*C*f*x^2/d^2 + 1/2*
A*e*arcsin(d*x)/d - 1/3*(-d^2*x^2 + 1)^(3/2)*B*e/d^2 - 1/3*(-d^2*x^2 + 1)^
(3/2)*A*f/d^2 - 1/4*(-d^2*x^2 + 1)^(3/2)*(C*e + B*f)*x/d^2 + 1/8*sqrt(-d^2
*x^2 + 1)*(C*e + B*f)*x/d^2 - 2/15*(-d^2*x^2 + 1)^(3/2)*C*f/d^4 + 1/8*(C*e
 + B*f)*arcsin(d*x)/d^3
```

3.3.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 631 vs. $2(151) = 302$.

Time = 0.34 (sec) , antiderivative size = 631, normalized size of antiderivative = 3.76

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)\left(A+Bx+Cx^2\right)\,dx$$
$$=\frac{60\left(\sqrt{dx+1}(dx-2)\sqrt{-dx+1}-2\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{dx+1}\right)\right)Ad^3e+120\left(\sqrt{dx+1}\sqrt{-dx+1}+2\arcsin\left(\right.\right.}{}$$

input
```
integrate((f*x+e)*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm=
"giac")
```

output
```
1/120*(60*(sqrt(d*x + 1)*(d*x - 2)*sqrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*s
qrt(d*x + 1)))*A*d^3*e + 120*(sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*arcsin(1/2*
sqrt(2)*sqrt(d*x + 1)))*A*d^3*e + 20*(((2*d*x - 5)*(d*x + 1) + 9)*sqrt(d*x
 + 1)*sqrt(-d*x + 1) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*B*d^2*e + 60*(
sqrt(d*x + 1)*(d*x - 2)*sqrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*sqrt(d*x + 1
)))*B*d^2*e + 20*(((2*d*x - 5)*(d*x + 1) + 9)*sqrt(d*x + 1)*sqrt(-d*x + 1)
 + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*A*d^2*f + 60*(sqrt(d*x + 1)*(d*x -
 2)*sqrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*A*d^2*f + 5*(((2
*(3*d*x - 10)*(d*x + 1) + 43)*(d*x + 1) - 39)*sqrt(d*x + 1)*sqrt(-d*x + 1)
 - 18*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*C*d*e + 20*(((2*d*x - 5)*(d*x + 1
) + 9)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))
*C*d*e + 5*(((2*(3*d*x - 10)*(d*x + 1) + 43)*(d*x + 1) - 39)*sqrt(d*x + 1)
*sqrt(-d*x + 1) - 18*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*B*d*f + 20*(((2*d*
x - 5)*(d*x + 1) + 9)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 6*arcsin(1/2*sqrt(2)*
sqrt(d*x + 1)))*B*d*f + (((2*(3*(4*d*x - 17)*(d*x + 1) + 133)*(d*x + 1) -
295)*(d*x + 1) + 195)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 90*arcsin(1/2*sqrt(2)
*sqrt(d*x + 1)))*C*f + 5*(((2*(3*d*x - 10)*(d*x + 1) + 43)*(d*x + 1) - 39)
*sqrt(d*x + 1)*sqrt(-d*x + 1) - 18*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*C*f)
/d^4
```

3.3. $\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)\left(A+Bx+Cx^2\right)\,dx$

3.3.9 Mupad [B] (verification not implemented)

Time = 13.44 (sec) , antiderivative size = 736, normalized size of antiderivative = 4.38

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)\left(A+Bx+Cx^2\right)\,dx$$
$$= \frac{\frac{B\,f\,(\sqrt{1-d\,x}-1)}{2\,(\sqrt{d\,x+1}-1)} - \frac{35\,B\,f\,(\sqrt{1-d\,x}-1)^3}{2\,(\sqrt{d\,x+1}-1)^3} + \frac{273\,B\,f\,(\sqrt{1-d\,x}-1)^5}{2\,(\sqrt{d\,x+1}-1)^5} - \frac{715\,B\,f\,(\sqrt{1-d\,x}-1)^7}{2\,(\sqrt{d\,x+1}-1)^7} + \frac{715\,B\,f\,(\sqrt{1-d\,x}-1)^9}{2\,(\sqrt{d\,x+1}-1)^9} - \frac{273\,B\,f\,(\sqrt{1-d\,x}}{2\,(\sqrt{d\,x+1}-1}}{d^3\left(\frac{(\sqrt{1-d\,x}-1)^2}{(\sqrt{d\,x+1}-1)^2}+1\right)^8}$$
$$- \sqrt{1-d\,x}\left(\frac{2\,C\,f\,\sqrt{d\,x+1}}{15\,d^4} - \frac{C\,f\,x^4\,\sqrt{d\,x+1}}{5} + \frac{C\,f\,x^2\,\sqrt{d\,x+1}}{15\,d^2}\right)$$
$$+ \frac{\frac{C\,e\,(\sqrt{1-d\,x}-1)}{2\,(\sqrt{d\,x+1}-1)} - \frac{35\,C\,e\,(\sqrt{1-d\,x}-1)^3}{2\,(\sqrt{d\,x+1}-1)^3} + \frac{273\,C\,e\,(\sqrt{1-d\,x}-1)^5}{2\,(\sqrt{d\,x+1}-1)^5} - \frac{715\,C\,e\,(\sqrt{1-d\,x}-1)^7}{2\,(\sqrt{d\,x+1}-1)^7} + \frac{715\,C\,e\,(\sqrt{1-d\,x}-1)^9}{2\,(\sqrt{d\,x+1}-1)^9} - \frac{273\,C\,e\,(\sqrt{1-d}{2\,(\sqrt{d\,x+1}-}}{d^3\left(\frac{(\sqrt{1-d\,x}-1)^2}{(\sqrt{d\,x+1}-1)^2}+1\right)^8}$$
$$- \frac{B\,f\,\mathrm{atan}\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)}{2\,d^3} - \frac{C\,e\,\mathrm{atan}\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)}{2\,d^3} + \frac{A\,e\,x\,\sqrt{1-d\,x}\,\sqrt{d\,x+1}}{2}$$
$$- \frac{A\,\sqrt{d}\,e\,\ln\left(\sqrt{-d}\,\sqrt{1-d\,x}\,\sqrt{d\,x+1}-d^{3/2}\,x\right)}{2\,(-d)^{3/2}}$$
$$+ \frac{A\,f\,(d^2\,x^2-1)\,\sqrt{1-d\,x}\,\sqrt{d\,x+1}}{3\,d^2} + \frac{B\,e\,(d^2\,x^2-1)\,\sqrt{1-d\,x}\,\sqrt{d\,x+1}}{3\,d^2}$$

input
```
int((e + f*x)*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)*(A + B*x + C*x^2),x)
```

output
```
((B*f*((1 - d*x)^(1/2) - 1))/(2*((d*x + 1)^(1/2) - 1)) - (35*B*f*((1 - d*x
)^(1/2) - 1)^3)/(2*((d*x + 1)^(1/2) - 1)^3) + (273*B*f*((1 - d*x)^(1/2) -
1)^5)/(2*((d*x + 1)^(1/2) - 1)^5) - (715*B*f*((1 - d*x)^(1/2) - 1)^7)/(2*(
(d*x + 1)^(1/2) - 1)^7) + (715*B*f*((1 - d*x)^(1/2) - 1)^9)/(2*((d*x + 1)^
(1/2) - 1)^9) - (273*B*f*((1 - d*x)^(1/2) - 1)^11)/(2*((d*x + 1)^(1/2) - 1
)^11) + (35*B*f*((1 - d*x)^(1/2) - 1)^13)/(2*((d*x + 1)^(1/2) - 1)^13) - (
B*f*((1 - d*x)^(1/2) - 1)^15)/(2*((d*x + 1)^(1/2) - 1)^15))/(d^3*(((1 - d*
x)^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 + 1)^8) - (1 - d*x)^(1/2)*((2*C*f*
(d*x + 1)^(1/2))/(15*d^4) - (C*f*x^4*(d*x + 1)^(1/2))/5 + (C*f*x^2*(d*x +
1)^(1/2))/(15*d^2)) + ((C*e*((1 - d*x)^(1/2) - 1))/(2*((d*x + 1)^(1/2) - 1
)) - (35*C*e*((1 - d*x)^(1/2) - 1)^3)/(2*((d*x + 1)^(1/2) - 1)^3) + (273*C
*e*((1 - d*x)^(1/2) - 1)^5)/(2*((d*x + 1)^(1/2) - 1)^5) - (715*C*e*((1 - d
*x)^(1/2) - 1)^7)/(2*((d*x + 1)^(1/2) - 1)^7) + (715*C*e*((1 - d*x)^(1/2)
- 1)^9)/(2*((d*x + 1)^(1/2) - 1)^9) - (273*C*e*((1 - d*x)^(1/2) - 1)^11)/(
2*((d*x + 1)^(1/2) - 1)^11) + (35*C*e*((1 - d*x)^(1/2) - 1)^13)/(2*((d*x +
 1)^(1/2) - 1)^13) - (C*e*((1 - d*x)^(1/2) - 1)^15)/(2*((d*x + 1)^(1/2) -
1)^15))/(d^3*(((1 - d*x)^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 + 1)^8) - (B
*f*atan(((1 - d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/(2*d^3) - (C*e*atan(
((1 - d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/(2*d^3) + (A*e*x*(1 - d*x)^(
1/2)*(d*x + 1)^(1/2))/2 - (A*d^(1/2)*e*log((-d)^(1/2)*(1 - d*x)^(1/2)*(...
```

3.3. $\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)\left(A+Bx+Cx^2\right)\,dx$

3.4 $\int \sqrt{1-dx}\sqrt{1+dx}(A+Bx+Cx^2)\,dx$

3.4.1 Optimal result

Integrand size = 30, antiderivative size = 95

$$\int \sqrt{1-dx}\sqrt{1+dx}\left(A+Bx+Cx^2\right)\,dx=\frac{\left(C+4Ad^2\right)x\sqrt{1-d^2x^2}}{8d^2}-\frac{B\left(1-d^2x^2\right)^{3/2}}{3d^2}-\frac{Cx\left(1-d^2x^2\right)^{3/2}}{4d^2}+\frac{\left(C+4Ad^2\right)\arcsin(dx)}{8d^3}$$

output
```
-1/3*B*(-d^2*x^2+1)^(3/2)/d^2-1/4*C*x*(-d^2*x^2+1)^(3/2)/d^2+1/8*(4*A*d^2+
C)*arcsin(d*x)/d^3+1/8*(4*A*d^2+C)*x*(-d^2*x^2+1)^(1/2)/d^2
```

3.4.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.94

$$\int \sqrt{1-dx}\sqrt{1+dx}\left(A+Bx+Cx^2\right)\,dx=\frac{d\sqrt{1-d^2x^2}\left(-8B-3Cx+12Ad^2x+8Bd^2x^2+6Cd^2x^3\right)+6\left(C+4Ad^2\right)\arctan\left(\frac{dx}{-1+\sqrt{1-d^2x^2}}\right)}{24d^3}$$

input
```
Integrate[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(A + B*x + C*x^2),x]
```

output
```
(d*Sqrt[1 - d^2*x^2]*(-8*B - 3*C*x + 12*A*d^2*x + 8*B*d^2*x^2 + 6*C*d^2*x^
3) + 6*(C + 4*A*d^2)*ArcTan[(d*x)/(-1 + Sqrt[1 - d^2*x^2])])/(24*d^3)
```

3.4.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1188, 2346, 25, 455, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-dx}\sqrt{dx+1}\left(A+Bx+Cx^2\right) \, dx$$

$$\Big\downarrow \ 1188$$

$$\int \sqrt{1-d^2x^2}\left(A+Bx+Cx^2\right)dx$$

$$\Big\downarrow \ 2346$$

$$-\frac{\int -\left(\left(4Ad^2+4Bxd^2+C\right)\sqrt{1-d^2x^2}\right)dx}{4d^2}-\frac{Cx\left(1-d^2x^2\right)^{3/2}}{4d^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \left(4Ad^2+4Bxd^2+C\right)\sqrt{1-d^2x^2}dx}{4d^2}-\frac{Cx\left(1-d^2x^2\right)^{3/2}}{4d^2}$$

$$\Big\downarrow \ 455$$

$$\frac{\left(4Ad^2+C\right)\int \sqrt{1-d^2x^2}dx-\frac{4}{3}B\left(1-d^2x^2\right)^{3/2}}{4d^2}-\frac{Cx\left(1-d^2x^2\right)^{3/2}}{4d^2}$$

$$\Big\downarrow \ 211$$

$$\frac{\left(4Ad^2+C\right)\left(\frac{1}{2}\int \frac{1}{\sqrt{1-d^2x^2}}dx+\frac{1}{2}x\sqrt{1-d^2x^2}\right)-\frac{4}{3}B\left(1-d^2x^2\right)^{3/2}}{4d^2}-\frac{Cx\left(1-d^2x^2\right)^{3/2}}{4d^2}$$

$$\Big\downarrow \ 223$$

$$\frac{\left(4Ad^2+C\right)\left(\frac{\arcsin(dx)}{2d}+\frac{1}{2}x\sqrt{1-d^2x^2}\right)-\frac{4}{3}B\left(1-d^2x^2\right)^{3/2}}{4d^2}-\frac{Cx\left(1-d^2x^2\right)^{3/2}}{4d^2}$$

input
```
Int[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(A + B*x + C*x^2),x]
```

output
```
-1/4*(C*x*(1 - d^2*x^2)^(3/2))/d^2 + ((-4*B*(1 - d^2*x^2)^(3/2))/3 + (C +
4*A*d^2)*((x*Sqrt[1 - d^2*x^2])/2 + ArcSin[d*x]/(2*d)))/(4*d^2)
```

3.4.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 1188
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2
)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m, n] && EqQ[e*f
 + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

3.4.4 Maple [A] (verified)

Time = 1.65 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.63

method	result
risch	$-\frac{(6C\,d^2x^3+8B\,d^2x^2+12A\,d^2x-3Cx-8B)\sqrt{dx+1}\,(dx-1)\sqrt{(-dx+1)(dx+1)}}{24d^2\sqrt{-(dx+1)(dx-1)}\,\sqrt{-dx+1}}+\frac{(4A\,d^2+C)\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+1}}\right)\sqrt{(-dx+1)(dx-}}{8d^2\sqrt{d^2}\,\sqrt{-dx+1}\,\sqrt{dx+1}}$
default	$\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\left(6C\operatorname{csgn}(d)d^3x^3\sqrt{-d^2x^2+1}+8B\operatorname{csgn}(d)d^3x^2\sqrt{-d^2x^2+1}+12A\operatorname{csgn}(d)d^3\sqrt{-d^2x^2+1}\,x-3C\operatorname{csgn}(d)d\sqrt{-d^2x^2+}\right.}{24\sqrt{-d^2x^2+1}\,d^3}$

input
```
int((C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(6*C*d^2*x^3+8*B*d^2*x^2+12*A*d^2*x-3*C*x-8*B)*(d*x+1)^(1/2)*(d*x-1)
/d^2/(-(d*x+1)*(d*x-1))^(1/2)*((-d*x+1)*(d*x+1))^(1/2)/(-d*x+1)^(1/2)+1/8*
(4*A*d^2+C)/d^2/(d^2)^(1/2)*arctan((d^2)^(1/2)*x/(-d^2*x^2+1)^(1/2))*((-d*
x+1)*(d*x+1))^(1/2)/(-d*x+1)^(1/2)/(d*x+1)^(1/2)
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00

$$\int \sqrt{1-dx}\sqrt{1+dx}\left(A+Bx+Cx^2\right)\,dx$$
$$= \frac{(6\,Cd^3x^3+8\,Bd^3x^2-8\,Bd+3\,(4\,Ad^3-Cd)x)\sqrt{dx+1}\sqrt{-dx+1}-6\,(4\,Ad^2+C)\arctan\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}}{dx}\right.}{24\,d^3}$$

input
```
integrate((C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm="fricas"
)
```

output
```
1/24*((6*C*d^3*x^3 + 8*B*d^3*x^2 - 8*B*d + 3*(4*A*d^3 - C*d)*x)*sqrt(d*x +
 1)*sqrt(-d*x + 1) - 6*(4*A*d^2 + C)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1)
- 1)/(d*x)))/d^3
```

3.4.6 Sympy [F]

$$\int \sqrt{1-dx}\sqrt{1+dx}\left(A+Bx+Cx^2\right)\,dx = \int \sqrt{-dx+1}\sqrt{dx+1}\left(A+Bx+Cx^2\right)\,dx$$

input
```
integrate((C*x**2+B*x+A)*(-d*x+1)**(1/2)*(d*x+1)**(1/2),x)
```

output
```
Integral(sqrt(-d*x + 1)*sqrt(d*x + 1)*(A + B*x + C*x**2), x)
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.98

$$\int \sqrt{1-dx}\sqrt{1+dx}(A+Bx+Cx^2)\,dx = \frac{1}{2}\sqrt{-d^2x^2+1}Ax - \frac{(-d^2x^2+1)^{\frac{3}{2}}Cx}{4\,d^2} + \frac{A\arcsin(dx)}{2\,d} - \frac{(-d^2x^2+1)^{\frac{3}{2}}B}{3\,d^2} + \frac{\sqrt{-d^2x^2+1}Cx}{8\,d^2} + \frac{C\arcsin(dx)}{8\,d^3}$$

input
```
integrate((C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm="maxima"
)
```

output
```
1/2*sqrt(-d^2*x^2 + 1)*A*x - 1/4*(-d^2*x^2 + 1)^(3/2)*C*x/d^2 + 1/2*A*arcs
in(d*x)/d - 1/3*(-d^2*x^2 + 1)^(3/2)*B/d^2 + 1/8*sqrt(-d^2*x^2 + 1)*C*x/d^
2 + 1/8*C*arcsin(d*x)/d^3
```

3.4.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. 2(81) = 162.

Time = 0.32 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.99

$$\int \sqrt{1-dx}\sqrt{1+dx}(A+Bx+Cx^2)\,dx$$
$$= \frac{12\left(\sqrt{dx+1}(dx-2)\sqrt{-dx+1} - 2\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{dx+1}\right)\right)Ad^2 + 24\left(\sqrt{dx+1}\sqrt{-dx+1} + 2\arcsin\left(\frac{1}{2}\right.\right.}{}$$

input
```
integrate((C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm="giac")
```

output
```
1/24*(12*(sqrt(d*x + 1)*(d*x - 2)*sqrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*sq
rt(d*x + 1)))*A*d^2 + 24*(sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*arcsin(1/2*sqrt
(2)*sqrt(d*x + 1)))*A*d^2 + 4*(((2*d*x - 5)*(d*x + 1) + 9)*sqrt(d*x + 1)*s
qrt(-d*x + 1) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*B*d + 12*(sqrt(d*x +
1)*(d*x - 2)*sqrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*B*d + (
((2*(3*d*x - 10)*(d*x + 1) + 43)*(d*x + 1) - 39)*sqrt(d*x + 1)*sqrt(-d*x +
 1) - 18*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*C + 4*(((2*d*x - 5)*(d*x + 1)
+ 9)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*C
)/d^3
```

3.4.9 Mupad [B] (verification not implemented)

Time = 7.87 (sec) , antiderivative size = 361, normalized size of antiderivative = 3.80

$$\int \sqrt{1-dx}\sqrt{1+dx}\left(A+Bx+Cx^2\right)\,dx=\frac{A\,x\,\sqrt{1-d\,x}\,\sqrt{d\,x+1}}{2}$$
$$-\frac{\frac{35\,C\,\left(\sqrt{1-d\,x}-1\right)^3}{2\,\left(\sqrt{d\,x+1}-1\right)^3}-\frac{273\,C\,\left(\sqrt{1-d\,x}-1\right)^5}{2\,\left(\sqrt{d\,x+1}-1\right)^5}+\frac{715\,C\,\left(\sqrt{1-d\,x}-1\right)^7}{2\,\left(\sqrt{d\,x+1}-1\right)^7}-\frac{715\,C\,\left(\sqrt{1-d\,x}-1\right)^9}{2\,\left(\sqrt{d\,x+1}-1\right)^9}+\frac{273\,C\,\left(\sqrt{1-d\,x}-1\right)^{11}}{2\,\left(\sqrt{d\,x+1}-1\right)^{11}}-\frac{35\,C\,\left(\sqrt{1-d\,x}-\right.}{2\,\left(\sqrt{d\,x+1}-1\right)}}{d^3\,\left(\frac{\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+1\right)^8}$$
$$-\frac{C\,\mathrm{atan}\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)}{2\,d^3}-\frac{A\,\sqrt{d}\,\ln\left(\sqrt{-d}\,\sqrt{1-d\,x}\,\sqrt{d\,x+1}-d^{3/2}\,x\right)}{2\,{\left(-d\right)}^{3/2}}$$
$$+\frac{B\,\left(d^2\,x^2-1\right)\,\sqrt{1-d\,x}\,\sqrt{d\,x+1}}{3\,d^2}$$

input

```
int((1 - d*x)^(1/2)*(d*x + 1)^(1/2)*(A + B*x + C*x^2),x)
```

output

```
(A*x*(1 - d*x)^(1/2)*(d*x + 1)^(1/2))/2 - ((35*C*((1 - d*x)^(1/2) - 1)^3)/
(2*((d*x + 1)^(1/2) - 1)^3) - (273*C*((1 - d*x)^(1/2) - 1)^5)/(2*((d*x + 1
)^(1/2) - 1)^5) + (715*C*((1 - d*x)^(1/2) - 1)^7)/(2*((d*x + 1)^(1/2) - 1)
^7) - (715*C*((1 - d*x)^(1/2) - 1)^9)/(2*((d*x + 1)^(1/2) - 1)^9) + (273*C
*((1 - d*x)^(1/2) - 1)^11)/(2*((d*x + 1)^(1/2) - 1)^11) - (35*C*((1 - d*x)
^(1/2) - 1)^13)/(2*((d*x + 1)^(1/2) - 1)^13) + (C*((1 - d*x)^(1/2) - 1)^15
)/(2*((d*x + 1)^(1/2) - 1)^15) - (C*((1 - d*x)^(1/2) - 1))/(2*((d*x + 1)^(
1/2) - 1)))/(d^3*(((1 - d*x)^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 + 1)^8)
- (C*atan(((1 - d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/(2*d^3) - (A*d^(1/
2)*log((-d)^(1/2)*(1 - d*x)^(1/2)*(d*x + 1)^(1/2) - d^(3/2)*x))/(2*(-d)^(3
/2)) + (B*(d^2*x^2 - 1)*(1 - d*x)^(1/2)*(d*x + 1)^(1/2))/(3*d^2)
```

3.5 $\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)} dx$

3.5.1 Optimal result

Integrand size = 37, antiderivative size = 122

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)}\,dx = -\frac{C\sqrt{1-d^2x^2}}{d^2f}-\frac{(Ce-Bf)\arcsin(dx)}{df^2}+\frac{(Ce^2-Bef+Af^2)\arctan\left(\frac{f+d^2ex}{\sqrt{d^2e^2-f^2}\sqrt{1-d^2x^2}}\right)}{f^2\sqrt{d^2e^2-f^2}}$$

output
```
-(-B*f+C*e)*arcsin(d*x)/d/f^2+(A*f^2-B*e*f+C*e^2)*arctan((d^2*e*x+f)/(d^2*
e^2-f^2)^(1/2)/(-d^2*x^2+1)^(1/2))/f^2/(d^2*e^2-f^2)^(1/2)-C*(-d^2*x^2+1)^
(1/2)/d^2/f
```

3.5.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.28

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)}\,dx$$
$$=\frac{-\frac{Cf\sqrt{1-d^2x^2}}{d^2}+\frac{2(-Ce+Bf)\arctan\left(\frac{dx}{-1+\sqrt{1-d^2x^2}}\right)}{d}-\frac{2\sqrt{d^2e^2-f^2}(Ce^2+f(-Be+Af))\arctan\left(\frac{\sqrt{d^2e^2-f^2}x}{e+fx-e\sqrt{1-d^2x^2}}\right)}{(de-f)(de+f)}}{f^2}$$

input
```
Integrate[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)),x]
```

output
```
(-((C*f*Sqrt[1 - d^2*x^2])/d^2) + (2*(-(C*e) + B*f)*ArcTan[(d*x)/(-1 + Sqr
t[1 - d^2*x^2])])/d - (2*Sqrt[d^2*e^2 - f^2]*(C*e^2 + f*(-(B*e) + A*f))*Ar
cTan[(Sqrt[d^2*e^2 - f^2]*x)/(e + f*x - e*Sqrt[1 - d^2*x^2])])/((d*e - f)*
(d*e + f)))/f^2
```

3.5.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {2112, 2185, 25, 27, 719, 223, 488, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{dx+1}(e+fx)}\,dx$$

$$\Big\downarrow \text{2112}$$

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-d^2x^2}(e+fx)}dx$$

$$\Big\downarrow \text{2185}$$

$$-\frac{\int -\frac{d^2f(Af-(Ce-Bf)x)}{(e+fx)\sqrt{1-d^2x^2}}dx}{d^2f^2}-\frac{C\sqrt{1-d^2x^2}}{d^2f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{d^2f(Af-(Ce-Bf)x)}{(e+fx)\sqrt{1-d^2x^2}}dx}{d^2f^2}-\frac{C\sqrt{1-d^2x^2}}{d^2f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{Af-(Ce-Bf)x}{(e+fx)\sqrt{1-d^2x^2}}dx}{f}-\frac{C\sqrt{1-d^2x^2}}{d^2f}$$

$$\Big\downarrow \text{719}$$

$$\frac{\frac{(Af^2-Bef+Ce^2)\int \frac{1}{(e+fx)\sqrt{1-d^2x^2}}dx}{f}-\frac{(Ce-Bf)\int \frac{1}{\sqrt{1-d^2x^2}}dx}{f}}{f}-\frac{C\sqrt{1-d^2x^2}}{d^2f}$$

$$\Big\downarrow \text{223}$$

$$\frac{\frac{(Af^2-Bef+Ce^2)\int \frac{1}{(e+fx)\sqrt{1-d^2x^2}}dx}{f}-\frac{\arcsin(dx)(Ce-Bf)}{df}}{f}-\frac{C\sqrt{1-d^2x^2}}{d^2f}$$

$$\Big\downarrow \text{488}$$

$$\frac{-\frac{(Af^2-Bef+Ce^2)\int \frac{1}{-d^2e^2+f^2-\frac{\left(exd^2+f\right)^2}{1-d^2x^2}}d\frac{exd^2+f}{\sqrt{1-d^2x^2}}}{f}-\frac{\arcsin(dx)(Ce-Bf)}{df}}{f}-\frac{C\sqrt{1-d^2x^2}}{d^2f}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{(Af^2-Bef+Ce^2)\arctan\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\sqrt{d^2e^2-f^2}}\right)}{f\sqrt{d^2e^2-f^2}}-\frac{\arcsin(dx)(Ce-Bf)}{df}}{f}-\frac{C\sqrt{1-d^2x^2}}{d^2f}$$

input
```
Int[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)),x]
```

output
```
-((C*Sqrt[1 - d^2*x^2])/(d^2*f)) + (-(((C*e - B*f)*ArcSin[d*x])/(d*f)) + (
(C*e^2 - B*e*f + A*f^2)*ArcTan[(f + d^2*e*x)/(Sqrt[d^2*e^2 - f^2]*Sqrt[1 -
 d^2*x^2])])/(f*Sqrt[d^2*e^2 - f^2]))/f
```

3.5.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2112
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.5.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. 2(114) = 228.

Time = 1.68 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.37

method	result
risch	$\frac{C\sqrt{dx+1}\,(dx-1)\sqrt{(-dx+1)(dx+1)}}{f\,d^2\sqrt{-(dx+1)(dx-1)}\,\sqrt{-dx+1}}+\frac{\left(\frac{(Bf-Ce)\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+1}}\right)}{f\sqrt{d^2}}-\frac{\left(A\,f^2-Bef+C\,e^2\right)\ln\left(-\frac{2\left(d^2e^2-f^2\right)}{f^2}+\frac{2d^2e\left(x+\frac{e}{f}\right)}{f}+2\sqrt{-\frac{d^2}{}}\right.}{f^2\sqrt{-\frac{d^2e^2-f}{f^2}}}\right.}{f\sqrt{-dx+1}\,\sqrt{dx+1}}$
default	$\left(-A\,\mathrm{csgn}(d)\ln\left(\frac{2d^2ex+2\sqrt{-d^2x^2+1}\,\sqrt{-\frac{d^2e^2-f^2}{f^2}}\,f+2f}{fx+e}\right)d^2f^2+B\,\mathrm{csgn}(d)\ln\left(\frac{2d^2ex+2\sqrt{-d^2x^2+1}\,\sqrt{-\frac{d^2e^2-f^2}{f^2}}\,f+2f}{fx+e}\right)d^2ef-C\,\mathrm{csgn}\right.$

input
```
int((C*x^2+B*x+A)/(f*x+e)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
C/f/d^2*(d*x+1)^(1/2)*(d*x-1)/(-(d*x+1)*(d*x-1))^(1/2)*((-d*x+1)*(d*x+1))^
(1/2)/(-d*x+1)^(1/2)+1/f*((B*f-C*e)/f/(d^2)^(1/2)*arctan((d^2)^(1/2)*x/(-d
^2*x^2+1)^(1/2))-(A*f^2-B*e*f+C*e^2)/f^2/(-(d^2*e^2-f^2)/f^2)^(1/2)*ln((-2
*(d^2*e^2-f^2)/f^2+2/f*d^2*e*(x+e/f)+2*(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*(x
+e/f)^2+2/f*d^2*e*(x+e/f)-(d^2*e^2-f^2)/f^2)^(1/2))/(x+e/f)))*((-d*x+1)*(d
*x+1))^(1/2)/(-d*x+1)^(1/2)/(d*x+1)^(1/2)
```

3.5.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(114) = 228$.

Time = 4.25 (sec) , antiderivative size = 493, normalized size of antiderivative = 4.04

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)}\,dx$$
$$=\left[-\frac{(Cd^2e^2-Bd^2ef+Ad^2f^2)\sqrt{-d^2e^2+f^2}\log\left(\frac{d^2efx+f^2-\sqrt{-d^2e^2+f^2}(d^2ex+f)-\left(\sqrt{-d^2e^2+f^2}\sqrt{-dx+1}f+(d^2e^2-f^2)\right.}{fx+e}\right.}{}\right.$$

input
```
integrate((C*x^2+B*x+A)/(f*x+e)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm=
"fricas")
```

output
```
[-((C*d^2*e^2 - B*d^2*e*f + A*d^2*f^2)*sqrt(-d^2*e^2 + f^2)*log((d^2*e*f*x
 + f^2 - sqrt(-d^2*e^2 + f^2)*(d^2*e*x + f) - (sqrt(-d^2*e^2 + f^2)*sqrt(-
d*x + 1)*f + (d^2*e^2 - f^2)*sqrt(-d*x + 1))*sqrt(d*x + 1))/(f*x + e)) + (
C*d^2*e^2*f - C*f^3)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 2*(C*d^3*e^3 - B*d^3*e
^2*f - C*d*e*f^2 + B*d*f^3)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x
)))/(d^4*e^2*f^2 - d^2*f^4), (2*(C*d^2*e^2 - B*d^2*e*f + A*d^2*f^2)*sqrt(d
^2*e^2 - f^2)*arctan(-(sqrt(d^2*e^2 - f^2)*sqrt(d*x + 1)*sqrt(-d*x + 1)*e
- sqrt(d^2*e^2 - f^2)*(f*x + e))/((d^2*e^2 - f^2)*x)) - (C*d^2*e^2*f - C*f
^3)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(C*d^3*e^3 - B*d^3*e^2*f - C*d*e*f^2
+ B*d*f^3)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/(d^4*e^2*f^2
- d^2*f^4)]
```

3.5.6 Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)}\,dx=\int \frac{A+Bx+Cx^2}{(e+fx)\sqrt{-dx+1}\sqrt{dx+1}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(f*x+e)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/((e + f*x)*sqrt(-d*x + 1)*sqrt(d*x + 1)), x)
```

3.5.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+B*x+A)/(f*x+e)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm=
"maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.5.8 Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((C*x^2+B*x+A)/(f*x+e)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

3.5.9 Mupad [B] (verification not implemented)

Time = 28.86 (sec) , antiderivative size = 5803, normalized size of antiderivative = 47.57

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/((e + f*x)*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

output
```
(4*C*e*atan((37748736*C^5*d^4*e^10*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2
) - 1)*(37748736*C^5*d^4*e^10 + 67108864*C^5*e^6*f^4 - 100663296*C^5*d^2*e
^8*f^2)) + (67108864*C^5*e^6*f^4*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2)
- 1)*(37748736*C^5*d^4*e^10 + 67108864*C^5*e^6*f^4 - 100663296*C^5*d^2*e^8
*f^2)) - (100663296*C^5*d^2*e^8*f^2*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/
2) - 1)*(37748736*C^5*d^4*e^10 + 67108864*C^5*e^6*f^4 - 100663296*C^5*d^2*
e^8*f^2))))/(d*f^2) - (4*B*atan((67108864*B^5*e*f^4*((1 - d*x)^(1/2) - 1))
/(((d*x + 1)^(1/2) - 1)*(67108864*B^5*e*f^4 + 37748736*B^5*d^4*e^5 - 10066
3296*B^5*d^2*e^3*f^2)) + (37748736*B^5*d^4*e^5*((1 - d*x)^(1/2) - 1))/(((d
*x + 1)^(1/2) - 1)*(67108864*B^5*e*f^4 + 37748736*B^5*d^4*e^5 - 100663296*
B^5*d^2*e^3*f^2)) - (100663296*B^5*d^2*e^3*f^2*((1 - d*x)^(1/2) - 1))/(((d
*x + 1)^(1/2) - 1)*(67108864*B^5*e*f^4 + 37748736*B^5*d^4*e^5 - 100663296*
B^5*d^2*e^3*f^2))))/(d*f) - (8*C*((1 - d*x)^(1/2) - 1)^2)/(f*((d*x + 1)^(1
/2) - 1)^2*(d^2 + (2*d^2*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2
+ (d^2*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4)) - (A*atan((f^2*1
i - d^2*e^2*1i - (f^2*((1 - d*x)^(1/2) - 1)^2*1i)/((d*x + 1)^(1/2) - 1)^2
+ (d^2*e^2*((1 - d*x)^(1/2) - 1)^2*1i)/((d*x + 1)^(1/2) - 1)^2)/(f*(f + d*
e)^(1/2)*(f - d*e)^(1/2) - (f*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(1/2)*(f -
 d*e)^(1/2))/((d*x + 1)^(1/2) - 1)^2 + (2*d*e*((1 - d*x)^(1/2) - 1)*(f + d
*e)^(1/2)*(f - d*e)^(1/2))/((d*x + 1)^(1/2) - 1)))*2i)/((f + d*e)^(1/2)...
```

3.5. $\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)}\,dx$

3.6 $\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^2}\,dx$

3.6.1 Optimal result

Integrand size = 37, antiderivative size = 163

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^2}\,dx = \frac{(Ce^2-Bef+Af^2)\sqrt{1-d^2x^2}}{f(d^2e^2-f^2)(e+fx)} + \frac{C\arcsin(dx)}{df^2} - \frac{(Cd^2e^3-2Cef^2-Ad^2ef^2+Bf^3)\arctan\left(\frac{f+d^2ex}{\sqrt{d^2e^2-f^2}\sqrt{1-d^2x^2}}\right)}{f^2(d^2e^2-f^2)^{3/2}}$$

output
```
C*arcsin(d*x)/d/f^2-(-A*d^2*e*f^2+C*d^2*e^3+B*f^3-2*C*e*f^2)*arctan((d^2*e
*x+f)/(d^2*e^2-f^2)^(1/2)/(-d^2*x^2+1)^(1/2))/f^2/(d^2*e^2-f^2)^(3/2)+(A*f
^2-B*e*f+C*e^2)*(-d^2*x^2+1)^(1/2)/f/(d^2*e^2-f^2)/(f*x+e)
```

3.6.2 Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^2}\,dx = \frac{\frac{f(Ce^2+f(-Be+Af))\sqrt{1-d^2x^2}}{(de-f)(de+f)(e+fx)} + \frac{2C\arctan\left(\frac{dx}{-1+\sqrt{1-d^2x^2}}\right)}{d} + \frac{2\sqrt{d^2e^2-f^2}(Cd^2e^3-2Cef^2-Ad^2ef^2+Bf^3)\arctan\left(\frac{\sqrt{d^2e^2-f^2}x}{e+fx-e\sqrt{1-d^2x^2}}\right)}{(-de+f)^2(de+f)^2}}{f^2}$$

input
```
Integrate[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^2),x]
```

output
```
((f*(C*e^2 + f*(-(B*e) + A*f))*Sqrt[1 - d^2*x^2])/((d*e - f)*(d*e + f)*(e
+ f*x)) + (2*C*ArcTan[(d*x)/(-1 + Sqrt[1 - d^2*x^2])])/d + (2*Sqrt[d^2*e^2
 - f^2]*(C*d^2*e^3 - 2*C*e*f^2 - A*d^2*e*f^2 + B*f^3)*ArcTan[(Sqrt[d^2*e^2
 - f^2]*x)/(e + f*x - e*Sqrt[1 - d^2*x^2])])/((-(d*e) + f)^2*(d*e + f)^2))
/f^2
```

3.6.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {2112, 2182, 719, 223, 488, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{dx+1}(e+fx)^2}dx$$

$$\Big\downarrow \text{2112}$$

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-d^2x^2}(e+fx)^2}dx$$

$$\Big\downarrow \text{2182}$$

$$\frac{\int \frac{Aed^2+Ce-Bf+C\left(\frac{d^2e^2}{f}-f\right)x}{(e+fx)\sqrt{1-d^2x^2}}dx}{d^2e^2-f^2}+\frac{\sqrt{1-d^2x^2}\left(Af^2-Bef+Ce^2\right)}{f\left(d^2e^2-f^2\right)(e+fx)}$$

$$\Big\downarrow \text{719}$$

$$\frac{\left(Ad^2e-Bf-\frac{Cd^2e^3}{f^2}+2Ce\right)\int \frac{1}{(e+fx)\sqrt{1-d^2x^2}}dx+\frac{C(de-f)(de+f)\int \frac{1}{\sqrt{1-d^2x^2}}dx}{f^2}}{d^2e^2-f^2}+\frac{\sqrt{1-d^2x^2}\left(Af^2-Bef+Ce^2\right)}{f\left(d^2e^2-f^2\right)(e+fx)}$$

$$\Big\downarrow \text{223}$$

$$\frac{\left(Ad^2e-Bf-\frac{Cd^2e^3}{f^2}+2Ce\right)\int \frac{1}{(e+fx)\sqrt{1-d^2x^2}}dx+\frac{C\arcsin(dx)(de-f)(de+f)}{df^2}}{d^2e^2-f^2}+\frac{\sqrt{1-d^2x^2}\left(Af^2-Bef+Ce^2\right)}{f\left(d^2e^2-f^2\right)(e+fx)}$$

$$\Big\downarrow \; 488$$

$$\frac{\frac{C\arcsin(dx)(de-f)(de+f)}{df^2}-\left(Ad^2e-Bf-\frac{Cd^2e^3}{f^2}+2Ce\right)\int \frac{1}{-d^2e^2+f^2-\frac{\left(exd^2+f\right)^2}{1-d^2x^2}}d\frac{exd^2+f}{\sqrt{1-d^2x^2}}}{d^2e^2-f^2}+\frac{\sqrt{1-d^2x^2}\left(Af^2-Bef+Ce^2\right)}{f\left(d^2e^2-f^2\right)(e+fx)}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{\arctan\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\sqrt{d^2e^2-f^2}}\right)\left(Ad^2e-Bf-\frac{Cd^2e^3}{f^2}+2Ce\right)}{\sqrt{d^2e^2-f^2}}+\frac{C\arcsin(dx)(de-f)(de+f)}{df^2}}{d^2e^2-f^2}+\frac{\sqrt{1-d^2x^2}\left(Af^2-Bef+Ce^2\right)}{f\left(d^2e^2-f^2\right)(e+fx)}$$

input
```
Int[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^2),x]
```

output
```
((C*e^2 - B*e*f + A*f^2)*Sqrt[1 - d^2*x^2])/(f*(d^2*e^2 - f^2)*(e + f*x))
+ ((C*(d*e - f)*(d*e + f)*ArcSin[d*x])/(d*f^2) + ((2*C*e + A*d^2*e - (C*d^
2*e^3)/f^2 - B*f)*ArcTan[(f + d^2*e*x)/(Sqrt[d^2*e^2 - f^2]*Sqrt[1 - d^2*x
^2])])/Sqrt[d^2*e^2 - f^2])/(d^2*e^2 - f^2)
```

3.6.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2112
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

3.6.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.67 (sec) , antiderivative size = 899, normalized size of antiderivative = 5.52

method	result
default	$\frac{\left(-A\,\operatorname{csgn}(d)\ln\left(\frac{2d^2ex+2\sqrt{-d^2x^2+1}\,\sqrt{-\frac{d^2e^2-f^2}{f^2}}\,f+2f}{fx+e}\right)d^3ef^3x+C\,\operatorname{csgn}(d)\ln\left(\frac{2d^2ex+2\sqrt{-d^2x^2+1}\,\sqrt{-\frac{d^2e^2-f^2}{f^2}}\,f+2f}{fx+e}\right)d^3e^3fx-A}{}$

input
```
int((C*x^2+B*x+A)/(f*x+e)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
(-A*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+
f)/(f*x+e))*d^3*e*f^3*x+C*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*
e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^3*e^3*f*x-A*csgn(d)*ln(2*(d^2*e*x+(-d^
2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^3*e^2*f^2+C*csgn
(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+
e))*d^3*e^4+C*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*d^2*e^2*f^2*x*(-(d^2*
e^2-f^2)/f^2)^(1/2)+A*csgn(d)*d*f^4*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2
)^(1/2)+B*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1
/2)*f+f)/(f*x+e))*d*f^4*x-B*csgn(d)*d*e*f^3*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-
f^2)/f^2)^(1/2)-2*C*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^
2)/f^2)^(1/2)*f+f)/(f*x+e))*d*e*f^3*x+C*csgn(d)*d*e^2*f^2*(-d^2*x^2+1)^(1/
2)*(-(d^2*e^2-f^2)/f^2)^(1/2)+C*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*d^2
*e^3*f*(-(d^2*e^2-f^2)/f^2)^(1/2)+B*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/
2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d*e*f^3-2*C*csgn(d)*ln(2*(d^2*
e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d*e^2*f^2-
C*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*f^4*x*(-(d^2*e^2-f^2)/f^2)^(1/2)-
C*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*e*f^3*(-(d^2*e^2-f^2)/f^2)^(1/2))
*csgn(d)*(-d*x+1)^(1/2)*(d*x+1)^(1/2)/(-d^2*x^2+1)^(1/2)/(d*e-f)/d/(d*e+f)
/(f*x+e)/(-(d^2*e^2-f^2)/f^2)^(1/2)/f^3
```

3.6.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 501 vs. $2(155) = 310$.

Time = 16.65 (sec) , antiderivative size = 1025, normalized size of antiderivative = 6.29

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^2}\,dx$$

$$= \left[\frac{Cd^3e^5f - Bd^3e^4f^2 + Bde^2f^4 - Adef^5 + (Ad^3 - Cd)e^3f^3 - (Cd^3e^5 + Bde^2f^3 - (Ad^3 + 2\,Cd)e^3f^2 + (\cdots}{\cdots}\right.$$

input
```
integrate((C*x^2+B*x+A)/(f*x+e)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorith
m="fricas")
```

output
```
[(C*d^3*e^5*f - B*d^3*e^4*f^2 + B*d*e^2*f^4 - A*d*e*f^5 + (A*d^3 - C*d)*e^
3*f^3 - (C*d^3*e^5 + B*d*e^2*f^3 - (A*d^3 + 2*C*d)*e^3*f^2 + (C*d^3*e^4*f
+ B*d*e*f^4 - (A*d^3 + 2*C*d)*e^2*f^3)*x)*sqrt(-d^2*e^2 + f^2)*log((d^2*e*
f*x + f^2 + sqrt(-d^2*e^2 + f^2)*(d^2*e*x + f) + (sqrt(-d^2*e^2 + f^2)*sqr
t(-d*x + 1)*f - (d^2*e^2 - f^2)*sqrt(-d*x + 1))*sqrt(d*x + 1))/(f*x + e))
+ (C*d^3*e^5*f - B*d^3*e^4*f^2 + B*d*e^2*f^4 - A*d*e*f^5 + (A*d^3 - C*d)*e
^3*f^3)*sqrt(d*x + 1)*sqrt(-d*x + 1) + (C*d^3*e^4*f^2 - B*d^3*e^3*f^3 + B*
d*e*f^5 - A*d*f^6 + (A*d^3 - C*d)*e^2*f^4)*x - 2*(C*d^4*e^6 - 2*C*d^2*e^4*
f^2 + C*e^2*f^4 + (C*d^4*e^5*f - 2*C*d^2*e^3*f^3 + C*e*f^5)*x)*arctan((sqr
t(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/(d^5*e^6*f^2 - 2*d^3*e^4*f^4 + d*e^
2*f^6 + (d^5*e^5*f^3 - 2*d^3*e^3*f^5 + d*e*f^7)*x), (C*d^3*e^5*f - B*d^3*e
^4*f^2 + B*d*e^2*f^4 - A*d*e*f^5 + (A*d^3 - C*d)*e^3*f^3 - 2*(C*d^3*e^5 +
B*d*e^2*f^3 - (A*d^3 + 2*C*d)*e^3*f^2 + (C*d^3*e^4*f + B*d*e*f^4 - (A*d^3
+ 2*C*d)*e^2*f^3)*x)*sqrt(d^2*e^2 - f^2)*arctan(-(sqrt(d^2*e^2 - f^2)*sqrt
(d*x + 1)*sqrt(-d*x + 1)*e - sqrt(d^2*e^2 - f^2)*(f*x + e))/((d^2*e^2 - f^
2)*x)) + (C*d^3*e^5*f - B*d^3*e^4*f^2 + B*d*e^2*f^4 - A*d*e*f^5 + (A*d^3 -
 C*d)*e^3*f^3)*sqrt(d*x + 1)*sqrt(-d*x + 1) + (C*d^3*e^4*f^2 - B*d^3*e^3*f
^3 + B*d*e*f^5 - A*d*f^6 + (A*d^3 - C*d)*e^2*f^4)*x - 2*(C*d^4*e^6 - 2*C*d
^2*e^4*f^2 + C*e^2*f^4 + (C*d^4*e^5*f - 2*C*d^2*e^3*f^3 + C*e*f^5)*x)*arct
an((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/(d^5*e^6*f^2 - 2*d^3*e^4*...
```

3.6.6 Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^2}\,dx = \int \frac{A+Bx+Cx^2}{(e+fx)^2\sqrt{-dx+1}\sqrt{dx+1}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(f*x+e)**2/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/((e + f*x)**2*sqrt(-d*x + 1)*sqrt(d*x + 1)), x
)
```

3.6.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A + Bx + Cx^2}{\sqrt{1 - dx}\sqrt{1 + dx}(e + fx)^2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+B*x+A)/(f*x+e)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.6.8 Giac [F(-2)]

Exception generated.

$$\int \frac{A + Bx + Cx^2}{\sqrt{1 - dx}\sqrt{1 + dx}(e + fx)^2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((C*x^2+B*x+A)/(f*x+e)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

3.6.9 Mupad [B] (verification not implemented)

Time = 57.67 (sec) , antiderivative size = 10198, normalized size of antiderivative = 62.56

$$\int \frac{A + Bx + Cx^2}{\sqrt{1 - dx}\sqrt{1 + dx}(e + fx)^2} \, dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/((e + f*x)^2*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

output
```
(A*d^5*e^5*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) -
1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d
^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*
e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(
1/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d
*x + 1)^(1/2) - 1)^2))*2i - A*d^3*e^3*f^2*atan(((f + d*e)^(3/2)*(f - d*e)^
(3/2)*1i - (((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((
d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((
d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2)
 - 1) + (2*d*e*f^2*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2
*f*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2))*2i + (4*A*f^2*((1 -
d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1) + (
A*d^5*e^5*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1
)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^
2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e
^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1
/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*
x + 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)^2*4i)/((d*x + 1)^(1/2) - 1)^2
+ (A*d^5*e^5*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2)
- 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^...
```

3.6. $\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^2}\,dx$

3.7 $\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^3} dx$

3.7.1 Optimal result

Integrand size = 37, antiderivative size = 248

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^3} dx &= \frac{(Ce^2-Bef+Af^2)\sqrt{1-d^2x^2}}{2f(d^2e^2-f^2)(e+fx)^2} \\ &\quad - \frac{(Cd^2e^3+Bd^2e^2f-4Cef^2-3Ad^2ef^2+2Bf^3)\sqrt{1-d^2x^2}}{2f(d^2e^2-f^2)^2(e+fx)} \\ &\quad + \frac{(C(d^2e^2+2f^2)-d^2(3Bef-A(2d^2e^2+f^2)))\arctan\left(\frac{f+d^2ex}{\sqrt{d^2e^2-f^2}\sqrt{1-d^2x^2}}\right)}{2(d^2e^2-f^2)^{5/2}}\end{aligned}$$

output
```
1/2*(C*(d^2*e^2+2*f^2)-d^2*(3*B*e*f-A*(2*d^2*e^2+f^2)))*arctan((d^2*e*x+f)
/(d^2*e^2-f^2)^(1/2)/(-d^2*x^2+1)^(1/2))/(d^2*e^2-f^2)^(5/2)+1/2*(A*f^2-B*
e*f+C*e^2)*(-d^2*x^2+1)^(1/2)/f/(d^2*e^2-f^2)/(f*x+e)^2-1/2*(-3*A*d^2*e*f^
2+B*d^2*e^2*f+C*d^2*e^3+2*B*f^3-4*C*e*f^2)*(-d^2*x^2+1)^(1/2)/f/(d^2*e^2-f
^2)^2/(f*x+e)
```

3.7.2 Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^3}\,dx = -\frac{\frac{(de-f)(de+f)\sqrt{1-d^2x^2}(Af^3+Bd^2e^2(2e+fx)+Bf^2(e+2fx)-Ad^2ef(4e+3fx)+Ce(-3ef+d^2e^2x-4f^2x))}{(e+fx)^2}+2\sqrt{d^2e^2-f^2}(C(d^2e^2}{2(de-f)^3(de+f)^3}$$

input
```
Integrate[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^3),x]
```

output
```
-1/2*(((d*e - f)*(d*e + f)*Sqrt[1 - d^2*x^2]*(A*f^3 + B*d^2*e^2*(2*e + f*x
) + B*f^2*(e + 2*f*x) - A*d^2*e*f*(4*e + 3*f*x) + C*e*(-3*e*f + d^2*e^2*x
- 4*f^2*x)))/(e + f*x)^2 + 2*Sqrt[d^2*e^2 - f^2]*(C*(d^2*e^2 + 2*f^2) + d^
2*(-3*B*e*f + A*(2*d^2*e^2 + f^2)))*ArcTan[(Sqrt[d^2*e^2 - f^2]*x)/(e + f*
x - e*Sqrt[1 - d^2*x^2])])/((d*e - f)^3*(d*e + f)^3)
```

3.7.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {2112, 2182, 679, 488, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{dx+1}(e+fx)^3}dx$$

$$\Big\downarrow \text{2112}$$

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-d^2x^2}(e+fx)^3}dx$$

$$\Big\downarrow \text{2182}$$

$$\frac{\int \frac{2(Aed^2+Ce-Bf)+\left(Bed^2-Afd^2+\frac{Ce^2d^2}{f}-2Cf\right)x}{(e+fx)^2\sqrt{1-d^2x^2}}dx}{2(d^2e^2-f^2)}+\frac{\sqrt{1-d^2x^2}(Af^2-Bef+Ce^2)}{2f(d^2e^2-f^2)(e+fx)^2}$$

$$\Big\downarrow \text{679}$$

$$\frac{\frac{\left(C\left(d^2e^2+2f^2\right)-d^2\left(3Bef-A\left(2d^2e^2+f^2\right)\right)\right)\int \frac{1}{(e+fx)\sqrt{1-d^2x^2}}dx}{d^2e^2-f^2}-\frac{\sqrt{1-d^2x^2}\left(-3Ad^2ef^2+Bd^2e^2f+2Bf^3+Cd^2e^3-4Cef^2\right)}{f(d^2e^2-f^2)(e+fx)}}{2\left(d^2e^2-f^2\right)}+\frac{\sqrt{1-d^2x^2}\left(Af^2-Bef+Ce^2\right)}{2f\left(d^2e^2-f^2\right)(e+fx)^2}$$

$\big\downarrow$ 488

$$\frac{-\frac{\left(C\left(d^2e^2+2f^2\right)-d^2\left(3Bef-A\left(2d^2e^2+f^2\right)\right)\right)\int \frac{1}{-d^2e^2+f^2-\frac{\left(exd^2+f\right)^2}{1-d^2x^2}}d\frac{exd^2+f}{\sqrt{1-d^2x^2}}}{d^2e^2-f^2}-\frac{\sqrt{1-d^2x^2}\left(-3Ad^2ef^2+Bd^2e^2f+2Bf^3+Cd^2e^3-4Cef^2\right)}{f(d^2e^2-f^2)(e+fx)}}{2\left(d^2e^2-f^2\right)}+\frac{\sqrt{1-d^2x^2}\left(Af^2-Bef+Ce^2\right)}{2f\left(d^2e^2-f^2\right)(e+fx)^2}$$

$\big\downarrow$ 217

$$\frac{\frac{\arctan\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\sqrt{d^2e^2-f^2}}\right)\left(C\left(d^2e^2+2f^2\right)-d^2\left(3Bef-A\left(2d^2e^2+f^2\right)\right)\right)}{\left(d^2e^2-f^2\right)^{3/2}}-\frac{\sqrt{1-d^2x^2}\left(-3Ad^2ef^2+Bd^2e^2f+2Bf^3+Cd^2e^3-4Cef^2\right)}{f(d^2e^2-f^2)(e+fx)}}{2\left(d^2e^2-f^2\right)}+\frac{\sqrt{1-d^2x^2}\left(Af^2-Bef+Ce^2\right)}{2f\left(d^2e^2-f^2\right)(e+fx)^2}$$

input
```
Int[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^3),x]
```

output
```
((C*e^2 - B*e*f + A*f^2)*Sqrt[1 - d^2*x^2])/(2*f*(d^2*e^2 - f^2)*(e + f*x)
^2) + (-(((C*d^2*e^3 + B*d^2*e^2*f - 4*C*e*f^2 - 3*A*d^2*e*f^2 + 2*B*f^3)*
Sqrt[1 - d^2*x^2])/(f*(d^2*e^2 - f^2)*(e + f*x))) + ((C*(d^2*e^2 + 2*f^2)
- d^2*(3*B*e*f - A*(2*d^2*e^2 + f^2)))*ArcTan[(f + d^2*e*x)/(Sqrt[d^2*e^2
- f^2]*Sqrt[1 - d^2*x^2])])/(d^2*e^2 - f^2)^(3/2))/(2*(d^2*e^2 - f^2))
```

3.7.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2112
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

3.7.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.64 (sec) , antiderivative size = 1449, normalized size of antiderivative = 5.84

method	result	size
default	Expression too large to display	1449

input
```
int((C*x^2+B*x+A)/(f*x+e)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
-1/2*(2*A*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)
/(f*x+e))*d^4*e^2*f^2*x^2-3*A*d^2*e*f^3*x*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^
2)/f^2)^(1/2)+B*d^2*e^2*f^2*x*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2
)+C*d^2*e^3*f*x*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)+2*A*ln(2*(d^
2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^4*e^4-
3*C*e^2*f^2*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)+2*B*f^4*x*(-d^2*
x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)+C*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)
*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^2*e^4+2*C*ln(2*(d^2*e*x+(-d^2*
x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*f^4*x^2+2*C*ln(2*(d^
2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*e^2*f^2+
B*e*f^3*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)+A*ln(2*(d^2*e*x+(-d^
2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^2*f^4*x^2+A*ln(2
*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^2*
e^2*f^2-3*B*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+
f)/(f*x+e))*d^2*e^3*f+4*C*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)
/f^2)^(1/2)*f+f)/(f*x+e))*e*f^3*x+A*f^4*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)
/f^2)^(1/2)+4*A*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2
)*f+f)/(f*x+e))*d^4*e^3*f*x-3*B*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^
2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^2*e*f^3*x^2+C*ln(2*(d^2*e*x+(-d^2*x^2+1)
^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^2*e^2*f^2*x^2+2*A*ln(...
```

3.7.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 778 vs. $2(232) = 464$.

Time $= 0.36$ (sec) , antiderivative size $= 1580$, normalized size of antiderivative $= 6.37$

$$\int \frac{A + Bx + Cx^2}{\sqrt{1 - dx}\sqrt{1 + dx}(e + fx)^3}\, dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(f*x+e)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorith
m="fricas")
```

output
```
[-1/2*(2*B*d^4*e^7 - B*d^2*e^5*f^2 - (4*A*d^4 + 3*C*d^2)*e^6*f + (5*A*d^2
+ 3*C)*e^4*f^3 - B*e^3*f^4 - A*e^2*f^5 + (2*B*d^4*e^5*f^2 - B*d^2*e^3*f^4
- (4*A*d^4 + 3*C*d^2)*e^4*f^3 + (5*A*d^2 + 3*C)*e^2*f^5 - B*e*f^6 - A*f^7)
*x^2 - (3*B*d^2*e^5*f - (2*A*d^4 + C*d^2)*e^6 - (A*d^2 + 2*C)*e^4*f^2 + (3
*B*d^2*e^3*f^3 - (2*A*d^4 + C*d^2)*e^4*f^2 - (A*d^2 + 2*C)*e^2*f^4)*x^2 +
2*(3*B*d^2*e^4*f^2 - (2*A*d^4 + C*d^2)*e^5*f - (A*d^2 + 2*C)*e^3*f^3)*x)*s
qrt(-d^2*e^2 + f^2)*log((d^2*e*f*x + f^2 - sqrt(-d^2*e^2 + f^2)*(d^2*e*x +
 f) - (sqrt(-d^2*e^2 + f^2)*sqrt(-d*x + 1)*f + (d^2*e^2 - f^2)*sqrt(-d*x +
 1))*sqrt(d*x + 1))/(f*x + e)) + (2*B*d^4*e^7 - B*d^2*e^5*f^2 - (4*A*d^4 +
 3*C*d^2)*e^6*f + (5*A*d^2 + 3*C)*e^4*f^3 - B*e^3*f^4 - A*e^2*f^5 + (C*d^4
*e^7 + B*d^4*e^6*f + B*d^2*e^4*f^3 - (3*A*d^4 + 5*C*d^2)*e^5*f^2 + (3*A*d^
2 + 4*C)*e^3*f^4 - 2*B*e^2*f^5)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(2*B*d
^4*e^6*f - B*d^2*e^4*f^3 - (4*A*d^4 + 3*C*d^2)*e^5*f^2 + (5*A*d^2 + 3*C)*e
^3*f^4 - B*e^2*f^5 - A*e*f^6)*x)/(d^6*e^10 - 3*d^4*e^8*f^2 + 3*d^2*e^6*f^4
 - e^4*f^6 + (d^6*e^8*f^2 - 3*d^4*e^6*f^4 + 3*d^2*e^4*f^6 - e^2*f^8)*x^2 +
 2*(d^6*e^9*f - 3*d^4*e^7*f^3 + 3*d^2*e^5*f^5 - e^3*f^7)*x), -1/2*(2*B*d^4
*e^7 - B*d^2*e^5*f^2 - (4*A*d^4 + 3*C*d^2)*e^6*f + (5*A*d^2 + 3*C)*e^4*f^3
 - B*e^3*f^4 - A*e^2*f^5 + (2*B*d^4*e^5*f^2 - B*d^2*e^3*f^4 - (4*A*d^4 + 3
*C*d^2)*e^4*f^3 + (5*A*d^2 + 3*C)*e^2*f^5 - B*e*f^6 - A*f^7)*x^2 + 2*(3*B*
d^2*e^5*f - (2*A*d^4 + C*d^2)*e^6 - (A*d^2 + 2*C)*e^4*f^2 + (3*B*d^2*e^...
```

3.7.6 Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^3}\,dx=\int \frac{A+Bx+Cx^2}{(e+fx)^3\sqrt{-dx+1}\sqrt{dx+1}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(f*x+e)**3/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/((e + f*x)**3*sqrt(-d*x + 1)*sqrt(d*x + 1)), x
)
```

3.7.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+B*x+A)/(f*x+e)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((f-d*e)*(f+d*e)>0)', see `assume
?` for mor
```

3.7.8 Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((C*x^2+B*x+A)/(f*x+e)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

3.7.9 Mupad [B] (verification not implemented)

Time = 66.29 (sec) , antiderivative size = 9097, normalized size of antiderivative = 36.68

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/((e + f*x)^3*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

output
```
((12*(2*C*f^3 + C*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^2)/(((d*x + 1)^(1/2) -
1)^2*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) - (24*(2*C*f^3 - C*d^2*e^2*f)*((1 -
d*x)^(1/2) - 1)^4)/(((d*x + 1)^(1/2) - 1)^4*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2
)) + (12*(2*C*f^3 + C*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^6)/(((d*x + 1)^(1/2
) - 1)^6*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) - (2*((1 - d*x)^(1/2) - 1)^7*(C*
d^3*e^3 + 2*C*d*e*f^2))/(((d*x + 1)^(1/2) - 1)^7*(f^4 + d^4*e^4 - 2*d^2*e^
2*f^2)) - (2*((1 - d*x)^(1/2) - 1)^3*(7*C*d^3*e^3 - 34*C*d*e*f^2))/(((d*x
+ 1)^(1/2) - 1)^3*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) + (2*((1 - d*x)^(1/2) -
 1)^5*(7*C*d^3*e^3 - 34*C*d*e*f^2))/(((d*x + 1)^(1/2) - 1)^5*(f^4 + d^4*e^
4 - 2*d^2*e^2*f^2)) + (2*d*e*((1 - d*x)^(1/2) - 1)*(2*C*f^2 + C*d^2*e^2))/
(((d*x + 1)^(1/2) - 1)*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)))/(d^2*e^2 + (((1 -
 d*x)^(1/2) - 1)^2*(16*f^2 + 4*d^2*e^2))/((d*x + 1)^(1/2) - 1)^2 + (((1 -
d*x)^(1/2) - 1)^6*(16*f^2 + 4*d^2*e^2))/((d*x + 1)^(1/2) - 1)^6 - (((1 - d
*x)^(1/2) - 1)^4*(32*f^2 - 6*d^2*e^2))/((d*x + 1)^(1/2) - 1)^4 + (d^2*e^2*
((1 - d*x)^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8 + (8*d*e*f*((1 - d*x)^(1/
2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 - (8*d*e*f*((1 - d*x)^(1/2) - 1)^5)/((d
*x + 1)^(1/2) - 1)^5 - (8*d*e*f*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2)
- 1)^7 + (8*d*e*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)) + ((4*((1
- d*x)^(1/2) - 1)^2*(4*A*d^4*e^4*f - 2*A*f^5 + 7*A*d^2*e^2*f^3))/(e^2*((d*
x + 1)^(1/2) - 1)^2*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) + (8*((1 - d*x)^(1...
```

3.7. $\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^3}\,dx$

3.8 $\int \frac{(e+fx)^3(A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

3.8.1 Optimal result

Integrand size = 37, antiderivative size = 340

$$\int \frac{(e+fx)^3(A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$$
$$= -\frac{(4(4C+5Ad^2)f^2-3d^2e(Ce-5Bf))(e+fx)^2\sqrt{1-d^2x^2}}{60d^4f}$$
$$+\frac{(Ce-5Bf)(e+fx)^3\sqrt{1-d^2x^2}}{20d^2f}-\frac{C(e+fx)^4\sqrt{1-d^2x^2}}{5d^2f}$$
$$+\frac{(4(C(3d^4e^4-52d^2e^2f^2-16f^4)-5d^2f(4Af(4d^2e^2+f^2)+3B(d^2e^3+4ef^2)))+d^2f(6Cd^2e^3-30Bd^2}{120d^6f}$$
$$+\frac{(4Cd^2e^3+8Ad^4e^3+12Bd^2e^2f+9Cef^2+12Ad^2ef^2+3Bf^3)\arcsin(dx)}{8d^5}$$

output
```
1/8*(8*A*d^4*e^3+12*A*d^2*e*f^2+12*B*d^2*e^2*f+4*C*d^2*e^3+3*B*f^3+9*C*e*f
^2)*arcsin(d*x)/d^5-1/60*(4*(5*A*d^2+4*C)*f^2-3*d^2*e*(-5*B*f+C*e))*(f*x+e
)^2*(-d^2*x^2+1)^(1/2)/d^4/f+1/20*(-5*B*f+C*e)*(f*x+e)^3*(-d^2*x^2+1)^(1/2
)/d^2/f-1/5*C*(f*x+e)^4*(-d^2*x^2+1)^(1/2)/d^2/f+1/120*(4*C*(3*d^4*e^4-52*
d^2*e^2*f^2-16*f^4)-20*d^2*f*(4*A*f*(4*d^2*e^2+f^2)+3*B*(d^2*e^3+4*e*f^2))
+d^2*f*(-100*A*d^2*e*f^2-30*B*d^2*e^2*f+6*C*d^2*e^3-45*B*f^3-71*C*e*f^2)*x
)*(-d^2*x^2+1)^(1/2)/d^6/f
```

3.8.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.76

$$\int \frac{(e+fx)^3 (A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$$
$$= \frac{-\sqrt{1-d^2x^2}(20Ad^2f(4f^2+d^2(18e^2+9efx+2f^2x^2))+15B(d^2f^2(16e+3fx)+2d^4(4e^3+6e^2fx+4ef}{}$$

input
```
Integrate[((e + f*x)^3*(A + B*x + C*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

output
```
(-(Sqrt[1 - d^2*x^2]*(20*A*d^2*f*(4*f^2 + d^2*(18*e^2 + 9*e*f*x + 2*f^2*x^
2)) + 15*B*(d^2*f^2*(16*e + 3*f*x) + 2*d^4*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^
2 + f^3*x^3)) + C*(64*f^3 + d^2*f*(240*e^2 + 135*e*f*x + 32*f^2*x^2) + 6*d
^4*x*(10*e^3 + 20*e^2*f*x + 15*e*f^2*x^2 + 4*f^3*x^3)))) + 30*d*(4*C*d^2*e
^3 + 8*A*d^4*e^3 + 12*B*d^2*e^2*f + 9*C*e*f^2 + 12*A*d^2*e*f^2 + 3*B*f^3)*
ArcTan[(d*x)/(-1 + Sqrt[1 - d^2*x^2])])/(120*d^6)
```

3.8.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.297$, Rules used = {2112, 2185, 25, 27, 687, 25, 27, 687, 25, 676, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3 (A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{dx+1}}\,dx$$
$$\Big\downarrow \text{ 2112}$$
$$\int \frac{(e+fx)^3 (A+Bx+Cx^2)}{\sqrt{1-d^2x^2}}dx$$
$$\Big\downarrow \text{ 2185}$$
$$-\frac{\int -\frac{f(e+fx)^3((5Ad^2+4C)f-d^2(Ce-5Bf)x)}{\sqrt{1-d^2x^2}}dx}{5d^2f^2}-\frac{C\sqrt{1-d^2x^2}(e+fx)^4}{5d^2f}$$
$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{f(e+fx)^3((5Ad^2+4C)f-d^2(Ce-5Bf)x)}{\sqrt{1-d^2x^2}}dx}{5d^2f^2}-\frac{C\sqrt{1-d^2x^2}(e+fx)^4}{5d^2f}$$

$\downarrow$ 27

$$\frac{\int \frac{(e+fx)^3((5Ad^2+4C)f-d^2(Ce-5Bf)x)}{\sqrt{1-d^2x^2}}dx}{5d^2f}-\frac{C\sqrt{1-d^2x^2}(e+fx)^4}{5d^2f}$$

$\downarrow$ 687

$$\frac{\frac{1}{4}\sqrt{1-d^2x^2}(e+fx)^3(Ce-5Bf)-\frac{\int -\frac{d^2(e+fx)^2\left(f\left(20Aed^2+13Ce+15Bf\right)+\left(4\left(5Ad^2+4C\right)f^2-3d^2e(Ce-5Bf)\right)x\right)}{\sqrt{1-d^2x^2}}dx}{4d^2}}{5d^2f}-\frac{C\sqrt{1-d^2x^2}(e+fx)^4}{5d^2f}$$

$\downarrow$ 25

$$\frac{\frac{\int \frac{d^2(e+fx)^2\left(f\left(20Aed^2+13Ce+15Bf\right)+\left(4\left(5Ad^2+4C\right)f^2-3d^2e(Ce-5Bf)\right)x\right)}{\sqrt{1-d^2x^2}}dx}{4d^2}+\frac{1}{4}\sqrt{1-d^2x^2}(e+fx)^3(Ce-5Bf)}{5d^2f}-\frac{C\sqrt{1-d^2x^2}(e+fx)^4}{5d^2f}$$

$\downarrow$ 27

$$\frac{\frac{1}{4}\int \frac{(e+fx)^2(f(20Aed^2+13Ce+15Bf)+(4(5Ad^2+4C)f^2-3d^2e(Ce-5Bf))x)}{\sqrt{1-d^2x^2}}dx+\frac{1}{4}\sqrt{1-d^2x^2}(e+fx)^3(Ce-5Bf)}{5d^2f}-\frac{C\sqrt{1-d^2x^2}(e+fx)^4}{5d^2f}$$

$\downarrow$ 687

$$\frac{\frac{1}{4}\left(-\frac{\int -\frac{(e+fx)\left(f\left(60Ae^2d^4+33Ce^2d^2+40Af^2d^2+75Befd^2+32Cf^2\right)-d^2\left(6Cd^2e^3-30Bd^2fe^2-100Ad^2f^2e-71Cf^2e-45Bf^3\right)x\right)}{\sqrt{1-d^2x^2}}dx}{3d^2}-\frac{1}{3}\sqrt{1-d^2x^2}(e\right.}{5d^2f}$$

$$\frac{C\sqrt{1-d^2x^2}(e+fx)^4}{5d^2f}$$

$\downarrow$ 25

3.8. $\int \frac{(e+fx)^3(A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

$$\frac{\frac{1}{4}\left(\frac{\int \frac{(e+fx)\left(f\left(60Ae^2d^4+33Ce^2d^2+40Af^2d^2+75Befd^2+32Cf^2\right)-d^2\left(6Cd^2e^3-30Bd^2fe^2-100Ad^2f^2e-71Cf^2e-45Bf^3\right)x\right)}{\sqrt{1-d^2x^2}}dx}{3d^2}-\frac{1}{3}\sqrt{1-d^2x^2}(e+\right.}{5d^2f}$$

$$\frac{C\sqrt{1-d^2x^2}(e+fx)^4}{5d^2f}$$

$\Big\downarrow$ 676

$$\frac{\frac{1}{4}\left(\frac{\frac{15}{2}f\left(8Ad^4e^3+12Ad^2ef^2+12Bd^2e^2f+3Bf^3+4Cd^2e^3+9Cef^2\right)\int \frac{1}{\sqrt{1-d^2x^2}}dx+\frac{1}{2}fx\sqrt{1-d^2x^2}\left(-100Ad^2ef^2-30Bd^2e^2f-45Bf^3+6Cd^2e^3-7\right.}{3d^2}\right.}{}$$

$$\frac{C\sqrt{1-d^2x^2}(e+fx)^4}{5d^2f}$$

$\Big\downarrow$ 223

$$\frac{\frac{1}{4}\left(\frac{\frac{15f\arcsin(dx)\left(8Ad^4e^3+12Ad^2ef^2+12Bd^2e^2f+3Bf^3+4Cd^2e^3+9Cef^2\right)}{2d}+\frac{1}{2}fx\sqrt{1-d^2x^2}\left(-100Ad^2ef^2-30Bd^2e^2f-45Bf^3+6Cd^2e^3-71Cef^2\right)+\frac{2\sqrt{}}{}}{3d^2}\right.}{}$$

$$\frac{C\sqrt{1-d^2x^2}(e+fx)^4}{5d^2f}$$

input
```
Int[((e + f*x)^3*(A + B*x + C*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

output
```
-1/5*(C*(e + f*x)^4*Sqrt[1 - d^2*x^2])/(d^2*f) + ((((C*e - 5*B*f)*(e + f*x)
^3*Sqrt[1 - d^2*x^2])/4 + (-1/3*((5*f*(3*B*e + 4*A*f) - C*(3*e^2 - (16*f^2
)/d^2))*(e + f*x)^2*Sqrt[1 - d^2*x^2]) + ((2*(C*(3*d^4*e^4 - 52*d^2*e^2*f^
2 - 16*f^4) - 5*d^2*f*(4*A*f*(4*d^2*e^2 + f^2) + 3*B*(d^2*e^3 + 4*e*f^2)))
*Sqrt[1 - d^2*x^2])/d^2 + (f*(6*C*d^2*e^3 - 30*B*d^2*e^2*f - 71*C*e*f^2 -
100*A*d^2*e*f^2 - 45*B*f^3)*x*Sqrt[1 - d^2*x^2])/2 + (15*f*(4*C*d^2*e^3 +
8*A*d^4*e^3 + 12*B*d^2*e^2*f + 9*C*e*f^2 + 12*A*d^2*e*f^2 + 3*B*f^3)*ArcSi
n[d*x])/(2*d))/(3*d^2))/4)/(5*d^2*f)
```

3.8. $\int \frac{(e+fx)^3\left(A+Bx+Cx^2\right)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

3.8.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2112
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.8.4 Maple [A] (verified)

Time = 1.63 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.06

method	result
risch	$\frac{(24C d^4 f^3 x^4+30B d^4 f^3 x^3+90C d^4 e f^2 x^3+40A d^4 f^3 x^2+120B d^4 e f^2 x^2+120C d^4 e^2 f x^2+180A d^4 e f^2 x+180B d^4 e^2 f x+60C d^4 e^3 x\cdots}{120 d^6 \sqrt{\cdots}}$
default	$-\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(24C\sqrt{-d^2x^2+1}\,\operatorname{csgn}(d) d^4 f^3 x^4+30B\sqrt{-d^2x^2+1}\,\operatorname{csgn}(d) d^4 f^3 x^3+90C\sqrt{-d^2x^2+1}\,\operatorname{csgn}(d) d^4 e f^2 x^3+40A\sqrt{\cdots}\right.}{\cdots}$

input
```
int((f*x+e)^3*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/120*(24*C*d^4*f^3*x^4+30*B*d^4*f^3*x^3+90*C*d^4*e*f^2*x^3+40*A*d^4*f^3*x
^2+120*B*d^4*e*f^2*x^2+120*C*d^4*e^2*f*x^2+180*A*d^4*e*f^2*x+180*B*d^4*e^2
*f*x+60*C*d^4*e^3*x+360*A*d^4*e^2*f+120*B*d^4*e^3+32*C*d^2*f^3*x^2+45*B*d^
2*f^3*x+135*C*d^2*e*f^2*x+80*A*d^2*f^3+240*B*d^2*e*f^2+240*C*d^2*e^2*f+64*
C*f^3)*(d*x+1)^(1/2)*(d*x-1)/d^6/(-(d*x+1)*(d*x-1))^(1/2)*((-d*x+1)*(d*x+1
))^(1/2)/(-d*x+1)^(1/2)+1/8*(8*A*d^4*e^3+12*A*d^2*e*f^2+12*B*d^2*e^2*f+4*C
*d^2*e^3+3*B*f^3+9*C*e*f^2)/d^4/(d^2)^(1/2)*arctan((d^2)^(1/2)*x/(-d^2*x^2
+1)^(1/2))*((-d*x+1)*(d*x+1))^(1/2)/(-d*x+1)^(1/2)/(d*x+1)^(1/2)
```

3.8.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.84

$$\int \frac{(e+fx)^3 (A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{(24\,Cd^4f^3x^4 + 120\,Bd^4e^3 + 240\,Bd^2ef^2 + 120\,(3\,Ad^4 + 2\,Cd^2)e^2f + 16\,(5\,Ad^2 + 4\,C)f^3 + 30\,(3\,Cd^4ef}{}$$

input
```
integrate((f*x+e)^3*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorith
m="fricas")
```

output
```
-1/120*((24*C*d^4*f^3*x^4 + 120*B*d^4*e^3 + 240*B*d^2*e*f^2 + 120*(3*A*d^4
 + 2*C*d^2)*e^2*f + 16*(5*A*d^2 + 4*C)*f^3 + 30*(3*C*d^4*e*f^2 + B*d^4*f^3
)*x^3 + 8*(15*C*d^4*e^2*f + 15*B*d^4*e*f^2 + (5*A*d^4 + 4*C*d^2)*f^3)*x^2
+ 15*(4*C*d^4*e^3 + 12*B*d^4*e^2*f + 3*B*d^2*f^3 + 3*(4*A*d^4 + 3*C*d^2)*e
*f^2)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 30*(12*B*d^3*e^2*f + 3*B*d*f^3 + 4
*(2*A*d^5 + C*d^3)*e^3 + 3*(4*A*d^3 + 3*C*d)*e*f^2)*arctan((sqrt(d*x + 1)*
sqrt(-d*x + 1) - 1)/(d*x)))/d^6
```

3.8.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 (A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*(C*x**2+B*x+A)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

output
```
Timed out
```

3.8.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.04

$$\int \frac{(e+fx)^3 (A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{\sqrt{-d^2x^2+1}Cf^3x^4}{5\,d^2} + \frac{Ae^3\arcsin{(dx)}}{d} - \frac{\sqrt{-d^2x^2+1}Be^3}{d^2} - \frac{3\sqrt{-d^2x^2+1}Ae^2f}{d^2} - \frac{4\sqrt{-d^2x^2+1}Cf^3x^2}{15\,d^4} - \frac{(3\,Cef^2+Bf^3)\sqrt{-d^2x^2+1}x^3}{4\,d^2} - \frac{(3\,Ce^2f+3\,Bef^2+Af^3)\sqrt{-d^2x^2+1}x^2}{3\,d^2} - \frac{(Ce^3+3\,Be^2f+3\,Aef^2)\sqrt{-d^2x^2+1}x}{2\,d^2} + \frac{(Ce^3+3\,Be^2f+3\,Aef^2)\arcsin{(dx)}}{2\,d^3} - \frac{8\sqrt{-d^2x^2+1}Cf^3}{15\,d^6} - \frac{3\,(3\,Cef^2+Bf^3)\sqrt{-d^2x^2+1}x}{8\,d^4} - \frac{2\,(3\,Ce^2f+3\,Bef^2+Af^3)\sqrt{-d^2x^2+1}}{3\,d^4} + \frac{3\,(3\,Cef^2+Bf^3)\arcsin{(dx)}}{8\,d^5}$$

input
```
integrate((f*x+e)^3*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorith
m="maxima")
```

output
```
-1/5*sqrt(-d^2*x^2 + 1)*C*f^3*x^4/d^2 + A*e^3*arcsin(d*x)/d - sqrt(-d^2*x^
2 + 1)*B*e^3/d^2 - 3*sqrt(-d^2*x^2 + 1)*A*e^2*f/d^2 - 4/15*sqrt(-d^2*x^2 +
 1)*C*f^3*x^2/d^4 - 1/4*(3*C*e*f^2 + B*f^3)*sqrt(-d^2*x^2 + 1)*x^3/d^2 - 1
/3*(3*C*e^2*f + 3*B*e*f^2 + A*f^3)*sqrt(-d^2*x^2 + 1)*x^2/d^2 - 1/2*(C*e^3
 + 3*B*e^2*f + 3*A*e*f^2)*sqrt(-d^2*x^2 + 1)*x/d^2 + 1/2*(C*e^3 + 3*B*e^2*
f + 3*A*e*f^2)*arcsin(d*x)/d^3 - 8/15*sqrt(-d^2*x^2 + 1)*C*f^3/d^6 - 3/8*(
3*C*e*f^2 + B*f^3)*sqrt(-d^2*x^2 + 1)*x/d^4 - 2/3*(3*C*e^2*f + 3*B*e*f^2 +
 A*f^3)*sqrt(-d^2*x^2 + 1)/d^4 + 3/8*(3*C*e*f^2 + B*f^3)*arcsin(d*x)/d^5
```

3.8.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.10

$$\int \frac{(e+fx)^3 (A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx =$$
$$-\frac{(120\,Bd^4e^3+360\,Ad^4e^2f-60\,Cd^3e^3-180\,Bd^3e^2f-180\,Ad^3ef^2+360\,Cd^2e^2f+360\,Bd^2ef^2+120\,A$$

input
```
integrate((f*x+e)^3*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorith
m="giac")
```

output
```
-1/120*((120*B*d^4*e^3 + 360*A*d^4*e^2*f - 60*C*d^3*e^3 - 180*B*d^3*e^2*f
- 180*A*d^3*e*f^2 + 360*C*d^2*e^2*f + 360*B*d^2*e*f^2 + 120*A*d^2*f^3 - 22
5*C*d*e*f^2 - 75*B*d*f^3 + 120*C*f^3 + (60*C*d^3*e^3 + 180*B*d^3*e^2*f + 1
80*A*d^3*e*f^2 - 240*C*d^2*e^2*f - 240*B*d^2*e*f^2 - 80*A*d^2*f^3 + 405*C*
d*e*f^2 + 135*B*d*f^3 - 160*C*f^3 + 2*(60*C*d^2*e^2*f + 60*B*d^2*e*f^2 + 2
0*A*d^2*f^3 - 135*C*d*e*f^2 - 45*B*d*f^3 + 88*C*f^3 + 3*(15*C*d*e*f^2 + 4*
(d*x + 1)*C*f^3 + 5*B*d*f^3 - 16*C*f^3)*(d*x + 1))*(d*x + 1))*(d*x + 1))*s
qrt(d*x + 1)*sqrt(-d*x + 1) - 30*(8*A*d^5*e^3 + 4*C*d^3*e^3 + 12*B*d^3*e^2
*f + 12*A*d^3*e*f^2 + 9*C*d*e*f^2 + 3*B*d*f^3)*arcsin(1/2*sqrt(2)*sqrt(d*x
 + 1)))/d^6
```

3.8.9 Mupad [B] (verification not implemented)

Time = 37.96 (sec) , antiderivative size = 2606, normalized size of antiderivative = 7.66

$$\int \frac{(e+fx)^3 (A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)^3*(A + B*x + C*x^2))/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

output

```
- ((((2048*C*f^3)/3 + 640*C*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^6)/((d*x + 1)
^(1/2) - 1)^6 + (((2048*C*f^3)/3 + 640*C*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^
14)/((d*x + 1)^(1/2) - 1)^14 - (((4096*C*f^3)/3 - 832*C*d^2*e^2*f)*((1 - d
*x)^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8 - (((4096*C*f^3)/3 - 832*C*d^2*e
^2*f)*((1 - d*x)^(1/2) - 1)^12)/((d*x + 1)^(1/2) - 1)^12 + (((12288*C*f^3)
/5 + 768*C*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^10)/((d*x + 1)^(1/2) - 1)^10 +
 (((1 - d*x)^(1/2) - 1)^3*(2*C*d^3*e^3 - (87*C*d*e*f^2)/2))/((d*x + 1)^(1/
2) - 1)^3 - (((1 - d*x)^(1/2) - 1)^17*(2*C*d^3*e^3 - (87*C*d*e*f^2)/2))/((
d*x + 1)^(1/2) - 1)^17 + (((1 - d*x)^(1/2) - 1)^7*(88*C*d^3*e^3 - 42*C*d*e
*f^2))/((d*x + 1)^(1/2) - 1)^7 - (((1 - d*x)^(1/2) - 1)^13*(88*C*d^3*e^3 -
 42*C*d*e*f^2))/((d*x + 1)^(1/2) - 1)^13 + (((1 - d*x)^(1/2) - 1)^5*(40*C*
d^3*e^3 + 426*C*d*e*f^2))/((d*x + 1)^(1/2) - 1)^5 - (((1 - d*x)^(1/2) - 1)
^15*(40*C*d^3*e^3 + 426*C*d*e*f^2))/((d*x + 1)^(1/2) - 1)^15 + (((1 - d*x)
^(1/2) - 1)^9*(52*C*d^3*e^3 - 507*C*d*e*f^2))/((d*x + 1)^(1/2) - 1)^9 - ((
(1 - d*x)^(1/2) - 1)^11*(52*C*d^3*e^3 - 507*C*d*e*f^2))/((d*x + 1)^(1/2) -
 1)^11 - (d*(4*C*d^2*e^3 + 9*C*e*f^2)*((1 - d*x)^(1/2) - 1))/(2*((d*x + 1)
^(1/2) - 1)) + (d*(4*C*d^2*e^3 + 9*C*e*f^2)*((1 - d*x)^(1/2) - 1)^19)/(2*(
(d*x + 1)^(1/2) - 1)^19) + (192*C*d^2*e^2*f*((1 - d*x)^(1/2) - 1)^4)/((d*x
 + 1)^(1/2) - 1)^4 + (192*C*d^2*e^2*f*((1 - d*x)^(1/2) - 1)^16)/((d*x + 1)
^(1/2) - 1)^16)/(d^6 + (10*d^6*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2...
```

3.8. $\int \frac{(e+fx)^3(A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

3.9 $\int \frac{(e+fx)^2(A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

3.9.1 Optimal result

Integrand size = 37, antiderivative size = 228

$$\begin{aligned}
&\int \frac{(e+fx)^2\,(A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx \\
&= \frac{(Ce-4Bf)(e+fx)^2\sqrt{1-d^2x^2}}{12d^2f} - \frac{C(e+fx)^3\sqrt{1-d^2x^2}}{4d^2f} \\
&\quad + \frac{(4(C(d^2e^3-8ef^2)-4f(3Ad^2ef+B(d^2e^2+f^2)))-f(3(3C+4Ad^2)\,f^2-2d^2e(Ce-4Bf))\,x)\,\sqrt{1-}}{24d^4f} \\
&\quad + \frac{(C(4d^2e^2+3f^2)+4d^2(2Bef+A(2d^2e^2+f^2)))\arcsin(dx)}{8d^5}
\end{aligned}$$

output
```
1/8*(C*(4*d^2*e^2+3*f^2)+4*d^2*(2*B*e*f+A*(2*d^2*e^2+f^2)))*arcsin(d*x)/d^
5+1/12*(-4*B*f+C*e)*(f*x+e)^2*(-d^2*x^2+1)^(1/2)/d^2/f-1/4*C*(f*x+e)^3*(-d
^2*x^2+1)^(1/2)/d^2/f+1/24*(4*C*(d^2*e^3-8*e*f^2)-16*f*(3*A*d^2*e*f+B*(d^2
*e^2+f^2))-f*(3*(4*A*d^2+3*C)*f^2-2*d^2*e*(-4*B*f+C*e))*x)*(-d^2*x^2+1)^(1
/2)/d^4/f
```

3.9.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.78

$$\int \frac{(e+fx)^2 (A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$$
$$= \frac{-d\sqrt{1-d^2x^2}(12Ad^2f(4e+fx)+C(12d^2e^2x+16ef(2+d^2x^2)+3f^2x(3+2d^2x^2))+8B(2f^2+d^2(3e^2}{24d^5}$$

input
```
Integrate[((e + f*x)^2*(A + B*x + C*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

output
```
(-(d*Sqrt[1 - d^2*x^2]*(12*A*d^2*f*(4*e + f*x) + C*(12*d^2*e^2*x + 16*e*f*
(2 + d^2*x^2) + 3*f^2*x*(3 + 2*d^2*x^2)) + 8*B*(2*f^2 + d^2*(3*e^2 + 3*e*f
*x + f^2*x^2)))) + 6*(C*(4*d^2*e^2 + 3*f^2) + 4*d^2*(2*B*e*f + A*(2*d^2*e^
2 + f^2)))*ArcTan[(d*x)/(-1 + Sqrt[1 - d^2*x^2])])/(24*d^5)
```

3.9.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.243$, Rules used = {2112, 2185, 25, 27, 687, 25, 27, 676, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2 (A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{dx+1}}dx$$
$$\Big\downarrow \; 2112$$
$$\int \frac{(e+fx)^2 (A+Bx+Cx^2)}{\sqrt{1-d^2x^2}}dx$$
$$\Big\downarrow \; 2185$$
$$-\frac{\int -\frac{f(e+fx)^2((4Ad^2+3C)f-d^2(Ce-4Bf)x)}{\sqrt{1-d^2x^2}}dx}{4d^2f^2}-\frac{C\sqrt{1-d^2x^2}(e+fx)^3}{4d^2f}$$
$$\Big\downarrow \; 25$$
$$\frac{\int \frac{f(e+fx)^2((4Ad^2+3C)f-d^2(Ce-4Bf)x)}{\sqrt{1-d^2x^2}}dx}{4d^2f^2}-\frac{C\sqrt{1-d^2x^2}(e+fx)^3}{4d^2f}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(e+fx)^2\left((4Ad^2+3C)f-d^2(Ce-4Bf)x\right)}{\sqrt{1-d^2x^2}}dx}{4d^2f}-\frac{C\sqrt{1-d^2x^2}(e+fx)^3}{4d^2f}$$

$$\Big\downarrow \; 687$$

$$\frac{\frac{1}{3}\sqrt{1-d^2x^2}(e+fx)^2(Ce-4Bf)-\frac{\int -\frac{d^2(e+fx)\left(f\left(12Aed^2+7Ce+8Bf\right)+\left(3\left(4Ad^2+3C\right)f^2-2d^2e(Ce-4Bf)\right)x\right)}{\sqrt{1-d^2x^2}}dx}{3d^2}}{4d^2f}-\frac{C\sqrt{1-d^2x^2}(e+fx)^3}{4d^2f}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{d^2(e+fx)\left(f\left(12Aed^2+7Ce+8Bf\right)+\left(3\left(4Ad^2+3C\right)f^2-2d^2e(Ce-4Bf)\right)x\right)}{\sqrt{1-d^2x^2}}dx}{3d^2}+\frac{1}{3}\sqrt{1-d^2x^2}(e+fx)^2(Ce-4Bf)}{4d^2f}-\frac{C\sqrt{1-d^2x^2}(e+fx)^3}{4d^2f}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{3}\int \frac{(e+fx)(f(12Aed^2+7Ce+8Bf)+(3(4Ad^2+3C)f^2-2d^2e(Ce-4Bf))x)}{\sqrt{1-d^2x^2}}dx+\frac{1}{3}\sqrt{1-d^2x^2}(e+fx)^2(Ce-4Bf)}{4d^2f}-\frac{C\sqrt{1-d^2x^2}(e+fx)^3}{4d^2f}$$

$$\Big\downarrow \; 676$$

$$\frac{\frac{1}{3}\left(\frac{3f\left(4d^2\left(A\left(2d^2e^2+f^2\right)+2Bef\right)+C\left(4d^2e^2+3f^2\right)\right)\int \frac{1}{\sqrt{1-d^2x^2}}dx}{2d^2}+\frac{1}{2}fx\sqrt{1-d^2x^2}\left(-12Af^2-8Bef-\frac{9Cf^2}{d^2}+2Ce^2\right)+\frac{2\sqrt{1-}\cdots}{\cdots}\right.}{4d^2f}-\frac{C\sqrt{1-d^2x^2}(e+fx)^3}{4d^2f}$$

$$\Big\downarrow \; 223$$

$$\frac{\frac{1}{3}\left(\frac{3f\arcsin(dx)\left(4d^2\left(A\left(2d^2e^2+f^2\right)+2Bef\right)+C\left(4d^2e^2+3f^2\right)\right)}{2d^3}+\frac{1}{2}fx\sqrt{1-d^2x^2}\left(-12Af^2-8Bef-\frac{9Cf^2}{d^2}+2Ce^2\right)+\frac{2\sqrt{1-d^2x}\cdots}{\cdots}\right.}{4d^2f}-\frac{C\sqrt{1-d^2x^2}(e+fx)^3}{4d^2f}$$

input
```
Int[((e + f*x)^2*(A + B*x + C*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

3.9. $\int \frac{(e+fx)^2(A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

output
```
-1/4*(C*(e + f*x)^3*Sqrt[1 - d^2*x^2])/(d^2*f) + (((C*e - 4*B*f)*(e + f*x)
^2*Sqrt[1 - d^2*x^2])/3 + ((2*(C*d^2*e^3 - 4*B*d^2*e^2*f - 8*C*e*f^2 - 12*
A*d^2*e*f^2 - 4*B*f^3)*Sqrt[1 - d^2*x^2])/d^2 + (f*(2*C*e^2 - 8*B*e*f - 12
*A*f^2 - (9*C*f^2)/d^2)*x*Sqrt[1 - d^2*x^2])/2 + (3*f*(C*(4*d^2*e^2 + 3*f^
2) + 4*d^2*(2*B*e*f + A*(2*d^2*e^2 + f^2)))*ArcSin[d*x])/(2*d^3))/3)/(4*d^
2*f)
```

3.9.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2112
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.9.4 Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.12

method	result
risch	$\frac{\left(6C\,d^{2} f^{2} x^{3}+8B\,d^{2} f^{2} x^{2}+16C\,d^{2} e f\,x^{2}+12A\,d^{2} f^{2} x+24B\,d^{2} e f x+12C\,d^{2} e^{2} x+48A\,d^{2} e f+24B\,d^{2} e^{2}+9C\,f^{2} x+16B\,f^{2}+32C e f\right)\sqrt{dx+}}{24d^{4}\sqrt{-\left(dx+1\right)\left(dx-1\right)}\,\sqrt{-dx+1}}$
default	$-\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(6C\sqrt{-d^{2}x^{2}+1}\,\operatorname{csgn}\left(d\right)d^{3} f^{2} x^{3}+8B\sqrt{-d^{2}x^{2}+1}\,\operatorname{csgn}\left(d\right)d^{3} f^{2} x^{2}+16C\sqrt{-d^{2}x^{2}+1}\,\operatorname{csgn}\left(d\right)d^{3} e f\,x^{2}+12A\sqrt{-d^{2}x^{}}\right.}{}$

input
```
int((f*x+e)^2*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/24*(6*C*d^2*f^2*x^3+8*B*d^2*f^2*x^2+16*C*d^2*e*f*x^2+12*A*d^2*f^2*x+24*B
*d^2*e*f*x+12*C*d^2*e^2*x+48*A*d^2*e*f+24*B*d^2*e^2+9*C*f^2*x+16*B*f^2+32*
C*e*f)*(d*x+1)^(1/2)*(d*x-1)/d^4/(-(d*x+1)*(d*x-1))^(1/2)*((-d*x+1)*(d*x+1
))^(1/2)/(-d*x+1)^(1/2)+1/8*(8*A*d^4*e^2+4*A*d^2*f^2+8*B*d^2*e*f+4*C*d^2*e
^2+3*C*f^2)/d^4/(d^2)^(1/2)*arctan((d^2)^(1/2)*x/(-d^2*x^2+1)^(1/2))*((-d*
x+1)*(d*x+1))^(1/2)/(-d*x+1)^(1/2)/(d*x+1)^(1/2)
```

3.9.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.84

$$\int \frac{(e+fx)^2 (A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx =$$
$$-\frac{(6\,Cd^3f^2x^3+24\,Bd^3e^2+16\,Bdf^2+16\,(3\,Ad^3+2\,Cd)ef+8\,(2\,Cd^3ef+Bd^3f^2)x^2+3\,(4\,Cd^3e^2+8\,B$$

input
```
integrate((f*x+e)^2*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorith
m="fricas")
```

output
```
-1/24*((6*C*d^3*f^2*x^3 + 24*B*d^3*e^2 + 16*B*d*f^2 + 16*(3*A*d^3 + 2*C*d)
*e*f + 8*(2*C*d^3*e*f + B*d^3*f^2)*x^2 + 3*(4*C*d^3*e^2 + 8*B*d^3*e*f + (4
*A*d^3 + 3*C*d)*f^2)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 6*(8*B*d^2*e*f + 4*
(2*A*d^4 + C*d^2)*e^2 + (4*A*d^2 + 3*C)*f^2)*arctan((sqrt(d*x + 1)*sqrt(-d
*x + 1) - 1)/(d*x)))/d^5
```

3.9.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2 (A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*(C*x**2+B*x+A)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

output
```
Timed out
```

3.9.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.01

$$\int \frac{(e+fx)^2 (A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{\sqrt{-d^2x^2+1}Cf^2x^3}{4\,d^2} + \frac{Ae^2\arcsin(dx)}{d} - \frac{\sqrt{-d^2x^2+1}Be^2}{d^2} - \frac{2\sqrt{-d^2x^2+1}Aef}{d^2} - \frac{\sqrt{-d^2x^2+1}(2\,Cef+Bf^2)x^2}{3\,d^2} - \frac{\sqrt{-d^2x^2+1}(Ce^2+2\,Bef+Af^2)x}{2\,d^2} - \frac{3\sqrt{-d^2x^2+1}Cf^2x}{8\,d^4} + \frac{(Ce^2+2\,Bef+Af^2)\arcsin(dx)}{2\,d^3} + \frac{3\,Cf^2\arcsin(dx)}{8\,d^5} - \frac{2\sqrt{-d^2x^2+1}(2\,Cef+Bf^2)}{3\,d^4}$$

input
```
integrate((f*x+e)^2*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorith
m="maxima")
```

output
```
-1/4*sqrt(-d^2*x^2 + 1)*C*f^2*x^3/d^2 + A*e^2*arcsin(d*x)/d - sqrt(-d^2*x^
2 + 1)*B*e^2/d^2 - 2*sqrt(-d^2*x^2 + 1)*A*e*f/d^2 - 1/3*sqrt(-d^2*x^2 + 1)
*(2*C*e*f + B*f^2)*x^2/d^2 - 1/2*sqrt(-d^2*x^2 + 1)*(C*e^2 + 2*B*e*f + A*f
^2)*x/d^2 - 3/8*sqrt(-d^2*x^2 + 1)*C*f^2*x/d^4 + 1/2*(C*e^2 + 2*B*e*f + A*
f^2)*arcsin(d*x)/d^3 + 3/8*C*f^2*arcsin(d*x)/d^5 - 2/3*sqrt(-d^2*x^2 + 1)*
(2*C*e*f + B*f^2)/d^4
```

3.9.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.02

$$\int \frac{(e+fx)^2 (A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{(24\,Bd^3e^2+48\,Ad^3ef-12\,Cd^2e^2-24\,Bd^2ef-12\,Ad^2f^2+48\,Cdef+24\,Bdf^2-15\,Cf^2+(12\,Cd^2e^2}{}$$

input
```
integrate((f*x+e)^2*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorith
m="giac")
```

output
```
-1/24*((24*B*d^3*e^2 + 48*A*d^3*e*f - 12*C*d^2*e^2 - 24*B*d^2*e*f - 12*A*d
^2*f^2 + 48*C*d*e*f + 24*B*d*f^2 - 15*C*f^2 + (12*C*d^2*e^2 + 24*B*d^2*e*f
 + 12*A*d^2*f^2 - 32*C*d*e*f - 16*B*d*f^2 + 27*C*f^2 + 2*(8*C*d*e*f + 3*(d
*x + 1)*C*f^2 + 4*B*d*f^2 - 9*C*f^2)*(d*x + 1))*(d*x + 1))*sqrt(d*x + 1)*s
qrt(-d*x + 1) - 6*(8*A*d^4*e^2 + 4*C*d^2*e^2 + 8*B*d^2*e*f + 4*A*d^2*f^2 +
 3*C*f^2)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))/d^5
```

3.9.9 Mupad [B] (verification not implemented)

Time = 35.09 (sec) , antiderivative size = 1732, normalized size of antiderivative = 7.60

$$\int \frac{(e+fx)^2 (A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)^2*(A + B*x + C*x^2))/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

output
```
- ((14*A*f^2*((1 - d*x)^(1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 - (2*A*f^2*(
(1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) - (14*A*f^2*((1 - d*x)^(1/2) -
 1)^5)/((d*x + 1)^(1/2) - 1)^5 + (2*A*f^2*((1 - d*x)^(1/2) - 1)^7)/((d*x +
 1)^(1/2) - 1)^7 + (16*A*d*e*f*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) -
 1)^2 + (32*A*d*e*f*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (16
*A*d*e*f*((1 - d*x)^(1/2) - 1)^6)/((d*x + 1)^(1/2) - 1)^6)/(d^3 + (4*d^3*(
(1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (6*d^3*((1 - d*x)^(1/2)
- 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (4*d^3*((1 - d*x)^(1/2) - 1)^6)/((d*x +
1)^(1/2) - 1)^6 + (d^3*((1 - d*x)^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8) -
 ((((1 - d*x)^(1/2) - 1)^4*(64*B*f^2 + 32*B*d^2*e^2))/((d*x + 1)^(1/2) - 1
)^4 + (((1 - d*x)^(1/2) - 1)^8*(64*B*f^2 + 32*B*d^2*e^2))/((d*x + 1)^(1/2)
 - 1)^8 - (((1 - d*x)^(1/2) - 1)^6*((128*B*f^2)/3 - 48*B*d^2*e^2))/((d*x +
 1)^(1/2) - 1)^6 + (8*B*d^2*e^2*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2)
- 1)^2 + (8*B*d^2*e^2*((1 - d*x)^(1/2) - 1)^10)/((d*x + 1)^(1/2) - 1)^10 +
 (20*B*d*e*f*((1 - d*x)^(1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 + (24*B*d*e*
f*((1 - d*x)^(1/2) - 1)^5)/((d*x + 1)^(1/2) - 1)^5 - (24*B*d*e*f*((1 - d*x
)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7 - (20*B*d*e*f*((1 - d*x)^(1/2) - 1
)^9)/((d*x + 1)^(1/2) - 1)^9 + (4*B*d*e*f*((1 - d*x)^(1/2) - 1)^11)/((d*x
+ 1)^(1/2) - 1)^11 - (4*B*d*e*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) -
1))/(d^4 + (6*d^4*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (1...
```

3.10 $\int \frac{(e+fx)(A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

3.10.1 Optimal result

Integrand size = 35, antiderivative size = 130

$$\int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{C(e+fx)^2\sqrt{1-d^2x^2}}{3d^2f} - \frac{(2(3d^2f(Be+Af)-C(d^2e^2-2f^2))-d^2f(Ce-3Bf)x)\sqrt{1-d^2x^2}}{6d^4f} + \frac{(Ce+2Ad^2e+Bf)\arcsin(dx)}{2d^3}$$

output
```
1/2*(2*A*d^2*e+B*f+C*e)*arcsin(d*x)/d^3-1/3*C*(f*x+e)^2*(-d^2*x^2+1)^(1/2)
/d^2/f-1/6*(6*d^2*f*(A*f+B*e)-2*C*(d^2*e^2-2*f^2)-d^2*f*(-3*B*f+C*e)*x)*(-
d^2*x^2+1)^(1/2)/d^4/f
```

3.10.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.82

$$\int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{1-d^2x^2}(-6Bd^2e-4Cf-6Ad^2f-3Cd^2ex-3Bd^2fx-2Cd^2fx^2)}{6d^4} + \frac{(Ce+2Ad^2e+Bf)\arctan\left(\frac{dx}{-1+\sqrt{1-d^2x^2}}\right)}{d^3}$$

input
```
Integrate[((e + f*x)*(A + B*x + C*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

output
```
(Sqrt[1 - d^2*x^2]*(-6*B*d^2*e - 4*C*f - 6*A*d^2*f - 3*C*d^2*e*x - 3*B*d^2
*f*x - 2*C*d^2*f*x^2))/(6*d^4) + ((C*e + 2*A*d^2*e + B*f)*ArcTan[(d*x)/(-1
 + Sqrt[1 - d^2*x^2])])/d^3
```

3.10.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2112, 2185, 25, 27, 676, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{1-dx}\sqrt{dx+1}}dx$$

$$\Big\downarrow \text{2112}$$

$$\int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{1-d^2x^2}}dx$$

$$\Big\downarrow \text{2185}$$

$$-\frac{\int -\frac{f(e+fx)\left(\left(3Ad^2+2C\right)f-d^2(Ce-3Bf)x\right)}{\sqrt{1-d^2x^2}}dx}{3d^2f^2}-\frac{C\sqrt{1-d^2x^2}(e+fx)^2}{3d^2f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{f(e+fx)\left(\left(3Ad^2+2C\right)f-d^2(Ce-3Bf)x\right)}{\sqrt{1-d^2x^2}}dx}{3d^2f^2}-\frac{C\sqrt{1-d^2x^2}(e+fx)^2}{3d^2f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(e+fx)\left(\left(3Ad^2+2C\right)f-d^2(Ce-3Bf)x\right)}{\sqrt{1-d^2x^2}}dx}{3d^2f}-\frac{C\sqrt{1-d^2x^2}(e+fx)^2}{3d^2f}$$

$$\Big\downarrow \text{676}$$

$$\frac{\frac{3}{2}f\left(2Ad^2e+Bf+Ce\right)\int \frac{1}{\sqrt{1-d^2x^2}}dx-\sqrt{1-d^2x^2}\left(3f(Af+Be)-C\left(e^2-\frac{2f^2}{d^2}\right)\right)+\frac{1}{2}fx\sqrt{1-d^2x^2}(Ce-3Bf)}{3d^2f}-\frac{C\sqrt{1-d^2x^2}(e+fx)^2}{3d^2f}$$

$$\Big\downarrow \text{223}$$

$$\frac{\frac{3f\arcsin(dx)(2Ad^2e+Bf+Ce)}{2d}-\sqrt{1-d^2x^2}\left(3f(Af+Be)-C\left(e^2-\frac{2f^2}{d^2}\right)\right)+\frac{1}{2}fx\sqrt{1-d^2x^2}(Ce-3Bf)}{3d^2f}-\frac{C\sqrt{1-d^2x^2}(e+fx)^2}{3d^2f}$$

input
```
Int[((e + f*x)*(A + B*x + C*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

output
```
-1/3*(C*(e + f*x)^2*Sqrt[1 - d^2*x^2])/(d^2*f) + (-((3*f*(B*e + A*f) - C*(
e^2 - (2*f^2)/d^2))*Sqrt[1 - d^2*x^2]) + (f*(C*e - 3*B*f)*x*Sqrt[1 - d^2*x
^2])/2 + (3*f*(C*e + 2*A*d^2*e + B*f)*ArcSin[d*x])/(2*d))/(3*d^2*f)
```

3.10.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 2112
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.10.4 Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.33

method	result
risch	$\frac{(2C\,d^2 f\,x^2+3B\,d^2 fx+3C\,d^2 ex+6A\,d^2 f+6B\,d^2 e+4fC)\sqrt{dx+1}\,(dx-1)\sqrt{(-dx+1)(dx+1)}}{6d^4\sqrt{-(dx+1)(dx-1)}\,\sqrt{-dx+1}}+\frac{(2A\,d^2e+Bf+Ce)\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+}}\right.}{2d^2\sqrt{d^2}\,\sqrt{-dx+1}\,\sqrt{}}$
default	$-\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\left(2C\sqrt{-d^2x^2+1}\,\operatorname{csgn}(d)d^2 f\,x^2+3B\sqrt{-d^2x^2+1}\,\operatorname{csgn}(d)d^2 fx+3C\sqrt{-d^2x^2+1}\,\operatorname{csgn}(d)d^2 ex+6A\sqrt{-d^2x^2+1}\,\operatorname{csg}\right.}{}$

input
```
int((f*x+e)*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
1/6*(2*C*d^2*f*x^2+3*B*d^2*f*x+3*C*d^2*e*x+6*A*d^2*f+6*B*d^2*e+4*C*f)*(d*x
+1)^(1/2)*(d*x-1)/d^4/(-(d*x+1)*(d*x-1))^(1/2)*((-d*x+1)*(d*x+1))^(1/2)/(-
d*x+1)^(1/2)+1/2*(2*A*d^2*e+B*f+C*e)/d^2/(d^2)^(1/2)*arctan((d^2)^(1/2)*x/
(-d^2*x^2+1)^(1/2))*((-d*x+1)*(d*x+1))^(1/2)/(-d*x+1)^(1/2)/(d*x+1)^(1/2)
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.88

$$\int \frac{(e+fx)\,(A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx =$$
$$-\frac{(2\,Cd^2fx^2+6\,Bd^2e+2\,(3\,Ad^2+2\,C)f+3\,(Cd^2e+Bd^2f)x)\sqrt{dx+1}\sqrt{-dx+1}+6\,(Bdf+(2\,Ad^3+}{6\,d^4}$$

input
```
integrate((f*x+e)*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm=
"fricas")
```

output
```
-1/6*((2*C*d^2*f*x^2 + 6*B*d^2*e + 2*(3*A*d^2 + 2*C)*f + 3*(C*d^2*e + B*d^
2*f)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 6*(B*d*f + (2*A*d^3 + C*d)*e)*arcta
n((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/d^4
```

3.10.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)(A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*(C*x**2+B*x+A)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

output
```
Timed out
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.01

$$\begin{aligned}\int \frac{(e+fx)(A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = & -\frac{\sqrt{-d^2x^2+1}Cfx^2}{3\,d^2}+\frac{Ae\arcsin(dx)}{d}-\frac{\sqrt{-d^2x^2+1}Be}{d^2} \\ & -\frac{\sqrt{-d^2x^2+1}Af}{d^2}-\frac{\sqrt{-d^2x^2+1}(Ce+Bf)x}{2\,d^2} \\ & +\frac{(Ce+Bf)\arcsin(dx)}{2\,d^3}-\frac{2\,\sqrt{-d^2x^2+1}Cf}{3\,d^4}\end{aligned}$$

input
```
integrate((f*x+e)*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm=
"maxima")
```

output
```
-1/3*sqrt(-d^2*x^2 + 1)*C*f*x^2/d^2 + A*e*arcsin(d*x)/d - sqrt(-d^2*x^2 +
1)*B*e/d^2 - sqrt(-d^2*x^2 + 1)*A*f/d^2 - 1/2*sqrt(-d^2*x^2 + 1)*(C*e + B*
f)*x/d^2 + 1/2*(C*e + B*f)*arcsin(d*x)/d^3 - 2/3*sqrt(-d^2*x^2 + 1)*C*f/d^
4
```

3.10.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.86

$$\int \frac{(e+fx)(A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{(6\,Bd^2e+6\,Ad^2f-3\,Cde-3\,Bdf+(3\,Cde+2\,(dx+1)Cf+3\,Bdf-4\,Cf)(dx+1)+6\,Cf)\sqrt{dx+}}{6\,d^4}$$

input
```
integrate((f*x+e)*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm=
"giac")
```

output
```
-1/6*((6*B*d^2*e + 6*A*d^2*f - 3*C*d*e - 3*B*d*f + (3*C*d*e + 2*(d*x + 1)*
C*f + 3*B*d*f - 4*C*f)*(d*x + 1) + 6*C*f)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 6
*(2*A*d^3*e + C*d*e + B*d*f)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))/d^4
```

3.10.9 Mupad [B] (verification not implemented)

Time = 14.68 (sec) , antiderivative size = 492, normalized size of antiderivative = 3.78

$$\int \frac{(e+fx)(A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$$

$$= \frac{\frac{2\,B\,f\,\left(\sqrt{1-d\,x}-1\right)}{\sqrt{d\,x+1}-1}-\frac{14\,B\,f\,{\left(\sqrt{1-d\,x}-1\right)}^3}{{\left(\sqrt{d\,x+1}-1\right)}^3}+\frac{14\,B\,f\,{\left(\sqrt{1-d\,x}-1\right)}^5}{{\left(\sqrt{d\,x+1}-1\right)}^5}-\frac{2\,B\,f\,{\left(\sqrt{1-d\,x}-1\right)}^7}{{\left(\sqrt{d\,x+1}-1\right)}^7}}{d^3\,{\left(\frac{{\left(\sqrt{1-d\,x}-1\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+1\right)}^4} - \frac{\sqrt{1-d\,x}\,\left(\frac{2\,C\,f}{3\,d^4}+\frac{2\,C\,f\,x}{3\,d^3}+\frac{C\,f\,x^3}{3\,d}+\frac{C\,f\,x^2}{3\,d^2}\right)}{\sqrt{d\,x+1}} + \frac{\frac{2\,C\,e\,\left(\sqrt{1-d\,x}-1\right)}{\sqrt{d\,x+1}-1}-\frac{14\,C\,e\,{\left(\sqrt{1-d\,x}-1\right)}^3}{{\left(\sqrt{d\,x+1}-1\right)}^3}+\frac{14\,C\,e\,{\left(\sqrt{1-d\,x}-1\right)}^5}{{\left(\sqrt{d\,x+1}-1\right)}^5}-\frac{2\,C\,e\,{\left(\sqrt{1-d\,x}-1\right)}^7}{{\left(\sqrt{d\,x+1}-1\right)}^7}}{d^3\,{\left(\frac{{\left(\sqrt{1-d\,x}-1\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+1\right)}^4} - \frac{\left(\frac{A\,f}{d^2}+\frac{A\,f\,x}{d}\right)\,\sqrt{1-d\,x}}{\sqrt{d\,x+1}} - \frac{\left(\frac{B\,e}{d^2}+\frac{B\,e\,x}{d}\right)\,\sqrt{1-d\,x}}{\sqrt{d\,x+1}} - \frac{4\,A\,e\,\mathrm{atan}\left(\frac{d\,\left(\sqrt{1-d\,x}-1\right)}{\left(\sqrt{d\,x+1}-1\right)\,\sqrt{d^2}}\right)}{\sqrt{d^2}} - \frac{2\,B\,f\,\mathrm{atan}\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)}{d^3} - \frac{2\,C\,e\,\mathrm{atan}\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)}{d^3}$$

input
```
int(((e + f*x)*(A + B*x + C*x^2))/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

output
```
((2*B*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) - (14*B*f*((1 - d*x)^
(1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 + (14*B*f*((1 - d*x)^(1/2) - 1)^5)/(
(d*x + 1)^(1/2) - 1)^5 - (2*B*f*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2)
- 1)^7)/(d^3*(((1 - d*x)^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 + 1)^4) - ((
1 - d*x)^(1/2)*((2*C*f)/(3*d^4) + (2*C*f*x)/(3*d^3) + (C*f*x^3)/(3*d) + (C
*f*x^2)/(3*d^2)))/(d*x + 1)^(1/2) + ((2*C*e*((1 - d*x)^(1/2) - 1))/((d*x +
 1)^(1/2) - 1) - (14*C*e*((1 - d*x)^(1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3
+ (14*C*e*((1 - d*x)^(1/2) - 1)^5)/((d*x + 1)^(1/2) - 1)^5 - (2*C*e*((1 -
d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7)/(d^3*(((1 - d*x)^(1/2) - 1)^2/
((d*x + 1)^(1/2) - 1)^2 + 1)^4) - (((A*f)/d^2 + (A*f*x)/d)*(1 - d*x)^(1/2)
)/(d*x + 1)^(1/2) - (((B*e)/d^2 + (B*e*x)/d)*(1 - d*x)^(1/2))/(d*x + 1)^(1
/2) - (4*A*e*atan((d*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2) - 1)*(d^2)^(
1/2))))/(d^2)^(1/2) - (2*B*f*atan(((1 - d*x)^(1/2) - 1)/((d*x + 1)^(1/2) -
 1)))/d^3 - (2*C*e*atan(((1 - d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/d^3
```

3.10. $\int \frac{(e+fx)(A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

3.11 $\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

3.11.1 Optimal result

Integrand size = 30, antiderivative size = 63

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx=-\frac{B\sqrt{1-d^2x^2}}{d^2}-\frac{Cx\sqrt{1-d^2x^2}}{2d^2}+\frac{(C+2Ad^2)\arcsin(dx)}{2d^3}$$

output
```
1/2*(2*A*d^2+C)*arcsin(d*x)/d^3-B*(-d^2*x^2+1)^(1/2)/d^2-1/2*C*x*(-d^2*x^2
+1)^(1/2)/d^2
```

3.11.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx=\frac{(-2B-Cx)\sqrt{1-d^2x^2}}{2d^2}+\frac{(C+2Ad^2)\arctan\left(\frac{dx}{-1+\sqrt{1-d^2x^2}}\right)}{d^3}$$

input
```
Integrate[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

output
```
((-2*B - C*x)*Sqrt[1 - d^2*x^2])/(2*d^2) + ((C + 2*A*d^2)*ArcTan[(d*x)/(-1
 + Sqrt[1 - d^2*x^2])])/d^3
```

3.11.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1188, 2346, 25, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{dx+1}}dx$$

$$\Big\downarrow \; 1188$$

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-d^2x^2}}dx$$

$$\Big\downarrow \; 2346$$

$$-\frac{\int -\frac{2Ad^2+2Bxd^2+C}{\sqrt{1-d^2x^2}}dx}{2d^2}-\frac{Cx\sqrt{1-d^2x^2}}{2d^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{2Ad^2+2Bxd^2+C}{\sqrt{1-d^2x^2}}dx}{2d^2}-\frac{Cx\sqrt{1-d^2x^2}}{2d^2}$$

$$\Big\downarrow \; 455$$

$$\frac{(2Ad^2+C)\int \frac{1}{\sqrt{1-d^2x^2}}dx-2B\sqrt{1-d^2x^2}}{2d^2}-\frac{Cx\sqrt{1-d^2x^2}}{2d^2}$$

$$\Big\downarrow \; 223$$

$$\frac{\frac{(2Ad^2+C)\arcsin(dx)}{d}-2B\sqrt{1-d^2x^2}}{2d^2}-\frac{Cx\sqrt{1-d^2x^2}}{2d^2}$$

input
```
Int[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

output
```
-1/2*(C*x*Sqrt[1 - d^2*x^2])/d^2 + (-2*B*Sqrt[1 - d^2*x^2] + ((C + 2*A*d^2
)*ArcSin[d*x])/d)/(2*d^2)
```

3.11.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 1188
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2
)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m, n] && EqQ[e*f
 + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

3.11.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.63 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.86

method	result
default	$\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(2A\arctan\left(\frac{\operatorname{csgn}(d)dx}{\sqrt{-d^2x^2+1}}\right)d^2-C\ \operatorname{csgn}(d)d\sqrt{-d^2x^2+1}\,x-2B\sqrt{-d^2x^2+1}\ \operatorname{csgn}(d)d+C\arctan\left(\frac{\operatorname{csgn}(d)dx}{\sqrt{-d^2x^2+1}}\right)\right)\operatorname{csgn}(}{2d^3\sqrt{-d^2x^2+1}}$
risch	$\frac{(Cx+2B)\sqrt{dx+1}\,(dx-1)\sqrt{(-dx+1)(dx+1)}}{2d^2\sqrt{-(dx+1)(dx-1)}\,\sqrt{-dx+1}}+\frac{(2A\,d^2+C)\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+1}}\right)\sqrt{(-dx+1)(dx+1)}}{2d^2\sqrt{d^2}\,\sqrt{-dx+1}\,\sqrt{dx+1}}$

input
```
int((C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-d*x+1)^(1/2)*(d*x+1)^(1/2)/d^3*(2*A*arctan(csgn(d)*d*x/(-d^2*x^2+1)^
(1/2))*d^2-C*csgn(d)*d*(-d^2*x^2+1)^(1/2)*x-2*B*(-d^2*x^2+1)^(1/2)*csgn(d)
*d+C*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2)))/(-d^2*x^2+1)^(1/2)*csgn(d)
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{(Cdx+2\,Bd)\sqrt{dx+1}\sqrt{-dx+1}+2\,(2\,Ad^2+C)\arctan\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{dx}\right)}{2\,d^3}$$

input
```
integrate((C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas"
)
```

output
```
-1/2*((C*d*x + 2*B*d)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(2*A*d^2 + C)*arcta
n((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/d^3
```

3.11.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

output
```
Timed out
```

3.11.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{A\arcsin(dx)}{d} - \frac{\sqrt{-d^2x^2+1}Cx}{2\,d^2} - \frac{\sqrt{-d^2x^2+1}B}{d^2} + \frac{C\arcsin(dx)}{2\,d^3}$$

input
```
integrate((C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima"
)
```

output
```
A*arcsin(d*x)/d - 1/2*sqrt(-d^2*x^2 + 1)*C*x/d^2 - sqrt(-d^2*x^2 + 1)*B/d^
2 + 1/2*C*arcsin(d*x)/d^3
```

3.11.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{((dx+1)C+2\,Bd-C)\sqrt{dx+1}\sqrt{-dx+1}-2\,(2\,Ad^2+C)\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{dx+1}\right)}{2\,d^3}$$

input
```
integrate((C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

output
```
-1/2*(((d*x + 1)*C + 2*B*d - C)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 2*(2*A*d^2
+ C)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))/d^3
```

3.11.9 Mupad [B] (verification not implemented)

Time = 8.15 (sec) , antiderivative size = 232, normalized size of antiderivative = 3.68

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{\frac{14\,C\,\left(\sqrt{1-d\,x}-1\right)^3}{\left(\sqrt{d\,x+1}-1\right)^3}-\frac{14\,C\,\left(\sqrt{1-d\,x}-1\right)^5}{\left(\sqrt{d\,x+1}-1\right)^5}+\frac{2\,C\,\left(\sqrt{1-d\,x}-1\right)^7}{\left(\sqrt{d\,x+1}-1\right)^7}-\frac{2\,C\,\left(\sqrt{1-d\,x}-1\right)}{\sqrt{d\,x+1}-1}}{d^3\,\left(\frac{\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+1\right)^4} - \frac{4\,A\,\mathrm{atan}\left(\frac{d\,\left(\sqrt{1-d\,x}-1\right)}{\left(\sqrt{d\,x+1}-1\right)\,\sqrt{d^2}}\right)}{\sqrt{d^2}} - \frac{2\,C\,\mathrm{atan}\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)}{d^3} - \frac{\sqrt{1-d\,x}\,\left(\frac{B}{d^2}+\frac{B\,x}{d}\right)}{\sqrt{d\,x+1}}$$

input
```
int((A + B*x + C*x^2)/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

output
```
- ((14*C*((1 - d*x)^(1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 - (14*C*((1 - d*
x)^(1/2) - 1)^5)/((d*x + 1)^(1/2) - 1)^5 + (2*C*((1 - d*x)^(1/2) - 1)^7)/(
(d*x + 1)^(1/2) - 1)^7 - (2*C*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)
)/(d^3*(((1 - d*x)^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 + 1)^4) - (4*A*ata
n((d*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2) - 1)*(d^2)^(1/2))))/(d^2)^(1
/2) - (2*C*atan(((1 - d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/d^3 - ((1 -
d*x)^(1/2)*(B/d^2 + (B*x)/d))/(d*x + 1)^(1/2)
```

3.11. $\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

3.12 $\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)}\,dx$

3.12.1 Optimal result

Integrand size = 37, antiderivative size = 122

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)}\,dx = -\frac{C\sqrt{1-d^2x^2}}{d^2f}-\frac{(Ce-Bf)\arcsin(dx)}{df^2}+\frac{(Ce^2-Bef+Af^2)\arctan\left(\frac{f+d^2ex}{\sqrt{d^2e^2-f^2}\sqrt{1-d^2x^2}}\right)}{f^2\sqrt{d^2e^2-f^2}}$$

output
```
-(-B*f+C*e)*arcsin(d*x)/d/f^2+(A*f^2-B*e*f+C*e^2)*arctan((d^2*e*x+f)/(d^2*
e^2-f^2)^(1/2)/(-d^2*x^2+1)^(1/2))/f^2/(d^2*e^2-f^2)^(1/2)-C*(-d^2*x^2+1)^
(1/2)/d^2/f
```

3.12.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.28

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)}\,dx = \frac{-\frac{Cf\sqrt{1-d^2x^2}}{d^2}+\frac{2(-Ce+Bf)\arctan\left(\frac{dx}{-1+\sqrt{1-d^2x^2}}\right)}{d}-\frac{2\sqrt{d^2e^2-f^2}(Ce^2+f(-Be+Af))\arctan\left(\frac{\sqrt{d^2e^2-f^2}x}{e+fx-e\sqrt{1-d^2x^2}}\right)}{(de-f)(de+f)}}{f^2}$$

input
```
Integrate[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)),x]
```

output
```
(-((C*f*Sqrt[1 - d^2*x^2])/d^2) + (2*(-(C*e) + B*f)*ArcTan[(d*x)/(-1 + Sqr
t[1 - d^2*x^2])])/d - (2*Sqrt[d^2*e^2 - f^2]*(C*e^2 + f*(-(B*e) + A*f))*Ar
cTan[(Sqrt[d^2*e^2 - f^2]*x)/(e + f*x - e*Sqrt[1 - d^2*x^2])])/((d*e - f)*
(d*e + f)))/f^2
```

3.12.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {2112, 2185, 25, 27, 719, 223, 488, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{dx+1}(e+fx)}dx$$

$$\Big\downarrow \text{2112}$$

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-d^2x^2}(e+fx)}dx$$

$$\Big\downarrow \text{2185}$$

$$-\frac{\int -\frac{d^2f(Af-(Ce-Bf)x)}{(e+fx)\sqrt{1-d^2x^2}}dx}{d^2f^2}-\frac{C\sqrt{1-d^2x^2}}{d^2f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{d^2f(Af-(Ce-Bf)x)}{(e+fx)\sqrt{1-d^2x^2}}dx}{d^2f^2}-\frac{C\sqrt{1-d^2x^2}}{d^2f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{Af-(Ce-Bf)x}{(e+fx)\sqrt{1-d^2x^2}}dx}{f}-\frac{C\sqrt{1-d^2x^2}}{d^2f}$$

$$\Big\downarrow \text{719}$$

$$\frac{\frac{(Af^2-Bef+Ce^2)\int \frac{1}{(e+fx)\sqrt{1-d^2x^2}}dx}{f}-\frac{(Ce-Bf)\int \frac{1}{\sqrt{1-d^2x^2}}dx}{f}}{f}-\frac{C\sqrt{1-d^2x^2}}{d^2f}$$

$$\Big\downarrow \text{223}$$

$$\frac{\frac{(Af^2-Bef+Ce^2)\int \frac{1}{(e+fx)\sqrt{1-d^2x^2}}dx}{f}-\frac{\arcsin(dx)(Ce-Bf)}{df}}{f}-\frac{C\sqrt{1-d^2x^2}}{d^2f}$$

$$\Big\downarrow \textbf{488}$$

$$\frac{-\frac{(Af^2-Bef+Ce^2)\int \frac{1}{-d^2e^2+f^2-\frac{\left(exd^2+f\right)^2}{1-d^2x^2}}d\frac{exd^2+f}{\sqrt{1-d^2x^2}}}{f}-\frac{\arcsin(dx)(Ce-Bf)}{df}}{f}-\frac{C\sqrt{1-d^2x^2}}{d^2f}$$

$$\Big\downarrow \textbf{217}$$

$$\frac{\frac{(Af^2-Bef+Ce^2)\arctan\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\sqrt{d^2e^2-f^2}}\right)}{f\sqrt{d^2e^2-f^2}}-\frac{\arcsin(dx)(Ce-Bf)}{df}}{f}-\frac{C\sqrt{1-d^2x^2}}{d^2f}$$

input
```
Int[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)),x]
```

output
```
-((C*Sqrt[1 - d^2*x^2])/(d^2*f)) + (-(((C*e - B*f)*ArcSin[d*x])/(d*f)) + (
(C*e^2 - B*e*f + A*f^2)*ArcTan[(f + d^2*e*x)/(Sqrt[d^2*e^2 - f^2]*Sqrt[1 -
 d^2*x^2])])/(f*Sqrt[d^2*e^2 - f^2]))/f
```

3.12.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2112
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.12.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. 2(114) = 228.

Time = 1.66 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.37

method	result
risch	$\frac{C\sqrt{dx+1}\,(dx-1)\sqrt{(-dx+1)(dx+1)}}{f\,d^2\sqrt{-(dx+1)(dx-1)}\,\sqrt{-dx+1}}+\frac{\left(\frac{(Bf-Ce)\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+1}}\right)}{f\sqrt{d^2}}-\frac{\left(Af^2-Bef+Ce^2\right)\ln\left(\frac{-\frac{2\left(d^2e^2-f^2\right)}{f^2}+\frac{2d^2e\left(x+\frac{e}{f}\right)}{f}+2\sqrt{-\frac{d^2 \cdots}{}}}{}\right)}{f^2\sqrt{-\frac{d^2e^2-f \cdots}{f^2}}}\right)}{f\sqrt{-dx+1}\,\sqrt{dx+1}}$
default	$\frac{\left(-A\,\mathrm{csgn}(d)\ln\left(\frac{2d^2ex+2\sqrt{-d^2x^2+1}\,\sqrt{-\frac{d^2e^2-f^2}{f^2}}\,f+2f}{fx+e}\right)d^2f^2+B\,\mathrm{csgn}(d)\ln\left(\frac{2d^2ex+2\sqrt{-d^2x^2+1}\,\sqrt{-\frac{d^2e^2-f^2}{f^2}}\,f+2f}{fx+e}\right)d^2ef-C\,\mathrm{csgn} \cdots}{}$

input
```
int((C*x^2+B*x+A)/(f*x+e)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
C/f/d^2*(d*x+1)^(1/2)*(d*x-1)/(-(d*x+1)*(d*x-1))^(1/2)*((-d*x+1)*(d*x+1))^
(1/2)/(-d*x+1)^(1/2)+1/f*((B*f-C*e)/f/(d^2)^(1/2)*arctan((d^2)^(1/2)*x/(-d
^2*x^2+1)^(1/2))-(A*f^2-B*e*f+C*e^2)/f^2/(-(d^2*e^2-f^2)/f^2)^(1/2)*ln((-2
*(d^2*e^2-f^2)/f^2+2/f*d^2*e*(x+e/f)+2*(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*(x
+e/f)^2+2/f*d^2*e*(x+e/f)-(d^2*e^2-f^2)/f^2)^(1/2))/(x+e/f)))*((-d*x+1)*(d
*x+1))^(1/2)/(-d*x+1)^(1/2)/(d*x+1)^(1/2)
```

3.12.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(114) = 228$.

Time = 4.22 (sec) , antiderivative size = 493, normalized size of antiderivative = 4.04

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)}\,dx$$
$$= \left[-\frac{(Cd^2e^2-Bd^2ef+Ad^2f^2)\sqrt{-d^2e^2+f^2}\log\left(\frac{d^2efx+f^2-\sqrt{-d^2e^2+f^2}(d^2ex+f)-\left(\sqrt{-d^2e^2+f^2}\sqrt{-dx+1}f+(d^2e^2-f^2)\cdots\right.}{fx+e}\right.}{\cdots}\right.$$

input
```
integrate((C*x^2+B*x+A)/(f*x+e)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm=
"fricas")
```

output
```
[-((C*d^2*e^2 - B*d^2*e*f + A*d^2*f^2)*sqrt(-d^2*e^2 + f^2)*log((d^2*e*f*x
 + f^2 - sqrt(-d^2*e^2 + f^2)*(d^2*e*x + f) - (sqrt(-d^2*e^2 + f^2)*sqrt(-
d*x + 1)*f + (d^2*e^2 - f^2)*sqrt(-d*x + 1))*sqrt(d*x + 1))/(f*x + e)) + (
C*d^2*e^2*f - C*f^3)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 2*(C*d^3*e^3 - B*d^3*e
^2*f - C*d*e*f^2 + B*d*f^3)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x
)))/(d^4*e^2*f^2 - d^2*f^4), (2*(C*d^2*e^2 - B*d^2*e*f + A*d^2*f^2)*sqrt(d
^2*e^2 - f^2)*arctan(-(sqrt(d^2*e^2 - f^2)*sqrt(d*x + 1)*sqrt(-d*x + 1)*e
- sqrt(d^2*e^2 - f^2)*(f*x + e))/((d^2*e^2 - f^2)*x)) - (C*d^2*e^2*f - C*f
^3)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(C*d^3*e^3 - B*d^3*e^2*f - C*d*e*f^2
+ B*d*f^3)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/(d^4*e^2*f^2
- d^2*f^4)]
```

3.12.6 Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)}\,dx = \int \frac{A+Bx+Cx^2}{(e+fx)\sqrt{-dx+1}\sqrt{dx+1}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(f*x+e)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/((e + f*x)*sqrt(-d*x + 1)*sqrt(d*x + 1)), x)
```

3.12.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+B*x+A)/(f*x+e)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm=
"maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.12.8 Giac [F(-2)]

Exception generated.

$$\int \frac{A + Bx + Cx^2}{\sqrt{1 - dx}\sqrt{1 + dx}(e + fx)}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((C*x^2+B*x+A)/(f*x+e)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

3.12.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 5803, normalized size of antiderivative = 47.57

$$\int \frac{A + Bx + Cx^2}{\sqrt{1 - dx}\sqrt{1 + dx}(e + fx)}\, dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/((e + f*x)*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

output

```
(4*C*e*atan((37748736*C^5*d^4*e^10*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2
) - 1)*(37748736*C^5*d^4*e^10 + 67108864*C^5*e^6*f^4 - 100663296*C^5*d^2*e
^8*f^2)) + (67108864*C^5*e^6*f^4*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2)
- 1)*(37748736*C^5*d^4*e^10 + 67108864*C^5*e^6*f^4 - 100663296*C^5*d^2*e^8
*f^2)) - (100663296*C^5*d^2*e^8*f^2*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/
2) - 1)*(37748736*C^5*d^4*e^10 + 67108864*C^5*e^6*f^4 - 100663296*C^5*d^2*
e^8*f^2))))/(d*f^2) - (4*B*atan((67108864*B^5*e*f^4*((1 - d*x)^(1/2) - 1))
/(((d*x + 1)^(1/2) - 1)*(67108864*B^5*e*f^4 + 37748736*B^5*d^4*e^5 - 10066
3296*B^5*d^2*e^3*f^2)) + (37748736*B^5*d^4*e^5*((1 - d*x)^(1/2) - 1))/(((d
*x + 1)^(1/2) - 1)*(67108864*B^5*e*f^4 + 37748736*B^5*d^4*e^5 - 100663296*
B^5*d^2*e^3*f^2)) - (100663296*B^5*d^2*e^3*f^2*((1 - d*x)^(1/2) - 1))/(((d
*x + 1)^(1/2) - 1)*(67108864*B^5*e*f^4 + 37748736*B^5*d^4*e^5 - 100663296*
B^5*d^2*e^3*f^2))))/(d*f) - (8*C*((1 - d*x)^(1/2) - 1)^2)/(f*((d*x + 1)^(1
/2) - 1)^2*(d^2 + (2*d^2*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2
+ (d^2*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4)) - (A*atan((f^2*1
i - d^2*e^2*1i - (f^2*((1 - d*x)^(1/2) - 1)^2*1i)/((d*x + 1)^(1/2) - 1)^2
+ (d^2*e^2*((1 - d*x)^(1/2) - 1)^2*1i)/((d*x + 1)^(1/2) - 1)^2)/(f*(f + d*
e)^(1/2)*(f - d*e)^(1/2) - (f*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(1/2)*(f -
 d*e)^(1/2))/((d*x + 1)^(1/2) - 1)^2 + (2*d*e*((1 - d*x)^(1/2) - 1)*(f + d
*e)^(1/2)*(f - d*e)^(1/2))/((d*x + 1)^(1/2) - 1)))*2i)/((f + d*e)^(1/2)...
```

3.12. $\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)}\,dx$

3.13 $\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^2}\,dx$

3.13.1 Optimal result

Integrand size = 37, antiderivative size = 163

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^2}\,dx = \frac{(Ce^2-Bef+Af^2)\sqrt{1-d^2x^2}}{f(d^2e^2-f^2)(e+fx)} + \frac{C\arcsin(dx)}{df^2} - \frac{(Cd^2e^3-2Cef^2-Ad^2ef^2+Bf^3)\arctan\left(\frac{f+d^2ex}{\sqrt{d^2e^2-f^2}\sqrt{1-d^2x^2}}\right)}{f^2(d^2e^2-f^2)^{3/2}}$$

output
```
C*arcsin(d*x)/d/f^2-(-A*d^2*e*f^2+C*d^2*e^3+B*f^3-2*C*e*f^2)*arctan((d^2*e
*x+f)/(d^2*e^2-f^2)^(1/2)/(-d^2*x^2+1)^(1/2))/f^2/(d^2*e^2-f^2)^(3/2)+(A*f
^2-B*e*f+C*e^2)*(-d^2*x^2+1)^(1/2)/f/(d^2*e^2-f^2)/(f*x+e)
```

3.13.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^2}\,dx = \frac{\frac{f(Ce^2+f(-Be+Af))\sqrt{1-d^2x^2}}{(de-f)(de+f)(e+fx)} + \frac{2C\arctan\left(\frac{dx}{-1+\sqrt{1-d^2x^2}}\right)}{d} + \frac{2\sqrt{d^2e^2-f^2}(Cd^2e^3-2Cef^2-Ad^2ef^2+Bf^3)\arctan\left(\frac{\sqrt{d^2e^2-f^2}x}{e+fx-e\sqrt{1-d^2x^2}}\right)}{(-de+f)^2(de+f)^2}}{f^2}$$

input
```
Integrate[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^2),x]
```

output
```
((f*(C*e^2 + f*(-(B*e) + A*f))*Sqrt[1 - d^2*x^2])/((d*e - f)*(d*e + f)*(e
+ f*x)) + (2*C*ArcTan[(d*x)/(-1 + Sqrt[1 - d^2*x^2])])/d + (2*Sqrt[d^2*e^2
 - f^2]*(C*d^2*e^3 - 2*C*e*f^2 - A*d^2*e*f^2 + B*f^3)*ArcTan[(Sqrt[d^2*e^2
 - f^2]*x)/(e + f*x - e*Sqrt[1 - d^2*x^2])])/((-(d*e) + f)^2*(d*e + f)^2))
/f^2
```

3.13.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {2112, 2182, 719, 223, 488, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{dx+1}(e+fx)^2}dx$$

$$\Big\downarrow \text{2112}$$

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-d^2x^2}(e+fx)^2}dx$$

$$\Big\downarrow \text{2182}$$

$$\frac{\int \frac{Aed^2+Ce-Bf+C\left(\frac{d^2e^2}{f}-f\right)x}{(e+fx)\sqrt{1-d^2x^2}}dx}{d^2e^2-f^2}+\frac{\sqrt{1-d^2x^2}\left(Af^2-Bef+Ce^2\right)}{f\left(d^2e^2-f^2\right)(e+fx)}$$

$$\Big\downarrow \text{719}$$

$$\frac{\left(Ad^2e-Bf-\frac{Cd^2e^3}{f^2}+2Ce\right)\int \frac{1}{(e+fx)\sqrt{1-d^2x^2}}dx+\frac{C(de-f)(de+f)\int \frac{1}{\sqrt{1-d^2x^2}}dx}{f^2}}{d^2e^2-f^2}+\frac{\sqrt{1-d^2x^2}\left(Af^2-Bef+Ce^2\right)}{f\left(d^2e^2-f^2\right)(e+fx)}$$

$$\Big\downarrow \text{223}$$

$$\frac{\left(Ad^2e-Bf-\frac{Cd^2e^3}{f^2}+2Ce\right)\int \frac{1}{(e+fx)\sqrt{1-d^2x^2}}dx+\frac{C\arcsin(dx)(de-f)(de+f)}{df^2}}{d^2e^2-f^2}+\frac{\sqrt{1-d^2x^2}\left(Af^2-Bef+Ce^2\right)}{f\left(d^2e^2-f^2\right)(e+fx)}$$

$$\Big\downarrow \text{488}$$

$$\frac{\frac{C\arcsin(dx)(de-f)(de+f)}{df^2}-\left(Ad^2e-Bf-\frac{Cd^2e^3}{f^2}+2Ce\right)\int\frac{1}{-d^2e^2+f^2-\frac{\left(exd^2+f\right)^2}{1-d^2x^2}}d\frac{exd^2+f}{\sqrt{1-d^2x^2}}}{d^2e^2-f^2}+\frac{\sqrt{1-d^2x^2}\left(Af^2-Bef+Ce^2\right)}{f\left(d^2e^2-f^2\right)(e+fx)}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{\arctan\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\sqrt{d^2e^2-f^2}}\right)\left(Ad^2e-Bf-\frac{Cd^2e^3}{f^2}+2Ce\right)}{\sqrt{d^2e^2-f^2}}+\frac{C\arcsin(dx)(de-f)(de+f)}{df^2}}{d^2e^2-f^2}+\frac{\sqrt{1-d^2x^2}\left(Af^2-Bef+Ce^2\right)}{f\left(d^2e^2-f^2\right)(e+fx)}$$

input
```
Int[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^2),x]
```

output
```
((C*e^2 - B*e*f + A*f^2)*Sqrt[1 - d^2*x^2])/(f*(d^2*e^2 - f^2)*(e + f*x))
+ ((C*(d*e - f)*(d*e + f)*ArcSin[d*x])/(d*f^2) + ((2*C*e + A*d^2*e - (C*d^
2*e^3)/f^2 - B*f)*ArcTan[(f + d^2*e*x)/(Sqrt[d^2*e^2 - f^2]*Sqrt[1 - d^2*x
^2])])/Sqrt[d^2*e^2 - f^2])/(d^2*e^2 - f^2)
```

3.13.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2112
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

3.13.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.66 (sec) , antiderivative size = 899, normalized size of antiderivative = 5.52

method	result
default	$\frac{\left(-A\,\operatorname{csgn}(d)\ln\left(\frac{2d^2ex+2\sqrt{-d^2x^2+1}\,\sqrt{-\frac{d^2e^2-f^2}{f^2}}\,f+2f}{fx+e}\right)d^3e\,f^3x+C\,\operatorname{csgn}(d)\ln\left(\frac{2d^2ex+2\sqrt{-d^2x^2+1}\,\sqrt{-\frac{d^2e^2-f^2}{f^2}}\,f+2f}{fx+e}\right)d^3e^3fx-A\right.}{}$

input
```
int((C*x^2+B*x+A)/(f*x+e)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
(-A*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+
f)/(f*x+e))*d^3*e*f^3*x+C*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*
e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^3*e^3*f*x-A*csgn(d)*ln(2*(d^2*e*x+(-d^
2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^3*e^2*f^2+C*csgn
(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+
e))*d^3*e^4+C*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*d^2*e^2*f^2*x*(-(d^2*
e^2-f^2)/f^2)^(1/2)+A*csgn(d)*d*f^4*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2
)^(1/2)+B*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1
/2)*f+f)/(f*x+e))*d*f^4*x-B*csgn(d)*d*e*f^3*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-
f^2)/f^2)^(1/2)-2*C*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^
2)/f^2)^(1/2)*f+f)/(f*x+e))*d*e*f^3*x+C*csgn(d)*d*e^2*f^2*(-d^2*x^2+1)^(1/
2)*(-(d^2*e^2-f^2)/f^2)^(1/2)+C*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*d^2
*e^3*f*(-(d^2*e^2-f^2)/f^2)^(1/2)+B*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/
2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d*e*f^3-2*C*csgn(d)*ln(2*(d^2*
e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d*e^2*f^2-
C*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*f^4*x*(-(d^2*e^2-f^2)/f^2)^(1/2)-
C*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*e*f^3*(-(d^2*e^2-f^2)/f^2)^(1/2))
*csgn(d)*(-d*x+1)^(1/2)*(d*x+1)^(1/2)/(-d^2*x^2+1)^(1/2)/(d*e-f)/d/(d*e+f)
/(f*x+e)/(-(d^2*e^2-f^2)/f^2)^(1/2)/f^3
```

3.13.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 501 vs. $2(155) = 310$.

Time = 16.95 (sec) , antiderivative size = 1025, normalized size of antiderivative = 6.29

$$\int \frac{A + Bx + Cx^2}{\sqrt{1 - dx}\sqrt{1 + dx}(e + fx)^2}\,dx$$

$$= \left[\frac{Cd^3e^5f - Bd^3e^4f^2 + Bde^2f^4 - Adef^5 + (Ad^3 - Cd)e^3f^3 - (Cd^3e^5 + Bde^2f^3 - (Ad^3 + 2\,Cd)e^3f^2 + (\cdots}{\cdots}\right.$$

input
```
integrate((C*x^2+B*x+A)/(f*x+e)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorith
m="fricas")
```

output
```
[(C*d^3*e^5*f - B*d^3*e^4*f^2 + B*d*e^2*f^4 - A*d*e*f^5 + (A*d^3 - C*d)*e^
3*f^3 - (C*d^3*e^5 + B*d*e^2*f^3 - (A*d^3 + 2*C*d)*e^3*f^2 + (C*d^3*e^4*f
+ B*d*e*f^4 - (A*d^3 + 2*C*d)*e^2*f^3)*x)*sqrt(-d^2*e^2 + f^2)*log((d^2*e*
f*x + f^2 + sqrt(-d^2*e^2 + f^2)*(d^2*e*x + f) + (sqrt(-d^2*e^2 + f^2)*sqr
t(-d*x + 1)*f - (d^2*e^2 - f^2)*sqrt(-d*x + 1))*sqrt(d*x + 1))/(f*x + e))
+ (C*d^3*e^5*f - B*d^3*e^4*f^2 + B*d*e^2*f^4 - A*d*e*f^5 + (A*d^3 - C*d)*e
^3*f^3)*sqrt(d*x + 1)*sqrt(-d*x + 1) + (C*d^3*e^4*f^2 - B*d^3*e^3*f^3 + B*
d*e*f^5 - A*d*f^6 + (A*d^3 - C*d)*e^2*f^4)*x - 2*(C*d^4*e^6 - 2*C*d^2*e^4*
f^2 + C*e^2*f^4 + (C*d^4*e^5*f - 2*C*d^2*e^3*f^3 + C*e*f^5)*x)*arctan((sqr
t(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/(d^5*e^6*f^2 - 2*d^3*e^4*f^4 + d*e^
2*f^6 + (d^5*e^5*f^3 - 2*d^3*e^3*f^5 + d*e*f^7)*x), (C*d^3*e^5*f - B*d^3*e
^4*f^2 + B*d*e^2*f^4 - A*d*e*f^5 + (A*d^3 - C*d)*e^3*f^3 - 2*(C*d^3*e^5 +
B*d*e^2*f^3 - (A*d^3 + 2*C*d)*e^3*f^2 + (C*d^3*e^4*f + B*d*e*f^4 - (A*d^3
+ 2*C*d)*e^2*f^3)*x)*sqrt(d^2*e^2 - f^2)*arctan(-(sqrt(d^2*e^2 - f^2)*sqrt
(d*x + 1)*sqrt(-d*x + 1)*e - sqrt(d^2*e^2 - f^2)*(f*x + e))/((d^2*e^2 - f^
2)*x)) + (C*d^3*e^5*f - B*d^3*e^4*f^2 + B*d*e^2*f^4 - A*d*e*f^5 + (A*d^3 -
 C*d)*e^3*f^3)*sqrt(d*x + 1)*sqrt(-d*x + 1) + (C*d^3*e^4*f^2 - B*d^3*e^3*f
^3 + B*d*e*f^5 - A*d*f^6 + (A*d^3 - C*d)*e^2*f^4)*x - 2*(C*d^4*e^6 - 2*C*d
^2*e^4*f^2 + C*e^2*f^4 + (C*d^4*e^5*f - 2*C*d^2*e^3*f^3 + C*e*f^5)*x)*arct
an((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/(d^5*e^6*f^2 - 2*d^3*e^4*...
```

3.13.6 Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^2}\,dx=\int \frac{A+Bx+Cx^2}{(e+fx)^2\sqrt{-dx+1}\sqrt{dx+1}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(f*x+e)**2/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/((e + f*x)**2*sqrt(-d*x + 1)*sqrt(d*x + 1)), x
)
```

3.13.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A + Bx + Cx^2}{\sqrt{1 - dx}\sqrt{1 + dx}(e + fx)^2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+B*x+A)/(f*x+e)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.13.8 Giac [F(-2)]

Exception generated.

$$\int \frac{A + Bx + Cx^2}{\sqrt{1 - dx}\sqrt{1 + dx}(e + fx)^2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((C*x^2+B*x+A)/(f*x+e)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

3.13.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 10198, normalized size of antiderivative = 62.56

$$\int \frac{A + Bx + Cx^2}{\sqrt{1 - dx}\sqrt{1 + dx}(e + fx)^2} \, dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/((e + f*x)^2*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

output
```
(A*d^5*e^5*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) -
1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d
^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*
e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(
1/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d
*x + 1)^(1/2) - 1)^2))*2i - A*d^3*e^3*f^2*atan(((f + d*e)^(3/2)*(f - d*e)^
(3/2)*1i - (((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((
d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((
d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2)
 - 1) + (2*d*e*f^2*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2
*f*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2))*2i + (4*A*f^2*((1 -
d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1) + (
A*d^5*e^5*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1
)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^
2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e
^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1
/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*
x + 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)^2*4i)/((d*x + 1)^(1/2) - 1)^2
+ (A*d^5*e^5*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2)
- 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^...
```

3.13. $\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^2}\,dx$

3.14 $\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^3}\,dx$

3.14.1 Optimal result

Integrand size = 37, antiderivative size = 248

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^3}\,dx &= \frac{(Ce^2-Bef+Af^2)\sqrt{1-d^2x^2}}{2f(d^2e^2-f^2)(e+fx)^2} \\ &\quad - \frac{(Cd^2e^3+Bd^2e^2f-4Cef^2-3Ad^2ef^2+2Bf^3)\sqrt{1-d^2x^2}}{2f(d^2e^2-f^2)^2(e+fx)} \\ &\quad + \frac{(C(d^2e^2+2f^2)-d^2(3Bef-A(2d^2e^2+f^2)))\arctan\left(\frac{f+d^2ex}{\sqrt{d^2e^2-f^2}\sqrt{1-d^2x^2}}\right)}{2(d^2e^2-f^2)^{5/2}}\end{aligned}$$

output
```
1/2*(C*(d^2*e^2+2*f^2)-d^2*(3*B*e*f-A*(2*d^2*e^2+f^2)))*arctan((d^2*e*x+f)
/(d^2*e^2-f^2)^(1/2)/(-d^2*x^2+1)^(1/2))/(d^2*e^2-f^2)^(5/2)+1/2*(A*f^2-B*
e*f+C*e^2)*(-d^2*x^2+1)^(1/2)/f/(d^2*e^2-f^2)/(f*x+e)^2-1/2*(-3*A*d^2*e*f^
2+B*d^2*e^2*f+C*d^2*e^3+2*B*f^3-4*C*e*f^2)*(-d^2*x^2+1)^(1/2)/f/(d^2*e^2-f
^2)^2/(f*x+e)
```

3.14.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^3}\,dx =$$
$$-\frac{\frac{(de-f)(de+f)\sqrt{1-d^2x^2}\left(Af^3+Bd^2e^2(2e+fx)+Bf^2(e+2fx)-Ad^2ef(4e+3fx)+Ce\left(-3ef+d^2e^2x-4f^2x\right)\right)}{(e+fx)^2}+2\sqrt{d^2e^2-f^2}(C(d^2e^2}{2(de-f)^3(de+f)^3}$$

input
```
Integrate[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^3),x]
```

output
```
-1/2*(((d*e - f)*(d*e + f)*Sqrt[1 - d^2*x^2]*(A*f^3 + B*d^2*e^2*(2*e + f*x
) + B*f^2*(e + 2*f*x) - A*d^2*e*f*(4*e + 3*f*x) + C*e*(-3*e*f + d^2*e^2*x
- 4*f^2*x)))/(e + f*x)^2 + 2*Sqrt[d^2*e^2 - f^2]*(C*(d^2*e^2 + 2*f^2) + d^
2*(-3*B*e*f + A*(2*d^2*e^2 + f^2)))*ArcTan[(Sqrt[d^2*e^2 - f^2]*x)/(e + f*
x - e*Sqrt[1 - d^2*x^2])])/((d*e - f)^3*(d*e + f)^3)
```

3.14.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {2112, 2182, 679, 488, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{dx+1}(e+fx)^3}\,dx$$

$$\Big\downarrow \text{ 2112}$$

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-d^2x^2}(e+fx)^3}dx$$

$$\Big\downarrow \text{ 2182}$$

$$\frac{\int \frac{2\left(Aed^2+Ce-Bf\right)+\left(Bed^2-Afd^2+\frac{Ce^2d^2}{f}-2Cf\right)x}{(e+fx)^2\sqrt{1-d^2x^2}}dx}{2\left(d^2e^2-f^2\right)}+\frac{\sqrt{1-d^2x^2}\left(Af^2-Bef+Ce^2\right)}{2f\left(d^2e^2-f^2\right)(e+fx)^2}$$

$$\Big\downarrow \text{ 679}$$

$$\frac{\frac{\left(C(d^2e^2+2f^2)-d^2(3Bef-A(2d^2e^2+f^2))\right)\int \frac{1}{(e+fx)\sqrt{1-d^2x^2}}dx}{d^2e^2-f^2}-\frac{\sqrt{1-d^2x^2}(-3Ad^2ef^2+Bd^2e^2f+2Bf^3+Cd^2e^3-4Cef^2)}{f(d^2e^2-f^2)(e+fx)}}{2\left(d^2e^2-f^2\right)}+\frac{\sqrt{1-d^2x^2}\left(Af^2-Bef+Ce^2\right)}{2f\left(d^2e^2-f^2\right)(e+fx)^2}$$

$\Big\downarrow$ 488

$$\frac{-\frac{\left(C(d^2e^2+2f^2)-d^2(3Bef-A(2d^2e^2+f^2))\right)\int \frac{1}{-d^2e^2+f^2-\frac{(exd^2+f)^2}{1-d^2x^2}}d\frac{exd^2+f}{\sqrt{1-d^2x^2}}}{d^2e^2-f^2}-\frac{\sqrt{1-d^2x^2}(-3Ad^2ef^2+Bd^2e^2f+2Bf^3+Cd^2e^3-4Cef^2)}{f(d^2e^2-f^2)(e+fx)}}{2\left(d^2e^2-f^2\right)}+\frac{\sqrt{1-d^2x^2}\left(Af^2-Bef+Ce^2\right)}{2f\left(d^2e^2-f^2\right)(e+fx)^2}$$

$\Big\downarrow$ 217

$$\frac{\frac{\arctan\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\sqrt{d^2e^2-f^2}}\right)\left(C(d^2e^2+2f^2)-d^2(3Bef-A(2d^2e^2+f^2))\right)}{(d^2e^2-f^2)^{3/2}}-\frac{\sqrt{1-d^2x^2}(-3Ad^2ef^2+Bd^2e^2f+2Bf^3+Cd^2e^3-4Cef^2)}{f(d^2e^2-f^2)(e+fx)}}{2\left(d^2e^2-f^2\right)}+\frac{\sqrt{1-d^2x^2}\left(Af^2-Bef+Ce^2\right)}{2f\left(d^2e^2-f^2\right)(e+fx)^2}$$

input
```
Int[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^3),x]
```

output
```
((C*e^2 - B*e*f + A*f^2)*Sqrt[1 - d^2*x^2])/(2*f*(d^2*e^2 - f^2)*(e + f*x)
^2) + (-(((C*d^2*e^3 + B*d^2*e^2*f - 4*C*e*f^2 - 3*A*d^2*e*f^2 + 2*B*f^3)*
Sqrt[1 - d^2*x^2])/(f*(d^2*e^2 - f^2)*(e + f*x))) + ((C*(d^2*e^2 + 2*f^2)
- d^2*(3*B*e*f - A*(2*d^2*e^2 + f^2)))*ArcTan[(f + d^2*e*x)/(Sqrt[d^2*e^2
- f^2]*Sqrt[1 - d^2*x^2])])/(d^2*e^2 - f^2)^(3/2))/(2*(d^2*e^2 - f^2))
```

3.14.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2112
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

3.14.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.65 (sec) , antiderivative size = 1449, normalized size of antiderivative = 5.84

method	result	size
default	Expression too large to display	1449

input
```
int((C*x^2+B*x+A)/(f*x+e)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
-1/2*(2*A*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)
/(f*x+e))*d^4*e^2*f^2*x^2-3*A*d^2*e*f^3*x*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^
2)/f^2)^(1/2)+B*d^2*e^2*f^2*x*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2
)+C*d^2*e^3*f*x*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)+2*A*ln(2*(d^
2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^4*e^4-
3*C*e^2*f^2*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)+2*B*f^4*x*(-d^2*
x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)+C*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)
*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^2*e^4+2*C*ln(2*(d^2*e*x+(-d^2*
x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*f^4*x^2+2*C*ln(2*(d^
2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*e^2*f^2+
B*e*f^3*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)+A*ln(2*(d^2*e*x+(-d^
2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^2*f^4*x^2+A*ln(2
*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^2*
e^2*f^2-3*B*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+
f)/(f*x+e))*d^2*e^3*f+4*C*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)
/f^2)^(1/2)*f+f)/(f*x+e))*e*f^3*x+A*f^4*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)
/f^2)^(1/2)+4*A*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2
)*f+f)/(f*x+e))*d^4*e^3*f*x-3*B*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^
2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^2*e*f^3*x^2+C*ln(2*(d^2*e*x+(-d^2*x^2+1)
^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^2*e^2*f^2*x^2+2*A*ln(...
```

3.14.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 778 vs. $2(232) = 464$.

Time = 0.42 (sec) , antiderivative size = 1580, normalized size of antiderivative = 6.37

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(f*x+e)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorith
m="fricas")
```

output
```
[-1/2*(2*B*d^4*e^7 - B*d^2*e^5*f^2 - (4*A*d^4 + 3*C*d^2)*e^6*f + (5*A*d^2
+ 3*C)*e^4*f^3 - B*e^3*f^4 - A*e^2*f^5 + (2*B*d^4*e^5*f^2 - B*d^2*e^3*f^4
- (4*A*d^4 + 3*C*d^2)*e^4*f^3 + (5*A*d^2 + 3*C)*e^2*f^5 - B*e*f^6 - A*f^7)
*x^2 - (3*B*d^2*e^5*f - (2*A*d^4 + C*d^2)*e^6 - (A*d^2 + 2*C)*e^4*f^2 + (3
*B*d^2*e^3*f^3 - (2*A*d^4 + C*d^2)*e^4*f^2 - (A*d^2 + 2*C)*e^2*f^4)*x^2 +
2*(3*B*d^2*e^4*f^2 - (2*A*d^4 + C*d^2)*e^5*f - (A*d^2 + 2*C)*e^3*f^3)*x)*s
qrt(-d^2*e^2 + f^2)*log((d^2*e*f*x + f^2 - sqrt(-d^2*e^2 + f^2)*(d^2*e*x +
 f) - (sqrt(-d^2*e^2 + f^2)*sqrt(-d*x + 1)*f + (d^2*e^2 - f^2)*sqrt(-d*x +
 1))*sqrt(d*x + 1))/(f*x + e)) + (2*B*d^4*e^7 - B*d^2*e^5*f^2 - (4*A*d^4 +
 3*C*d^2)*e^6*f + (5*A*d^2 + 3*C)*e^4*f^3 - B*e^3*f^4 - A*e^2*f^5 + (C*d^4
*e^7 + B*d^4*e^6*f + B*d^2*e^4*f^3 - (3*A*d^4 + 5*C*d^2)*e^5*f^2 + (3*A*d^
2 + 4*C)*e^3*f^4 - 2*B*e^2*f^5)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(2*B*d
^4*e^6*f - B*d^2*e^4*f^3 - (4*A*d^4 + 3*C*d^2)*e^5*f^2 + (5*A*d^2 + 3*C)*e
^3*f^4 - B*e^2*f^5 - A*e*f^6)*x)/(d^6*e^10 - 3*d^4*e^8*f^2 + 3*d^2*e^6*f^4
 - e^4*f^6 + (d^6*e^8*f^2 - 3*d^4*e^6*f^4 + 3*d^2*e^4*f^6 - e^2*f^8)*x^2 +
 2*(d^6*e^9*f - 3*d^4*e^7*f^3 + 3*d^2*e^5*f^5 - e^3*f^7)*x), -1/2*(2*B*d^4
*e^7 - B*d^2*e^5*f^2 - (4*A*d^4 + 3*C*d^2)*e^6*f + (5*A*d^2 + 3*C)*e^4*f^3
 - B*e^3*f^4 - A*e^2*f^5 + (2*B*d^4*e^5*f^2 - B*d^2*e^3*f^4 - (4*A*d^4 + 3
*C*d^2)*e^4*f^3 + (5*A*d^2 + 3*C)*e^2*f^5 - B*e*f^6 - A*f^7)*x^2 + 2*(3*B*
d^2*e^5*f - (2*A*d^4 + C*d^2)*e^6 - (A*d^2 + 2*C)*e^4*f^2 + (3*B*d^2*e^...
```

3.14.6 Sympy [F]

$$\int \frac{A + Bx + Cx^2}{\sqrt{1 - dx}\sqrt{1 + dx}(e + fx)^3}\, dx = \int \frac{A + Bx + Cx^2}{(e + fx)^3 \sqrt{-dx + 1}\sqrt{dx + 1}}\, dx$$

input
```
integrate((C*x**2+B*x+A)/(f*x+e)**3/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/((e + f*x)**3*sqrt(-d*x + 1)*sqrt(d*x + 1)), x
)
```

3.14.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A + Bx + Cx^2}{\sqrt{1 - dx}\sqrt{1 + dx}(e + fx)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+B*x+A)/(f*x+e)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((f-d*e)*(f+d*e)>0)', see `assume
?` for mor
```

3.14.8 Giac [F(-2)]

Exception generated.

$$\int \frac{A + Bx + Cx^2}{\sqrt{1 - dx}\sqrt{1 + dx}(e + fx)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((C*x^2+B*x+A)/(f*x+e)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

3.14.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 9097, normalized size of antiderivative = 36.68

$$\int \frac{A + Bx + Cx^2}{\sqrt{1 - dx}\sqrt{1 + dx}(e + fx)^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/((e + f*x)^3*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

output
```
((12*(2*C*f^3 + C*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^2)/(((d*x + 1)^(1/2) -
1)^2*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) - (24*(2*C*f^3 - C*d^2*e^2*f)*((1 -
d*x)^(1/2) - 1)^4)/(((d*x + 1)^(1/2) - 1)^4*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2
)) + (12*(2*C*f^3 + C*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^6)/(((d*x + 1)^(1/2
) - 1)^6*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) - (2*((1 - d*x)^(1/2) - 1)^7*(C*
d^3*e^3 + 2*C*d*e*f^2))/(((d*x + 1)^(1/2) - 1)^7*(f^4 + d^4*e^4 - 2*d^2*e^
2*f^2)) - (2*((1 - d*x)^(1/2) - 1)^3*(7*C*d^3*e^3 - 34*C*d*e*f^2))/(((d*x
+ 1)^(1/2) - 1)^3*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) + (2*((1 - d*x)^(1/2) -
 1)^5*(7*C*d^3*e^3 - 34*C*d*e*f^2))/(((d*x + 1)^(1/2) - 1)^5*(f^4 + d^4*e^
4 - 2*d^2*e^2*f^2)) + (2*d*e*((1 - d*x)^(1/2) - 1)*(2*C*f^2 + C*d^2*e^2))/
(((d*x + 1)^(1/2) - 1)*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)))/(d^2*e^2 + (((1 -
 d*x)^(1/2) - 1)^2*(16*f^2 + 4*d^2*e^2))/((d*x + 1)^(1/2) - 1)^2 + (((1 -
d*x)^(1/2) - 1)^6*(16*f^2 + 4*d^2*e^2))/((d*x + 1)^(1/2) - 1)^6 - (((1 - d
*x)^(1/2) - 1)^4*(32*f^2 - 6*d^2*e^2))/((d*x + 1)^(1/2) - 1)^4 + (d^2*e^2*
((1 - d*x)^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8 + (8*d*e*f*((1 - d*x)^(1/
2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 - (8*d*e*f*((1 - d*x)^(1/2) - 1)^5)/((d
*x + 1)^(1/2) - 1)^5 - (8*d*e*f*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2)
- 1)^7 + (8*d*e*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)) + ((4*((1
- d*x)^(1/2) - 1)^2*(4*A*d^4*e^4*f - 2*A*f^5 + 7*A*d^2*e^2*f^3))/(e^2*((d*
x + 1)^(1/2) - 1)^2*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) + (8*((1 - d*x)^(1...
```

3.14. $\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^3}\,dx$

3.15 $\int \frac{x(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

3.15.1 Optimal result

Integrand size = 31, antiderivative size = 79

$$\int \frac{x(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{cx^2\sqrt{1-d^2x^2}}{3d^2} - \frac{(2(2c+3ad^2)+3bd^2x)\sqrt{1-d^2x^2}}{6d^4} + \frac{b\arcsin(dx)}{2d^3}$$

output
```
1/2*b*arcsin(d*x)/d^3-1/3*c*x^2*(-d^2*x^2+1)^(1/2)/d^2-1/6*(3*b*d^2*x+6*a*
d^2+4*c)*(-d^2*x^2+1)^(1/2)/d^4
```

3.15.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.95

$$\int \frac{x(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{1-d^2x^2}(-4c-6ad^2-3bd^2x-2cd^2x^2)}{6d^4} + \frac{b\arctan\left(\frac{dx}{-1+\sqrt{1-d^2x^2}}\right)}{d^3}$$

input
```
Integrate[(x*(a + b*x + c*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

output
```
(Sqrt[1 - d^2*x^2]*(-4*c - 6*a*d^2 - 3*b*d^2*x - 2*c*d^2*x^2))/(6*d^4) + (
b*ArcTan[(d*x)/(-1 + Sqrt[1 - d^2*x^2])])/d^3
```

3.15.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.20, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2112, 2340, 25, 533, 27, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{dx+1}}dx$$

$$\Big\downarrow \text{2112}$$

$$\int \frac{x(a+bx+cx^2)}{\sqrt{1-d^2x^2}}dx$$

$$\Big\downarrow \text{2340}$$

$$-\frac{\int -\frac{x(3ad^2+3bxd^2+2c)}{\sqrt{1-d^2x^2}}dx}{3d^2}-\frac{cx^2\sqrt{1-d^2x^2}}{3d^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{x(3ad^2+3bxd^2+2c)}{\sqrt{1-d^2x^2}}dx}{3d^2}-\frac{cx^2\sqrt{1-d^2x^2}}{3d^2}$$

$$\Big\downarrow \text{533}$$

$$\frac{\frac{\int \frac{d^2(3b+2(3ad^2+2c)x)}{\sqrt{1-d^2x^2}}dx}{2d^2}-\frac{3}{2}bx\sqrt{1-d^2x^2}}{3d^2}-\frac{cx^2\sqrt{1-d^2x^2}}{3d^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{2}\int \frac{3b+2(3ad^2+2c)x}{\sqrt{1-d^2x^2}}dx-\frac{3}{2}bx\sqrt{1-d^2x^2}}{3d^2}-\frac{cx^2\sqrt{1-d^2x^2}}{3d^2}$$

$$\Big\downarrow \text{455}$$

$$\frac{\frac{1}{2}\left(3b\int \frac{1}{\sqrt{1-d^2x^2}}dx-2\sqrt{1-d^2x^2}\left(3a+\frac{2c}{d^2}\right)\right)-\frac{3}{2}bx\sqrt{1-d^2x^2}}{3d^2}-\frac{cx^2\sqrt{1-d^2x^2}}{3d^2}$$

$$\Big\downarrow \text{223}$$

$$\frac{\frac{1}{2}\left(\frac{3b\arcsin(dx)}{d}-2\sqrt{1-d^2x^2}\left(3a+\frac{2c}{d^2}\right)\right)-\frac{3}{2}bx\sqrt{1-d^2x^2}}{3d^2}-\frac{cx^2\sqrt{1-d^2x^2}}{3d^2}$$

input
```
Int[(x*(a + b*x + c*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

output
```
-1/3*(c*x^2*Sqrt[1 - d^2*x^2])/d^2 + ((-3*b*x*Sqrt[1 - d^2*x^2])/2 + (-2*(
3*a + (2*c)/d^2)*Sqrt[1 - d^2*x^2] + (3*b*ArcSin[d*x])/d)/2)/(3*d^2)
```

3.15.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 2112
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

3.15.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 5.67 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.76

method	result
default	$-\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(2\,\mathrm{csgn}(d) c\, d^{2} x^{2} \sqrt{-d^{2}x^{2}+1}+3\sqrt{-d^{2}x^{2}+1}\,\mathrm{csgn}(d) b\, d^{2} x+6\,\mathrm{csgn}(d)\sqrt{-d^{2}x^{2}+1}\,a\, d^{2}+4\,\mathrm{csgn}(d)\sqrt{-d^{2}x^{2}+1}\,c-3}{6d^{4}\sqrt{-d^{2}x^{2}+1}}$
risch	$\frac{\left(2c\, d^{2}x^{2}+3b\, d^{2}x+6a\, d^{2}+4c\right)\sqrt{dx+1}\,(dx-1)\sqrt{(-dx+1)(dx+1)}}{6d^{4}\sqrt{-(dx+1)(dx-1)}\,\sqrt{-dx+1}}+\frac{b\arctan\left(\frac{\sqrt{d^{2}}\,x}{\sqrt{-d^{2}x^{2}+1}}\right)\sqrt{(-dx+1)(dx+1)}}{2d^{2}\sqrt{d^{2}}\,\sqrt{-dx+1}\,\sqrt{dx+1}}$

input
```
int(x*(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*(2*csgn(d)*c*d^2*x^2*(-d^2*x^2+1)^(1/2)+
3*(-d^2*x^2+1)^(1/2)*csgn(d)*b*d^2*x+6*csgn(d)*(-d^2*x^2+1)^(1/2)*a*d^2+4*
csgn(d)*(-d^2*x^2+1)^(1/2)*c-3*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*b*d)
*csgn(d)/d^4/(-d^2*x^2+1)^(1/2)
```

3.15.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99

$$\int \frac{x(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{6\,bd\arctan\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{dx}\right)+\left(2\,cd^2x^2+3\,bd^2x+6\,ad^2+4\,c\right)\sqrt{dx+1}\sqrt{-dx+1}}{6\,d^4}$$

input
```
integrate(x*(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="frica
s")
```

output
```
-1/6*(6*b*d*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)) + (2*c*d^2*x^
2 + 3*b*d^2*x + 6*a*d^2 + 4*c)*sqrt(d*x + 1)*sqrt(-d*x + 1))/d^4
```

3.15.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \text{Timed out}$$

input
```
integrate(x*(c*x**2+b*x+a)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

output
```
Timed out
```

3.15.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.10

$$\int \frac{x(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{\sqrt{-d^2x^2+1}cx^2}{3\,d^2} - \frac{\sqrt{-d^2x^2+1}bx}{2\,d^2} - \frac{\sqrt{-d^2x^2+1}a}{d^2} + \frac{b\arcsin{(dx)}}{2\,d^3} - \frac{2\,\sqrt{-d^2x^2+1}c}{3\,d^4}$$

input
```
integrate(x*(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxim
a")
```

output
```
-1/3*sqrt(-d^2*x^2 + 1)*c*x^2/d^2 - 1/2*sqrt(-d^2*x^2 + 1)*b*x/d^2 - sqrt(
-d^2*x^2 + 1)*a/d^2 + 1/2*b*arcsin(d*x)/d^3 - 2/3*sqrt(-d^2*x^2 + 1)*c/d^4
```

3.15.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.96

$$\int \frac{x(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{6\,bd\arcsin\left(\frac{1}{2}\,\sqrt{2}\sqrt{dx+1}\right) - (6\,ad^2 + (2\,(dx+1)c + 3\,bd - 4\,c)(dx+1) - 3\,bd + 6\,c)\sqrt{dx+1}\sqrt{-dx+1}}{6\,d^4}$$

input
```
integrate(x*(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac"
)
```

output
```
1/6*(6*b*d*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)) - (6*a*d^2 + (2*(d*x + 1)*c +
 3*b*d - 4*c)*(d*x + 1) - 3*b*d + 6*c)*sqrt(d*x + 1)*sqrt(-d*x + 1))/d^4
```

3.15.9 Mupad [B] (verification not implemented)

Time = 8.58 (sec) , antiderivative size = 244, normalized size of antiderivative = 3.09

$$\int \frac{x(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{\sqrt{1-d\,x}\,\left(\frac{a}{d^2}+\frac{a\,x}{d}\right)}{\sqrt{d\,x+1}} - \frac{2\,b\,\mathrm{atan}\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)}{d^3} - \frac{\frac{14\,b\,\left(\sqrt{1-d\,x}-1\right)^3}{\left(\sqrt{d\,x+1}-1\right)^3}-\frac{14\,b\,\left(\sqrt{1-d\,x}-1\right)^5}{\left(\sqrt{d\,x+1}-1\right)^5}+\frac{2\,b\,\left(\sqrt{1-d\,x}-1\right)^7}{\left(\sqrt{d\,x+1}-1\right)^7}-\frac{2\,b\,\left(\sqrt{1-d\,x}-1\right)}{\sqrt{d\,x+1}-1}}{d^3\,{\left(\frac{{\left(\sqrt{1-d\,x}-1\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+1\right)}^4} - \frac{\sqrt{1-d\,x}\,\left(\frac{2\,c}{3\,d^4}+\frac{c\,x^3}{3\,d}+\frac{c\,x^2}{3\,d^2}+\frac{2\,c\,x}{3\,d^3}\right)}{\sqrt{d\,x+1}}$$

input
```
int((x*(a + b*x + c*x^2))/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

output
```
- ((1 - d*x)^(1/2)*(a/d^2 + (a*x)/d))/(d*x + 1)^(1/2) - (2*b*atan(((1 - d*
x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/d^3 - ((14*b*((1 - d*x)^(1/2) - 1)^3
)/((d*x + 1)^(1/2) - 1)^3 - (14*b*((1 - d*x)^(1/2) - 1)^5)/((d*x + 1)^(1/2
) - 1)^5 + (2*b*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7 - (2*b*((
1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1))/(d^3*(((1 - d*x)^(1/2) - 1)^2/
((d*x + 1)^(1/2) - 1)^2 + 1)^4) - ((1 - d*x)^(1/2)*((2*c)/(3*d^4) + (c*x^3
)/(3*d) + (c*x^2)/(3*d^2) + (2*c*x)/(3*d^3)))/(d*x + 1)^(1/2)
```

3.16 $\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}} dx$

3.16.1 Optimal result

Integrand size = 30, antiderivative size = 63

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx=-\frac{b\sqrt{1-d^2x^2}}{d^2}-\frac{cx\sqrt{1-d^2x^2}}{2d^2}+\frac{(c+2ad^2)\arcsin(dx)}{2d^3}$$

output
```
1/2*(2*a*d^2+c)*arcsin(d*x)/d^3-b*(-d^2*x^2+1)^(1/2)/d^2-1/2*c*x*(-d^2*x^2
+1)^(1/2)/d^2
```

3.16.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx=\frac{(-2b-cx)\sqrt{1-d^2x^2}}{2d^2}+\frac{(c+2ad^2)\arctan\left(\frac{dx}{-1+\sqrt{1-d^2x^2}}\right)}{d^3}$$

input
```
Integrate[(a + b*x + c*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

output
```
((-2*b - c*x)*Sqrt[1 - d^2*x^2])/(2*d^2) + ((c + 2*a*d^2)*ArcTan[(d*x)/(-1
 + Sqrt[1 - d^2*x^2])])/d^3
```

3.16.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1188, 2346, 25, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{dx+1}}dx$$

$$\Big\downarrow \text{1188}$$

$$\int \frac{a+bx+cx^2}{\sqrt{1-d^2x^2}}dx$$

$$\Big\downarrow \text{2346}$$

$$-\frac{\int -\frac{2ad^2+2bxd^2+c}{\sqrt{1-d^2x^2}}dx}{2d^2}-\frac{cx\sqrt{1-d^2x^2}}{2d^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2ad^2+2bxd^2+c}{\sqrt{1-d^2x^2}}dx}{2d^2}-\frac{cx\sqrt{1-d^2x^2}}{2d^2}$$

$$\Big\downarrow \text{455}$$

$$\frac{(2ad^2+c)\int \frac{1}{\sqrt{1-d^2x^2}}dx-2b\sqrt{1-d^2x^2}}{2d^2}-\frac{cx\sqrt{1-d^2x^2}}{2d^2}$$

$$\Big\downarrow \text{223}$$

$$\frac{\frac{(2ad^2+c)\arcsin(dx)}{d}-2b\sqrt{1-d^2x^2}}{2d^2}-\frac{cx\sqrt{1-d^2x^2}}{2d^2}$$

input
```
Int[(a + b*x + c*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

output
```
-1/2*(c*x*Sqrt[1 - d^2*x^2])/d^2 + (-2*b*Sqrt[1 - d^2*x^2] + ((c + 2*a*d^2
)*ArcSin[d*x])/d)/(2*d^2)
```

3.16.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 1188
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2
)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m, n] && EqQ[e*f
 + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

3.16.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.64 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.86

method	result
default	$-\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\left(\sqrt{-d^{2}x^{2}+1}\,\operatorname{csgn}(d)dcx-2\arctan\left(\frac{\operatorname{csgn}(d)dx}{\sqrt{-d^{2}x^{2}+1}}\right)a\,d^{2}+2\,\operatorname{csgn}(d)d\sqrt{-d^{2}x^{2}+1}\,b-\arctan\left(\frac{\operatorname{csgn}(d)dx}{\sqrt{-d^{2}x^{2}+1}}\right)c\right)\operatorname{csgn}(d}{2d^{3}\sqrt{-d^{2}x^{2}+1}}$
risch	$\frac{(cx+2b)\sqrt{dx+1}\,(dx-1)\sqrt{(-dx+1)(dx+1)}}{2d^{2}\sqrt{-(dx+1)(dx-1)}\,\sqrt{-dx+1}}+\frac{(2a\,d^{2}+c)\arctan\left(\frac{\sqrt{d^{2}}\,x}{\sqrt{-d^{2}x^{2}+1}}\right)\sqrt{(-dx+1)(dx+1)}}{2d^{2}\sqrt{d^{2}}\,\sqrt{-dx+1}\,\sqrt{dx+1}}$

input
```
int((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-d*x+1)^(1/2)*(d*x+1)^(1/2)/d^3*((-d^2*x^2+1)^(1/2)*csgn(d)*d*c*x-2*
arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*a*d^2+2*csgn(d)*d*(-d^2*x^2+1)^(1/2
)*b-arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*c)/(-d^2*x^2+1)^(1/2)*csgn(d)
```

3.16.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.06

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{(cdx+2\,bd)\sqrt{dx+1}\sqrt{-dx+1}+2\,(2\,ad^2+c)\arctan\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{dx}\right)}{2\,d^3}$$

input
```
integrate((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas"
)
```

output
```
-1/2*((c*d*x + 2*b*d)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(2*a*d^2 + c)*arcta
n((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/d^3
```

3.16.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

output
```
Timed out
```

3.16.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{a\arcsin(dx)}{d} - \frac{\sqrt{-d^2x^2+1}cx}{2\,d^2} - \frac{\sqrt{-d^2x^2+1}b}{d^2} + \frac{c\arcsin(dx)}{2\,d^3}$$

input
```
integrate((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima"
)
```

output
```
a*arcsin(d*x)/d - 1/2*sqrt(-d^2*x^2 + 1)*c*x/d^2 - sqrt(-d^2*x^2 + 1)*b/d^
2 + 1/2*c*arcsin(d*x)/d^3
```

3.16.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{((dx+1)c+2\,bd-c)\sqrt{dx+1}\sqrt{-dx+1}-2\,(2\,ad^2+c)\arcsin\left(\frac{1}{2}\,\sqrt{2}\sqrt{dx+1}\right)}{2\,d^3}$$

input
```
integrate((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

output
```
-1/2*(((d*x + 1)*c + 2*b*d - c)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 2*(2*a*d^2
+ c)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))/d^3
```

3.16.9 Mupad [B] (verification not implemented)

Time = 8.11 (sec) , antiderivative size = 232, normalized size of antiderivative = 3.68

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{\sqrt{1-d\,x}\left(\frac{b}{d^2}+\frac{b\,x}{d}\right)}{\sqrt{d\,x+1}} - \frac{4\,a\,\mathrm{atan}\left(\frac{d\left(\sqrt{1-d\,x}-1\right)}{\left(\sqrt{d\,x+1}-1\right)\sqrt{d^2}}\right)}{\sqrt{d^2}} - \frac{2\,c\,\mathrm{atan}\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)}{d^3} - \frac{\frac{14\,c\left(\sqrt{1-d\,x}-1\right)^3}{\left(\sqrt{d\,x+1}-1\right)^3}-\frac{14\,c\left(\sqrt{1-d\,x}-1\right)^5}{\left(\sqrt{d\,x+1}-1\right)^5}+\frac{2\,c\left(\sqrt{1-d\,x}-1\right)^7}{\left(\sqrt{d\,x+1}-1\right)^7}-\frac{2\,c\left(\sqrt{1-d\,x}-1\right)}{\sqrt{d\,x+1}-1}}{d^3\left(\frac{\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+1\right)^4}$$

input
```
int((a + b*x + c*x^2)/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

output
```
- ((1 - d*x)^(1/2)*(b/d^2 + (b*x)/d))/(d*x + 1)^(1/2) - (4*a*atan((d*((1 -
 d*x)^(1/2) - 1))/(((d*x + 1)^(1/2) - 1)*(d^2)^(1/2))))/(d^2)^(1/2) - (2*c
*atan(((1 - d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/d^3 - ((14*c*((1 - d*x
)^(1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 - (14*c*((1 - d*x)^(1/2) - 1)^5)/(
(d*x + 1)^(1/2) - 1)^5 + (2*c*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) -
1)^7 - (2*c*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1))/(d^3*(((1 - d*x)
^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 + 1)^4)
```

3.16. $\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

3.17 $\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\sqrt{1+dx}}\,dx$

3.17.1 Optimal result

Integrand size = 33, antiderivative size = 48

$$\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{c\sqrt{1-d^2x^2}}{d^2}+\frac{b\arcsin(dx)}{d}-a\,\text{arctanh}\left(\sqrt{1-d^2x^2}\right)$$

output
```
b*arcsin(d*x)/d-a*arctanh((-d^2*x^2+1)^(1/2))-c*(-d^2*x^2+1)^(1/2)/d^2
```

3.17.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.52

$$\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{c\sqrt{1-d^2x^2}}{d^2}+\frac{2b\arctan\left(\frac{dx}{-1+\sqrt{1-d^2x^2}}\right)}{d} - a\log(x)+a\log\left(-1+\sqrt{1-d^2x^2}\right)$$

input
```
Integrate[(a + b*x + c*x^2)/(x*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

output
```
-((c*Sqrt[1 - d^2*x^2])/d^2) + (2*b*ArcTan[(d*x)/(-1 + Sqrt[1 - d^2*x^2])]
)/d - a*Log[x] + a*Log[-1 + Sqrt[1 - d^2*x^2]]
```

3.17.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2112, 2340, 25, 27, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\sqrt{dx+1}}dx$$

$$\Big\downarrow \text{ 2112}$$

$$\int \frac{a+bx+cx^2}{x\sqrt{1-d^2x^2}}dx$$

$$\Big\downarrow \text{ 2340}$$

$$-\frac{\int -\frac{d^2(a+bx)}{x\sqrt{1-d^2x^2}}dx}{d^2}-\frac{c\sqrt{1-d^2x^2}}{d^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{d^2(a+bx)}{x\sqrt{1-d^2x^2}}dx}{d^2}-\frac{c\sqrt{1-d^2x^2}}{d^2}$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{a+bx}{x\sqrt{1-d^2x^2}}dx-\frac{c\sqrt{1-d^2x^2}}{d^2}$$

$$\Big\downarrow \text{ 538}$$

$$a\int \frac{1}{x\sqrt{1-d^2x^2}}dx+b\int \frac{1}{\sqrt{1-d^2x^2}}dx-\frac{c\sqrt{1-d^2x^2}}{d^2}$$

$$\Big\downarrow \text{ 223}$$

$$a\int \frac{1}{x\sqrt{1-d^2x^2}}dx+\frac{b\arcsin(dx)}{d}-\frac{c\sqrt{1-d^2x^2}}{d^2}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}a\int \frac{1}{x^2\sqrt{1-d^2x^2}}dx^2+\frac{b\arcsin(dx)}{d}-\frac{c\sqrt{1-d^2x^2}}{d^2}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{a\int \frac{1}{\frac{1}{d^2}-\frac{x^4}{d^2}}d\sqrt{1-d^2x^2}}{d^2}+\frac{b\arcsin(dx)}{d}-\frac{c\sqrt{1-d^2x^2}}{d^2}$$

$$\Big\downarrow 221$$

$$-a\text{arctanh}\left(\sqrt{1-d^2x^2}\right)+\frac{b\arcsin(dx)}{d}-\frac{c\sqrt{1-d^2x^2}}{d^2}$$

input
```
Int[(a + b*x + c*x^2)/(x*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

output
```
-((c*Sqrt[1 - d^2*x^2])/d^2) + (b*ArcSin[d*x])/d - a*ArcTanh[Sqrt[1 - d^2*
x^2]]
```

3.17.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2112
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

3.17.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.60 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.00

method	result	size
default	$\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(-\operatorname{csgn}\left(d\right)\operatorname{arctanh}\left(\frac{1}{\sqrt{-d^{2}x^{2}+1}}\right)a\,d^{2}-\operatorname{csgn}\left(d\right)\sqrt{-d^{2}x^{2}+1}\,c+\arctan\left(\frac{\operatorname{csgn}\left(d\right)dx}{\sqrt{-\left(dx+1\right)\left(dx-1\right)}}\right)bd\right)\operatorname{csgn}\left(d\right)}{d^{2}\sqrt{-d^{2}x^{2}+1}}$	96

input
```
int((c*x^2+b*x+a)/x/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-d*x+1)^(1/2)*(d*x+1)^(1/2)/d^2*(-csgn(d)*arctanh(1/(-d^2*x^2+1)^(1/2))*a
*d^2-csgn(d)*(-d^2*x^2+1)^(1/2)*c+arctan(csgn(d)*d*x/(-(d*x+1)*(d*x-1))^(1
/2))*b*d)*csgn(d)/(-d^2*x^2+1)^(1/2)
```

3.17.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.69

$$\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{ad^2\log\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{x}\right) - 2\,bd\arctan\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{dx}\right) - \sqrt{dx+1}\sqrt{-dx+1}c}{d^2}$$

input
```
integrate((c*x^2+b*x+a)/x/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="frica
s")
```

output
```
(a*d^2*log((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/x) - 2*b*d*arctan((sqrt(d*x
+ 1)*sqrt(-d*x + 1) - 1)/(d*x)) - sqrt(d*x + 1)*sqrt(-d*x + 1)*c)/d^2
```

3.17.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 28.59 (sec) , antiderivative size = 245, normalized size of antiderivative = 5.10

$$\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{iaG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2} \\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0 \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{aG_{6,6}^{2,6}\left(\begin{matrix} 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0 \end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{ibG_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1 \\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d} + \frac{bG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0 \end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d} - \frac{icG_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1 \\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2} - \frac{cG_{6,6}^{2,6}\left(\begin{matrix} -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0 \end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}$$

input
```
integrate((c*x**2+b*x+a)/x/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

output
```
I*a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)),
 1/(d**2*x**2))/(4*pi**(3/2)) - a*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()),
((1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi**(3/
2)) - I*b*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0
), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d) + b*meijerg(((-1/2, -1/4, 0, 1/4, 1
/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(-2*I*pi)/(d**2*x**2
))/(4*pi**(3/2)*d) - I*c*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1
/4, 0, 1/4, 1/2, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**2) - c*meijerg(((
-1, -3/4, -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp
_polar(-2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**2)
```

3.17. $\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\sqrt{1+dx}}\,dx$

3.17.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\sqrt{1+dx}}\,dx = -a\log\left(\frac{2\sqrt{-d^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{b\arcsin(dx)}{d}-\frac{\sqrt{-d^2x^2+1}c}{d^2}$$

input
```
integrate((c*x^2+b*x+a)/x/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxim
a")
```

output
```
-a*log(2*sqrt(-d^2*x^2 + 1)/abs(x) + 2/abs(x)) + b*arcsin(d*x)/d - sqrt(-d
^2*x^2 + 1)*c/d^2
```

3.17.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(44) = 88.

Time = 0.35 (sec) , antiderivative size = 196, normalized size of antiderivative = 4.08

$$\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\sqrt{1+dx}}\,dx =$$

$$-\frac{ad^2\log\left(\left|-\frac{\sqrt{2}-\sqrt{-dx+1}}{\sqrt{dx+1}}+\frac{\sqrt{dx+1}}{\sqrt{2}-\sqrt{-dx+1}}+2\right|\right)-ad^2\log\left(\left|-\frac{\sqrt{2}-\sqrt{-dx+1}}{\sqrt{dx+1}}+\frac{\sqrt{dx+1}}{\sqrt{2}-\sqrt{-dx+1}}-2\right|\right)-\left(\pi+2\,\mathrm{arct}\right.}{d^2}$$

input
```
integrate((c*x^2+b*x+a)/x/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac"
)
```

output
```
-(a*d^2*log(abs(-(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + sqrt(d*x + 1)/
(sqrt(2) - sqrt(-d*x + 1)) + 2)) - a*d^2*log(abs(-(sqrt(2) - sqrt(-d*x + 1
))/sqrt(d*x + 1) + sqrt(d*x + 1)/(sqrt(2) - sqrt(-d*x + 1)) - 2)) - (pi +
2*arctan(1/2*sqrt(d*x + 1)*((sqrt(2) - sqrt(-d*x + 1))^2/(d*x + 1) - 1)/(s
qrt(2) - sqrt(-d*x + 1))))*b*d + sqrt(d*x + 1)*sqrt(-d*x + 1)*c)/d^2
```

3.17.9 Mupad [B] (verification not implemented)

Time = 4.51 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.54

$$\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\sqrt{1+dx}}\,dx = a\left(\ln\left(\frac{\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}-1\right)-\ln\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)\right) - \frac{\sqrt{1-d\,x}\left(\frac{c}{d^2}+\frac{c\,x}{d}\right)}{\sqrt{d\,x+1}} - \frac{4\,b\,\mathrm{atan}\left(\frac{d\left(\sqrt{1-d\,x}-1\right)}{\left(\sqrt{d\,x+1}-1\right)\sqrt{d^2}}\right)}{\sqrt{d^2}}$$

input
```
int((a + b*x + c*x^2)/(x*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

output
```
a*(log(((1 - d*x)^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 - 1) - log(((1 - d*
x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1))) - ((1 - d*x)^(1/2)*(c/d^2 + (c*x)/d)
)/(d*x + 1)^(1/2) - (4*b*atan((d*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2)
- 1)*(d^2)^(1/2))))/(d^2)^(1/2)
```

3.18 $\int \frac{a+bx+cx^2}{x^2\sqrt{1-dx}\sqrt{1+dx}}\,dx$

3.18.1 Optimal result

Integrand size = 33, antiderivative size = 48

$$\int \frac{a+bx+cx^2}{x^2\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{a\sqrt{1-d^2x^2}}{x}+\frac{c\arcsin(dx)}{d}-b\,\mathrm{arctanh}\left(\sqrt{1-d^2x^2}\right)$$

output
```
c*arcsin(d*x)/d-b*arctanh((-d^2*x^2+1)^(1/2))-a*(-d^2*x^2+1)^(1/2)/x
```

3.18.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.52

$$\int \frac{a+bx+cx^2}{x^2\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{a\sqrt{1-d^2x^2}}{x}+\frac{2c\arctan\left(\frac{dx}{-1+\sqrt{1-d^2x^2}}\right)}{d} - b\log(x)+b\log\left(-1+\sqrt{1-d^2x^2}\right)$$

input
```
Integrate[(a + b*x + c*x^2)/(x^2*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

output
```
-((a*Sqrt[1 - d^2*x^2])/x) + (2*c*ArcTan[(d*x)/(-1 + Sqrt[1 - d^2*x^2])])/
d - b*Log[x] + b*Log[-1 + Sqrt[1 - d^2*x^2]]
```

3.18.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2112, 2338, 25, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{x^2\sqrt{1-dx}\sqrt{dx+1}}dx$$

$$\Big\downarrow \text{ 2112}$$

$$\int \frac{a+bx+cx^2}{x^2\sqrt{1-d^2x^2}}dx$$

$$\Big\downarrow \text{ 2338}$$

$$-\int -\frac{b+cx}{x\sqrt{1-d^2x^2}}dx-\frac{a\sqrt{1-d^2x^2}}{x}$$

$$\Big\downarrow \text{ 25}$$

$$\int \frac{b+cx}{x\sqrt{1-d^2x^2}}dx-\frac{a\sqrt{1-d^2x^2}}{x}$$

$$\Big\downarrow \text{ 538}$$

$$b\int \frac{1}{x\sqrt{1-d^2x^2}}dx+c\int \frac{1}{\sqrt{1-d^2x^2}}dx-\frac{a\sqrt{1-d^2x^2}}{x}$$

$$\Big\downarrow \text{ 223}$$

$$b\int \frac{1}{x\sqrt{1-d^2x^2}}dx-\frac{a\sqrt{1-d^2x^2}}{x}+\frac{c\arcsin(dx)}{d}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}b\int \frac{1}{x^2\sqrt{1-d^2x^2}}dx^2-\frac{a\sqrt{1-d^2x^2}}{x}+\frac{c\arcsin(dx)}{d}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{b\int \frac{1}{\frac{1}{d^2}-\frac{x^4}{d^2}}d\sqrt{1-d^2x^2}}{d^2}-\frac{a\sqrt{1-d^2x^2}}{x}+\frac{c\arcsin(dx)}{d}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{a\sqrt{1-d^2x^2}}{x}+\frac{c\arcsin(dx)}{d}-b\text{arctanh}\left(\sqrt{1-d^2x^2}\right)$$

input
```
Int[(a + b*x + c*x^2)/(x^2*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

output
```
-((a*Sqrt[1 - d^2*x^2])/x) + (c*ArcSin[d*x])/d - b*ArcTanh[Sqrt[1 - d^2*x^
2]]
```

3.18.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2112
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

3.18.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.64 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.02

method	result	size
default	$\frac{\left(-\operatorname{arctanh}\left(\frac{1}{\sqrt{-d^{2} x^{2}+1}}\right) \operatorname{csgn}(d) d b x-\sqrt{-d^{2} x^{2}+1}\, \operatorname{csgn}(d) d a+\arctan\left(\frac{\operatorname{csgn}(d) d x}{\sqrt{-d^{2} x^{2}+1}}\right) c x\right) \sqrt{-d x+1}\, \sqrt{d x+1}\, \operatorname{csgn}(d)}{\sqrt{-d^{2} x^{2}+1}\, x d}$	97
risch	$\frac{a \sqrt{d x+1}\, (d x-1) \sqrt{(-d x+1)(d x+1)}}{x \sqrt{-(d x+1)(d x-1)}\, \sqrt{-d x+1}}+\frac{\left(\frac{c \arctan\left(\frac{\sqrt{d^{2}}\, x}{\sqrt{-d^{2} x^{2}+1}}\right)}{\sqrt{d^{2}}}-b \operatorname{arctanh}\left(\frac{1}{\sqrt{-d^{2} x^{2}+1}}\right)\right) \sqrt{(-d x+1)(d x+1)}}{\sqrt{-d x+1}\, \sqrt{d x+1}}$	129

input
```
int((c*x^2+b*x+a)/x^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
(-arctanh(1/(-d^2*x^2+1)^(1/2))*csgn(d)*d*b*x-(-d^2*x^2+1)^(1/2)*csgn(d)*d
*a+arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*c*x)*(-d*x+1)^(1/2)*(d*x+1)^(1/2
)*csgn(d)/(-d^2*x^2+1)^(1/2)/x/d
```

3.18.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.75

$$\int \frac{a+bx+cx^2}{x^2\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{bdx\log\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{x}\right)-\sqrt{dx+1}\sqrt{-dx+1}ad-2\,cx\arctan\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{dx}\right)}{dx}$$

input
```
integrate((c*x^2+b*x+a)/x^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fri
cas")
```

output
```
(b*d*x*log((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/x) - sqrt(d*x + 1)*sqrt(-d*x
 + 1)*a*d - 2*c*x*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/(d*x)
```

3.18.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 28.22 (sec) , antiderivative size = 221, normalized size of antiderivative = 4.60

$$\int \frac{a+bx+cx^2}{x^2\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{iadG_{6,6}^{5,3}\left(\begin{matrix} \frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2 \\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0 \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{adG_{6,6}^{2,6}\left(\begin{matrix} \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0 \end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{ibG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2} \\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0 \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{bG_{6,6}^{2,6}\left(\begin{matrix} 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0 \end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{icG_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1 \\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d} + \frac{cG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0 \end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}$$

input
```
integrate((c*x**2+b*x+a)/x**2/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

output
```
I*a*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,)
), 1/(d**2*x**2))/(4*pi**(3/2)) + a*d*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1),
 ()), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi*
*(3/2)) + I*b*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/
2), (0,)), 1/(d**2*x**2))/(4*pi**(3/2)) - b*meijerg(((0, 1/4, 1/2, 3/4, 1,
 1), ()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/
(4*pi**(3/2)) - I*c*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2,
 3/4, 1, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d) + c*meijerg(((-1/2, -1/4,
 0, 1/4, 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(-2*I*pi)/
(d**2*x**2))/(4*pi**(3/2)*d)
```

3.18.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int \frac{a+bx+cx^2}{x^2\sqrt{1-dx}\sqrt{1+dx}}\,dx = -b\log\left(\frac{2\sqrt{-d^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{c\arcsin(dx)}{d}-\frac{\sqrt{-d^2x^2+1}a}{x}$$

input
```
integrate((c*x^2+b*x+a)/x^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="max
ima")
```

output
```
-b*log(2*sqrt(-d^2*x^2 + 1)/abs(x) + 2/abs(x)) + c*arcsin(d*x)/d - sqrt(-d
^2*x^2 + 1)*a/x
```

3.18.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. 2(44) = 88.

Time = 0.38 (sec) , antiderivative size = 282, normalized size of antiderivative = 5.88

$$\int \frac{a+bx+cx^2}{x^2\sqrt{1-dx}\sqrt{1+dx}}\,dx =$$

$$-\frac{\frac{4ad^2\left(\frac{\sqrt{2}-\sqrt{-dx+1}}{\sqrt{dx+1}}-\frac{\sqrt{dx+1}}{\sqrt{2}-\sqrt{-dx+1}}\right)}{\left(\frac{\sqrt{2}-\sqrt{-dx+1}}{\sqrt{dx+1}}-\frac{\sqrt{dx+1}}{\sqrt{2}-\sqrt{-dx+1}}\right)^2-4}+bd\log\left(\left|-\frac{\sqrt{2}-\sqrt{-dx+1}}{\sqrt{dx+1}}+\frac{\sqrt{dx+1}}{\sqrt{2}-\sqrt{-dx+1}}+2\right|\right)-bd\log\left(\left|-\frac{\sqrt{2}-\sqrt{-dx+1}}{\sqrt{dx+1}}+\frac{\sqrt{d}}{\sqrt{2}-\sqrt{}}\right.\right.}{d}$$

input
```
integrate((c*x^2+b*x+a)/x^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="gia
c")
```

output
```
-(4*a*d^2*((sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) - sqrt(d*x + 1)/(sqrt(
2) - sqrt(-d*x + 1)))/(((sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) - sqrt(d*
x + 1)/(sqrt(2) - sqrt(-d*x + 1)))^2 - 4) + b*d*log(abs(-(sqrt(2) - sqrt(-
d*x + 1))/sqrt(d*x + 1) + sqrt(d*x + 1)/(sqrt(2) - sqrt(-d*x + 1)) + 2)) -
 b*d*log(abs(-(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + sqrt(d*x + 1)/(sq
rt(2) - sqrt(-d*x + 1)) - 2)) - (pi + 2*arctan(1/2*sqrt(d*x + 1)*((sqrt(2)
 - sqrt(-d*x + 1))^2/(d*x + 1) - 1)/(sqrt(2) - sqrt(-d*x + 1))))*c)/d
```

3.18.9 Mupad [B] (verification not implemented)

Time = 4.94 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.38

$$\int \frac{a+bx+cx^2}{x^2\sqrt{1-dx}\sqrt{1+dx}}\,dx = b\left(\ln\left(\frac{\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}-1\right)-\ln\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)\right) - \frac{4\,c\,\mathrm{atan}\left(\frac{d\,\left(\sqrt{1-d\,x}-1\right)}{\left(\sqrt{d\,x+1}-1\right)\,\sqrt{d^2}}\right)}{\sqrt{d^2}} - \frac{a\,\sqrt{1-d\,x}\,\sqrt{d\,x+1}}{x}$$

input
```
int((a + b*x + c*x^2)/(x^2*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

output
```
b*(log(((1 - d*x)^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 - 1) - log(((1 - d*
x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1))) - (4*c*atan((d*((1 - d*x)^(1/2) - 1)
)/(((d*x + 1)^(1/2) - 1)*(d^2)^(1/2))))/(d^2)^(1/2) - (a*(1 - d*x)^(1/2)*(
d*x + 1)^(1/2))/x
```

3.19 $\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\sqrt{1+dx}}\,dx$

3.19.1 Optimal result

Integrand size = 33, antiderivative size = 71

$$\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\sqrt{1+dx}}\,dx=-\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{b\sqrt{1-d^2x^2}}{x}-\frac{1}{2}\left(2c+ad^2\right)\operatorname{arctanh}\left(\sqrt{1-d^2x^2}\right)$$

output
```
-1/2*(a*d^2+2*c)*arctanh((-d^2*x^2+1)^(1/2))-1/2*a*(-d^2*x^2+1)^(1/2)/x^2-
b*(-d^2*x^2+1)^(1/2)/x
```

3.19.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99

$$\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\sqrt{1+dx}}\,dx=\frac{1}{2}\left(-\frac{(a+2bx)\sqrt{1-d^2x^2}}{x^2}-\left(2c+ad^2\right)\log(x)+\left(2c+ad^2\right)\log\left(-1+\sqrt{1-d^2x^2}\right)\right)$$

input
```
Integrate[(a + b*x + c*x^2)/(x^3*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

output
```
(-(((a + 2*b*x)*Sqrt[1 - d^2*x^2])/x^2) - (2*c + a*d^2)*Log[x] + (2*c + a*
d^2)*Log[-1 + Sqrt[1 - d^2*x^2]])/2
```

3.19.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2112, 2338, 25, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\sqrt{dx+1}}dx$$

$$\Big\downarrow \text{2112}$$

$$\int \frac{a+bx+cx^2}{x^3\sqrt{1-d^2x^2}}dx$$

$$\Big\downarrow \text{2338}$$

$$-\frac{1}{2}\int -\frac{2b+\left(ad^2+2c\right)x}{x^2\sqrt{1-d^2x^2}}dx-\frac{a\sqrt{1-d^2x^2}}{2x^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int \frac{2b+\left(ad^2+2c\right)x}{x^2\sqrt{1-d^2x^2}}dx-\frac{a\sqrt{1-d^2x^2}}{2x^2}$$

$$\Big\downarrow \text{534}$$

$$\frac{1}{2}\left(\left(ad^2+2c\right)\int \frac{1}{x\sqrt{1-d^2x^2}}dx-\frac{2b\sqrt{1-d^2x^2}}{x}\right)-\frac{a\sqrt{1-d^2x^2}}{2x^2}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(ad^2+2c\right)\int \frac{1}{x^2\sqrt{1-d^2x^2}}dx^2-\frac{2b\sqrt{1-d^2x^2}}{x}\right)-\frac{a\sqrt{1-d^2x^2}}{2x^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(-\frac{\left(ad^2+2c\right)\int \frac{1}{\frac{1}{d^2}-\frac{x^4}{d^2}}d\sqrt{1-d^2x^2}}{d^2}-\frac{2b\sqrt{1-d^2x^2}}{x}\right)-\frac{a\sqrt{1-d^2x^2}}{2x^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(-\left(ad^2+2c\right)\text{arctanh}\left(\sqrt{1-d^2x^2}\right)-\frac{2b\sqrt{1-d^2x^2}}{x}\right)-\frac{a\sqrt{1-d^2x^2}}{2x^2}$$

input
```
Int[(a + b*x + c*x^2)/(x^3*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

output
```
-1/2*(a*Sqrt[1 - d^2*x^2])/x^2 + ((-2*b*Sqrt[1 - d^2*x^2])/x - (2*c + a*d^
2)*ArcTanh[Sqrt[1 - d^2*x^2]])/2
```

3.19.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 2112
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

3.19.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.62 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.52

method	result	siz
default	$-\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\operatorname{csgn}\left(d\right)^{2}\left(\operatorname{arctanh}\left(\frac{1}{\sqrt{-d^{2}x^{2}+1}}\right)a\,d^{2}x^{2}+2\,\operatorname{arctanh}\left(\frac{1}{\sqrt{-d^{2}x^{2}+1}}\right)c\,x^{2}+2\sqrt{-d^{2}x^{2}+1}\,bx+\sqrt{-d^{2}x^{2}+1}\,a\right)}{2\sqrt{-d^{2}x^{2}+1}\,x^{2}}$	108
risch	$\frac{\sqrt{dx+1}\,\left(dx-1\right)\left(2bx+a\right)\sqrt{\left(-dx+1\right)\left(dx+1\right)}}{2x^{2}\sqrt{-\left(dx+1\right)\left(dx-1\right)}\,\sqrt{-dx+1}}-\frac{\left(c+\frac{a\,d^{2}}{2}\right)\operatorname{arctanh}\left(\frac{1}{\sqrt{-d^{2}x^{2}+1}}\right)\sqrt{\left(-dx+1\right)\left(dx+1\right)}}{\sqrt{-dx+1}\,\sqrt{dx+1}}$	113

input
```
int((c*x^2+b*x+a)/x^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
-1/2*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*csgn(d)^2*(arctanh(1/(-d^2*x^2+1)^(1/2))
*a*d^2*x^2+2*arctanh(1/(-d^2*x^2+1)^(1/2))*c*x^2+2*(-d^2*x^2+1)^(1/2)*b*x+
(-d^2*x^2+1)^(1/2)*a)/(-d^2*x^2+1)^(1/2)/x^2
```

3.19.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.92

$$\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{(ad^2+2\,c)x^2\log\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{x}\right)-(2\,bx+a)\sqrt{dx+1}\sqrt{-dx+1}}{2\,x^2}$$

input
```
integrate((c*x^2+b*x+a)/x^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fri
cas")
```

output
```
1/2*((a*d^2 + 2*c)*x^2*log((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/x) - (2*b*x
+ a)*sqrt(d*x + 1)*sqrt(-d*x + 1))/x^2
```

3.19.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\sqrt{1+dx}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)/x**3/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

output
```
Timed out
```

3.19.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.38

$$\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{1}{2}\,ad^2\log\left(\frac{2\sqrt{-d^2x^2+1}}{|x|}+\frac{2}{|x|}\right) - c\log\left(\frac{2\sqrt{-d^2x^2+1}}{|x|}+\frac{2}{|x|}\right) - \frac{\sqrt{-d^2x^2+1}b}{x} - \frac{\sqrt{-d^2x^2+1}a}{2\,x^2}$$

input
```
integrate((c*x^2+b*x+a)/x^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="max
ima")
```

output
```
-1/2*a*d^2*log(2*sqrt(-d^2*x^2 + 1)/abs(x) + 2/abs(x)) - c*log(2*sqrt(-d^2
*x^2 + 1)/abs(x) + 2/abs(x)) - sqrt(-d^2*x^2 + 1)*b/x - 1/2*sqrt(-d^2*x^2
+ 1)*a/x^2
```

3.19.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. $2(61) = 122$.

Time = 0.37 (sec) , antiderivative size = 407, normalized size of antiderivative = 5.73

$$\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\sqrt{1+dx}}\,dx =$$

$$-\frac{(ad^3+2\,cd)\log\left(\left|-\frac{\sqrt{2}-\sqrt{-dx+1}}{\sqrt{dx+1}}+\frac{\sqrt{dx+1}}{\sqrt{2}-\sqrt{-dx+1}}+2\right|\right)-(ad^3+2\,cd)\log\left(\left|-\frac{\sqrt{2}-\sqrt{-dx+1}}{\sqrt{dx+1}}+\frac{\sqrt{dx+1}}{\sqrt{2}-\sqrt{-dx+1}}-2\right.\right.}{}$$

input
```
integrate((c*x^2+b*x+a)/x^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="gia
c")
```

output
```
-1/2*((a*d^3 + 2*c*d)*log(abs(-(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) +
sqrt(d*x + 1)/(sqrt(2) - sqrt(-d*x + 1)) + 2)) - (a*d^3 + 2*c*d)*log(abs(-
(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + sqrt(d*x + 1)/(sqrt(2) - sqrt(-
d*x + 1)) - 2)) - 4*(a*d^3*((sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) - sqr
t(d*x + 1)/(sqrt(2) - sqrt(-d*x + 1)))^3 - 2*b*d^2*((sqrt(2) - sqrt(-d*x +
 1))/sqrt(d*x + 1) - sqrt(d*x + 1)/(sqrt(2) - sqrt(-d*x + 1)))^3 + 4*a*d^3
*((sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) - sqrt(d*x + 1)/(sqrt(2) - sqrt
(-d*x + 1))) + 8*b*d^2*((sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) - sqrt(d*
x + 1)/(sqrt(2) - sqrt(-d*x + 1))))/(((sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x
+ 1) - sqrt(d*x + 1)/(sqrt(2) - sqrt(-d*x + 1)))^2 - 4)^2)/d
```

3.19.9 Mupad [B] (verification not implemented)

Time = 7.63 (sec) , antiderivative size = 312, normalized size of antiderivative = 4.39

$$\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\sqrt{1+dx}}\,dx = c\left(\ln\left(\frac{\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}-1\right)-\ln\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)\right)$$
$$-\frac{\frac{a\,d^2\,\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}-\frac{a\,d^2}{2}+\frac{15\,a\,d^2\,\left(\sqrt{1-d\,x}-1\right)^4}{2\,\left(\sqrt{d\,x+1}-1\right)^4}}{\frac{16\,\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}-\frac{32\,\left(\sqrt{1-d\,x}-1\right)^4}{\left(\sqrt{d\,x+1}-1\right)^4}+\frac{16\,\left(\sqrt{1-d\,x}-1\right)^6}{\left(\sqrt{d\,x+1}-1\right)^6}}$$
$$+\frac{a\,d^2\,\ln\left(\frac{\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}-1\right)}{2}-\frac{a\,d^2\,\ln\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)}{2}$$
$$-\frac{b\,\sqrt{1-d\,x}\,\sqrt{d\,x+1}}{x}+\frac{a\,d^2\,\left(\sqrt{1-d\,x}-1\right)^2}{32\,\left(\sqrt{d\,x+1}-1\right)^2}$$

input
```
int((a + b*x + c*x^2)/(x^3*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

output
```
c*(log(((1 - d*x)^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 - 1) - log(((1 - d*
x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1))) - ((a*d^2*((1 - d*x)^(1/2) - 1)^2)/(
(d*x + 1)^(1/2) - 1)^2 - (a*d^2)/2 + (15*a*d^2*((1 - d*x)^(1/2) - 1)^4)/(2
*((d*x + 1)^(1/2) - 1)^4))/((16*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2)
- 1)^2 - (32*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (16*((1 -
d*x)^(1/2) - 1)^6)/((d*x + 1)^(1/2) - 1)^6) + (a*d^2*log(((1 - d*x)^(1/2)
- 1)^2/((d*x + 1)^(1/2) - 1)^2 - 1))/2 - (a*d^2*log(((1 - d*x)^(1/2) - 1)/
((d*x + 1)^(1/2) - 1)))/2 - (b*(1 - d*x)^(1/2)*(d*x + 1)^(1/2))/x + (a*d^2
*((1 - d*x)^(1/2) - 1)^2)/(32*((d*x + 1)^(1/2) - 1)^2)
```

3.19. $\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\sqrt{1+dx}}\,dx$

3.20 $\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx$

3.20.1 Optimal result

Integrand size = 40, antiderivative size = 591

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx$$
$$=\frac{(A(8b^4e^3+6a^2b^2ef^2)+a^2(a^2f^2(3Ce+Bf)+2b^2e^2(Ce+3Bf)))\,x\sqrt{a+bx}\sqrt{ac-bcx}}{16b^4}$$
$$-\frac{(8a^2Cf^2-b^2(3Ce^2-7f(Be+2Af)))\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\left(a^2-b^2x^2\right)}{70b^4f}$$
$$+\frac{(3Ce-7Bf)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3\left(a^2-b^2x^2\right)}{42b^2f}$$
$$-\frac{C\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^4\left(a^2-b^2x^2\right)}{7b^2f}$$
$$-\frac{\sqrt{a+bx}\sqrt{ac-bcx}(8(8a^4Cf^4+2a^2b^2f^2(15Ce^2+7f(3Be+Af))-b^4e^2(3Ce^2-7f(Be+12Af)))+}{840b^6f}$$
$$+\frac{a^2\sqrt{c}(A(8b^4e^3+6a^2b^2ef^2)+a^2(a^2f^2(3Ce+Bf)+2b^2e^2(Ce+3Bf)))\sqrt{a+bx}\sqrt{ac-bcx}\arctan\left(\frac{}{\sqrt{a}}\right.}{16b^5\sqrt{a^2c-b^2cx^2}}$$

output
```
1/16*(A*(6*a^2*b^2*e*f^2+8*b^4*e^3)+a^2*(a^2*f^2*(B*f+3*C*e)+2*b^2*e^2*(3*
B*f+C*e)))*x*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)/b^4-1/70*(8*a^2*C*f^2-b^2*(3
*C*e^2-7*f*(2*A*f+B*e)))*(f*x+e)^2*(-b^2*x^2+a^2)*(b*x+a)^(1/2)*(-b*c*x+a*
c)^(1/2)/b^4/f+1/42*(-7*B*f+3*C*e)*(f*x+e)^3*(-b^2*x^2+a^2)*(b*x+a)^(1/2)*
(-b*c*x+a*c)^(1/2)/b^2/f-1/7*C*(f*x+e)^4*(-b^2*x^2+a^2)*(b*x+a)^(1/2)*(-b*
c*x+a*c)^(1/2)/b^2/f-1/840*(64*a^4*C*f^4+16*a^2*b^2*f^2*(15*C*e^2+7*f*(A*f
+3*B*e))-8*b^4*e^2*(3*C*e^2-7*f*(12*A*f+B*e))+3*b^2*f*(a^2*f^2*(35*B*f+41*
C*e)-2*b^2*e*(3*C*e^2-7*f*(7*A*f+B*e)))*x)*(-b^2*x^2+a^2)*(b*x+a)^(1/2)*(-
b*c*x+a*c)^(1/2)/b^6/f+1/16*a^2*(A*(6*a^2*b^2*e*f^2+8*b^4*e^3)+a^2*(a^2*f^
2*(B*f+3*C*e)+2*b^2*e^2*(3*B*f+C*e)))*arctan(b*x*c^(1/2)/(-b^2*c*x^2+a^2*c
)^(1/2))*c^(1/2)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)/b^5/(-b^2*c*x^2+a^2*c)^(
1/2)
```

3.20.2 Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 402, normalized size of antiderivative = 0.68

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx$$
$$= \frac{\sqrt{c(a-bx)}\left(-\sqrt{a-bx}\sqrt{a+bx}(128a^6Cf^3+a^4b^2f(7f(96Be+32Af+15Bfx)+C(672e^2+315efx+\right.}{}$$

input
```
Integrate[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^3*(A + B*x + C*x^2),x]
```

output
```
(Sqrt[c*(a - b*x)]*(-(Sqrt[a - b*x]*Sqrt[a + b*x]*(128*a^6*C*f^3 + a^4*b^2
*f*(7*f*(96*B*e + 32*A*f + 15*B*f*x) + C*(672*e^2 + 315*e*f*x + 64*f^2*x^2
)) + 2*a^2*b^4*(7*A*f*(120*e^2 + 45*e*f*x + 8*f^2*x^2) + 7*B*(40*e^3 + 45*
e^2*f*x + 24*e*f^2*x^2 + 5*f^3*x^3) + 3*C*x*(35*e^3 + 56*e^2*f*x + 35*e*f^
2*x^2 + 8*f^3*x^3)) - 4*b^6*x*(21*A*(10*e^3 + 20*e^2*f*x + 15*e*f^2*x^2 +
4*f^3*x^3) + x*(7*B*(20*e^3 + 45*e^2*f*x + 36*e*f^2*x^2 + 10*f^3*x^3) + 3*
C*x*(35*e^3 + 84*e^2*f*x + 70*e*f^2*x^2 + 20*f^3*x^3)))) + 210*a^2*b*(a^4
*f^2*(3*C*e + B*f) + 2*a^2*b^2*e^2*(C*e + 3*B*f) + A*(8*b^4*e^3 + 6*a^2*b^
2*e*f^2))*ArcTan[Sqrt[a + b*x]/Sqrt[a - b*x]]))/(1680*b^6*Sqrt[a - b*x])
```

3.20.3 Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 512, normalized size of antiderivative = 0.87, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.325, Rules used = {2113, 2185, 25, 27, 687, 27, 687, 25, 27, 676, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx}(e+fx)^3\sqrt{ac-bcx}\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{2113}$$

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\int (e+fx)^3\sqrt{a^2c-b^2cx^2}\left(Cx^2+Bx+A\right)dx}{\sqrt{a^2c-b^2cx^2}}$$

$$\Big\downarrow \text{2185}$$

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(-\frac{\int -cf(e+fx)^3\left(\left(4Ca^2+7Ab^2\right)f-b^2(3Ce-7Bf)x\right)\sqrt{a^2c-b^2cx^2}dx}{7b^2cf^2}-\frac{C(e+fx)^4\left(a^2c-b^2cx^2\right)^{3/2}}{7b^2cf}\right)}{\sqrt{a^2c-b^2cx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\int cf(e+fx)^3\left(\left(4Ca^2+7Ab^2\right)f-b^2(3Ce-7Bf)x\right)\sqrt{a^2c-b^2cx^2}dx}{7b^2cf^2}-\frac{C(e+fx)^4\left(a^2c-b^2cx^2\right)^{3/2}}{7b^2cf}\right)}{\sqrt{a^2c-b^2cx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\int (e+fx)^3\left(\left(4Ca^2+7Ab^2\right)f-b^2(3Ce-7Bf)x\right)\sqrt{a^2c-b^2cx^2}dx}{7b^2f}-\frac{C(e+fx)^4\left(a^2c-b^2cx^2\right)^{3/2}}{7b^2cf}\right)}{\sqrt{a^2c-b^2cx^2}}$$

$$\Big\downarrow \text{687}$$

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\frac{(e+fx)^3\left(a^2c-b^2cx^2\right)^{3/2}(3Ce-7Bf)}{6c}-\frac{\int -3b^2c(e+fx)^2\left(f\left(\left(5Ce+7Bf\right)a^2+14Ab^2e\right)+\left(8a^2Cf^2-b^2\left(3Ce^2-7f(Be+2Af)\right)\right)x\right)\sqrt{a^2}}{6b^2c}}{7b^2f}\right.}{\sqrt{a^2c-b^2cx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\frac{1}{2}\int (e+fx)^2\left(f\left(\left(5Ce+7Bf\right)a^2+14Ab^2e\right)+\left(8a^2Cf^2-b^2\left(3Ce^2-7f(Be+2Af)\right)\right)x\right)\sqrt{a^2c-b^2cx^2}dx+\frac{(e+fx)^3\left(a^2c-b^2cx^2\right)}{6c}}{7b^2f}\right.}{\sqrt{a^2c-b^2cx^2}}$$

$\downarrow$ 687

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\Bigg(\frac{\frac{1}{2}\left(-\frac{\int -c(e+fx)\left(\left(a^2f^2(41Ce+35Bf)-b^2\left(6Ce^3-14ef(Be+7Af)\right)\right)xb^2+f\left(16Cf^2a^4+b^2e(19Ce+49Bf)a^2+14A\left(5e^2b^4+2a^2f^2b^2\right.\right.\right.}{5b^2c}}{7b^2 f}}{\sqrt{a^2c}}$$

$\downarrow$ 25

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\Bigg(\frac{\frac{1}{2}\left(\frac{\int c(e+fx)\left(\left(a^2f^2(41Ce+35Bf)-b^2\left(6Ce^3-14ef(Be+7Af)\right)\right)xb^2+f\left(16Cf^2a^4+b^2e(19Ce+49Bf)a^2+14A\left(5e^2b^4+2a^2f^2b^2\right)\right)\right)}{5b^2c}\right.}{7b^2f}}{\sqrt{a^2c-}}$$

$\downarrow$ 27

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\Bigg(\frac{\frac{1}{2}\left(\frac{\int (e+fx)\left(\left(a^2f^2(41Ce+35Bf)-b^2\left(6Ce^3-14ef(Be+7Af)\right)\right)xb^2+f\left(16Cf^2a^4+b^2e(19Ce+49Bf)a^2+14A\left(5e^2b^4+2a^2f^2b^2\right)\right)\right)\sqrt{}}{5b^2}\right.}{7b^2f}}{\sqrt{a^2c-}}$$

$\downarrow$ 676

$$\sqrt{a+bx}\sqrt{ac-bcx}\Bigg(\frac{\frac{1}{2}\left(\frac{\frac{35}{4}f\left(a^4f^2(Bf+3Ce)+A\left(6a^2b^2ef^2+8b^4e^3\right)+2a^2b^2e^2(3Bf+Ce)\right)\int\sqrt{a^2c-b^2cx^2}dx-\frac{fx\left(a^2c-b^2cx^2\right)^{3/2}\left(a^2f^2(35Bf+41Ce)\right.}{4c}}{5}\right.}{}$$

$\downarrow$ 211

$$\sqrt{a+bx}\sqrt{ac-bcx}\Bigg(\frac{\frac{1}{2}\left(\frac{\frac{35}{4}f\left(a^4f^2(Bf+3Ce)+A\left(6a^2b^2ef^2+8b^4e^3\right)+2a^2b^2e^2(3Bf+Ce)\right)\left(\frac{1}{2}a^2c\int\frac{1}{\sqrt{a^2c-b^2cx^2}}dx+\frac{1}{2}x\sqrt{a^2c-b^2cx^2}\right)-\frac{fx\left(a^2c-b^2cx\right.}{}}{}\right.}{}$$

3.20. $\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx$

$$\Big\downarrow \; 224$$

$$\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\frac{1}{2}\left(\frac{\frac{35}{4}f\left(a^4f^2(Bf+3Ce)+A\left(6a^2b^2ef^2+8b^4e^3\right)+2a^2b^2e^2(3Bf+Ce)\right)\left(\frac{1}{2}a^2c\int\frac{1}{\frac{b^2cx^2}{a^2c-b^2cx^2}+1}d\frac{x}{\sqrt{a^2c-b^2cx^2}}+\frac{1}{2}x\sqrt{a^2c-b^2cx^2}\right)\cdots}{\cdots}\right.}{\cdots}\right.$$

$$\Big\downarrow \; 216$$

$$\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\frac{(e+fx)^3\left(a^2c-b^2cx^2\right)^{3/2}(3Ce-7Bf)}{6c}+\frac{1}{2}\left(\frac{-\frac{fx\left(a^2c-b^2cx^2\right)^{3/2}\left(a^2f^2(35Bf+41Ce)-b^2\left(6Ce^3-14ef(7Af+Be)\right)\right)}{4c}+\frac{35}{4}f\left(\frac{a^2\sqrt{c}\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

input
```
Int[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^3*(A + B*x + C*x^2),x]
```

output
```
(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(-1/7*(C*(e + f*x)^4*(a^2*c - b^2*c*x^2)^
(3/2))/(b^2*c*f) + (((3*C*e - 7*B*f)*(e + f*x)^3*(a^2*c - b^2*c*x^2)^(3/2)
)/(6*c) + (-1/5*((8*a^2*C*f^2 - b^2*(3*C*e^2 - 7*f*(B*e + 2*A*f)))*(e + f*
x)^2*(a^2*c - b^2*c*x^2)^(3/2))/(b^2*c) + ((-2*(8*a^4*C*f^4 + 2*a^2*b^2*f^
2*(15*C*e^2 + 7*f*(3*B*e + A*f)) - b^4*(3*C*e^4 - 7*e^2*f*(B*e + 12*A*f)))
*(a^2*c - b^2*c*x^2)^(3/2))/(3*b^2*c) - (f*(a^2*f^2*(41*C*e + 35*B*f) - b^
2*(6*C*e^3 - 14*e*f*(B*e + 7*A*f)))*x*(a^2*c - b^2*c*x^2)^(3/2))/(4*c) + (
35*f*(a^4*f^2*(3*C*e + B*f) + 2*a^2*b^2*e^2*(C*e + 3*B*f) + A*(8*b^4*e^3 +
 6*a^2*b^2*e*f^2))*((x*Sqrt[a^2*c - b^2*c*x^2])/2 + (a^2*Sqrt[c]*ArcTan[(b
*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(2*b)))/4)/(5*b^2))/2)/(7*b^2*f)))/S
qrt[a^2*c - b^2*c*x^2]
```

3.20. $\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx$

3.20.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2113
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m])   Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a
*d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.20.4 Maple [A] (verified)

Time = 1.69 (sec) , antiderivative size = 575, normalized size of antiderivative = 0.97

method	result
risch	$-\frac{(-240C f^3x^6b^6-280B b^6f^3x^5-840C b^6e f^2x^5-336A b^6f^3x^4-1008B b^6e f^2x^4+48C a^2b^4f^3x^4-1008C b^6e^2f x^4-1260A b^6e f^2x^3}{}$
default	Expression too large to display

input
```
int((f*x+e)^3*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/1680*(-240*C*b^6*f^3*x^6-280*B*b^6*f^3*x^5-840*C*b^6*e*f^2*x^5-336*A*b^
6*f^3*x^4-1008*B*b^6*e*f^2*x^4+48*C*a^2*b^4*f^3*x^4-1008*C*b^6*e^2*f*x^4-1
260*A*b^6*e*f^2*x^3+70*B*a^2*b^4*f^3*x^3-1260*B*b^6*e^2*f*x^3+210*C*a^2*b^
4*e*f^2*x^3-420*C*b^6*e^3*x^3+112*A*a^2*b^4*f^3*x^2-1680*A*b^6*e^2*f*x^2+3
36*B*a^2*b^4*e*f^2*x^2-560*B*b^6*e^3*x^2+64*C*a^4*b^2*f^3*x^2+336*C*a^2*b^
4*e^2*f*x^2+630*A*a^2*b^4*e*f^2*x-840*A*b^6*e^3*x+105*B*a^4*b^2*f^3*x+630*
B*a^2*b^4*e^2*f*x+315*C*a^4*b^2*e*f^2*x+210*C*a^2*b^4*e^3*x+224*A*a^4*b^2*
f^3+1680*A*a^2*b^4*e^2*f+672*B*a^4*b^2*e*f^2+560*B*a^2*b^4*e^3+128*C*a^6*f
^3+672*C*a^4*b^2*e^2*f)*(b*x+a)^(1/2)/b^6*(-b*x+a)/(-c*(b*x-a))^(1/2)*c+1/
16*a^2/b^4*(6*A*a^2*b^2*e*f^2+8*A*b^4*e^3+B*a^4*f^3+6*B*a^2*b^2*e^2*f+3*C*
a^4*e*f^2+2*C*a^2*b^2*e^3)/(b^2*c)^(1/2)*arctan((b^2*c)^(1/2)*x/(-b^2*c*x^
2+a^2*c)^(1/2))*(-(b*x+a)*c*(b*x-a))^(1/2)/(b*x+a)^(1/2)/(-c*(b*x-a))^(1/2
)*c
```

3.20.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 1001, normalized size of antiderivative = 1.69

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx$$

$$= \left[\frac{105\left(6\,Ba^4b^3e^2f+Ba^6bf^3+2\left(Ca^4b^3+4\,Aa^2b^5\right)e^3+3\left(Ca^6b+2\,Aa^4b^3\right)ef^2\right)\sqrt{-c}\log\left(2\,b^2cx^2+2\,\sqrt{-b\right.}{}}{}\right.$$

$$-\frac{105\left(6\,Ba^4b^3e^2f+Ba^6bf^3+2\left(Ca^4b^3+4\,Aa^2b^5\right)e^3+3\left(Ca^6b+2\,Aa^4b^3\right)ef^2\right)\sqrt{c}\arctan\left(\frac{\sqrt{-bcx+ac}\sqrt{bx+}}{b^2cx^2-a^2c}\right.}{}$$

input
```
integrate((f*x+e)^3*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algo
rithm="fricas")
```

output
```
[1/3360*(105*(6*B*a^4*b^3*e^2*f + B*a^6*b*f^3 + 2*(C*a^4*b^3 + 4*A*a^2*b^5
)*e^3 + 3*(C*a^6*b + 2*A*a^4*b^3)*e*f^2)*sqrt(-c)*log(2*b^2*c*x^2 + 2*sqrt
(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(-c)*x - a^2*c) + 2*(240*C*b^6*f^3*x^6
- 560*B*a^2*b^4*e^3 - 672*B*a^4*b^2*e*f^2 + 280*(3*C*b^6*e*f^2 + B*b^6*f^3
)*x^5 + 48*(21*C*b^6*e^2*f + 21*B*b^6*e*f^2 - (C*a^2*b^4 - 7*A*b^6)*f^3)*x
^4 - 336*(2*C*a^4*b^2 + 5*A*a^2*b^4)*e^2*f - 32*(4*C*a^6 + 7*A*a^4*b^2)*f^
3 + 70*(6*C*b^6*e^3 + 18*B*b^6*e^2*f - B*a^2*b^4*f^3 - 3*(C*a^2*b^4 - 6*A*
b^6)*e*f^2)*x^3 + 16*(35*B*b^6*e^3 - 21*B*a^2*b^4*e*f^2 - 21*(C*a^2*b^4 -
5*A*b^6)*e^2*f - (4*C*a^4*b^2 + 7*A*a^2*b^4)*f^3)*x^2 - 105*(6*B*a^2*b^4*e
^2*f + B*a^4*b^2*f^3 + 2*(C*a^2*b^4 - 4*A*b^6)*e^3 + 3*(C*a^4*b^2 + 2*A*a^
2*b^4)*e*f^2)*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/b^6, -1/1680*(105*(6*B*
a^4*b^3*e^2*f + B*a^6*b*f^3 + 2*(C*a^4*b^3 + 4*A*a^2*b^5)*e^3 + 3*(C*a^6*b
 + 2*A*a^4*b^3)*e*f^2)*sqrt(c)*arctan(sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*s
qrt(c)*x/(b^2*c*x^2 - a^2*c)) - (240*C*b^6*f^3*x^6 - 560*B*a^2*b^4*e^3 - 6
72*B*a^4*b^2*e*f^2 + 280*(3*C*b^6*e*f^2 + B*b^6*f^3)*x^5 + 48*(21*C*b^6*e^
2*f + 21*B*b^6*e*f^2 - (C*a^2*b^4 - 7*A*b^6)*f^3)*x^4 - 336*(2*C*a^4*b^2 +
 5*A*a^2*b^4)*e^2*f - 32*(4*C*a^6 + 7*A*a^4*b^2)*f^3 + 70*(6*C*b^6*e^3 + 1
8*B*b^6*e^2*f - B*a^2*b^4*f^3 - 3*(C*a^2*b^4 - 6*A*b^6)*e*f^2)*x^3 + 16*(3
5*B*b^6*e^3 - 21*B*a^2*b^4*e*f^2 - 21*(C*a^2*b^4 - 5*A*b^6)*e^2*f - (4*C*a
^4*b^2 + 7*A*a^2*b^4)*f^3)*x^2 - 105*(6*B*a^2*b^4*e^2*f + B*a^4*b^2*f^3...
```

3.20.6 Sympy [F]

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx$$
$$=\int \sqrt{-c\left(-a+bx\right)}\sqrt{a+bx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx$$

input
```
integrate((f*x+e)**3*(C*x**2+B*x+A)*(b*x+a)**(1/2)*(-b*c*x+a*c)**(1/2),x)
```

output
```
Integral(sqrt(-c*(-a + b*x))*sqrt(a + b*x)*(e + f*x)**3*(A + B*x + C*x**2)
, x)
```

3.20.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 584, normalized size of antiderivative = 0.99

$$
\begin{aligned}
&\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx \\
&= -\frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}Cf^3x^4}{7\,b^2c} + \frac{Aa^2\sqrt{c}e^3\arcsin\left(\frac{bx}{a}\right)}{2\,b} + \frac{1}{2}\sqrt{-b^2cx^2+a^2c}Ae^3x \\
&\quad - \frac{4\,(-b^2cx^2+a^2c)^{\frac{3}{2}}Ca^2f^3x^2}{35\,b^4c} + \frac{(3\,Cef^2+Bf^3)a^6\sqrt{c}\arcsin\left(\frac{bx}{a}\right)}{16\,b^5} \\
&\quad + \frac{(Ce^3+3\,Be^2f+3\,Aef^2)a^4\sqrt{c}\arcsin\left(\frac{bx}{a}\right)}{8\,b^3} - \frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}Be^3}{3\,b^2c} \\
&\quad - \frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}Ae^2f}{b^2c} - \frac{8\,(-b^2cx^2+a^2c)^{\frac{3}{2}}Ca^4f^3}{105\,b^6c} \\
&\quad + \frac{\sqrt{-b^2cx^2+a^2c}(3\,Cef^2+Bf^3)a^4x}{16\,b^4} + \frac{\sqrt{-b^2cx^2+a^2c}(Ce^3+3\,Be^2f+3\,Aef^2)a^2x}{8\,b^2} \\
&\quad - \frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}(3\,Cef^2+Bf^3)x^3}{6\,b^2c} - \frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}(3\,Ce^2f+3\,Bef^2+Af^3)x^2}{5\,b^2c} \\
&\quad - \frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}(3\,Cef^2+Bf^3)a^2x}{8\,b^4c} - \frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}(Ce^3+3\,Be^2f+3\,Aef^2)x}{4\,b^2c} \\
&\quad - \frac{2\,(-b^2cx^2+a^2c)^{\frac{3}{2}}(3\,Ce^2f+3\,Bef^2+Af^3)a^2}{15\,b^4c}
\end{aligned}
$$

input

```
integrate((f*x+e)^3*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algo
rithm="maxima")
```

output

```
-1/7*(-b^2*c*x^2 + a^2*c)^(3/2)*C*f^3*x^4/(b^2*c) + 1/2*A*a^2*sqrt(c)*e^3*
arcsin(b*x/a)/b + 1/2*sqrt(-b^2*c*x^2 + a^2*c)*A*e^3*x - 4/35*(-b^2*c*x^2
+ a^2*c)^(3/2)*C*a^2*f^3*x^2/(b^4*c) + 1/16*(3*C*e*f^2 + B*f^3)*a^6*sqrt(c
)*arcsin(b*x/a)/b^5 + 1/8*(C*e^3 + 3*B*e^2*f + 3*A*e*f^2)*a^4*sqrt(c)*arcs
in(b*x/a)/b^3 - 1/3*(-b^2*c*x^2 + a^2*c)^(3/2)*B*e^3/(b^2*c) - (-b^2*c*x^2
 + a^2*c)^(3/2)*A*e^2*f/(b^2*c) - 8/105*(-b^2*c*x^2 + a^2*c)^(3/2)*C*a^4*f
^3/(b^6*c) + 1/16*sqrt(-b^2*c*x^2 + a^2*c)*(3*C*e*f^2 + B*f^3)*a^4*x/b^4 +
 1/8*sqrt(-b^2*c*x^2 + a^2*c)*(C*e^3 + 3*B*e^2*f + 3*A*e*f^2)*a^2*x/b^2 -
1/6*(-b^2*c*x^2 + a^2*c)^(3/2)*(3*C*e*f^2 + B*f^3)*x^3/(b^2*c) - 1/5*(-b^2
*c*x^2 + a^2*c)^(3/2)*(3*C*e^2*f + 3*B*e*f^2 + A*f^3)*x^2/(b^2*c) - 1/8*(-
b^2*c*x^2 + a^2*c)^(3/2)*(3*C*e*f^2 + B*f^3)*a^2*x/(b^4*c) - 1/4*(-b^2*c*x
^2 + a^2*c)^(3/2)*(C*e^3 + 3*B*e^2*f + 3*A*e*f^2)*x/(b^2*c) - 2/15*(-b^2*c
*x^2 + a^2*c)^(3/2)*(3*C*e^2*f + 3*B*e*f^2 + A*f^3)*a^2/(b^4*c)
```

3.20.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2671 vs. $2(550) = 1100$.

Time = 1.02 (sec) , antiderivative size = 2671, normalized size of antiderivative = 4.52

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algo
rithm="giac")
```

output
```
-1/1680*(1680*(2*a*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c +
 2*a*c)))/sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*A*a*b^5*e^3
 - 840*(2*a^2*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*
c)))/sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(b*x - 2*a))*B*a*
b^4*e^3 - 840*(2*a^2*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c
 + 2*a*c)))/sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(b*x - 2*a
))*A*b^5*e^3 - 2520*(2*a^2*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x
+ a)*c + 2*a*c)))/sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(b*x
 - 2*a))*A*a*b^4*e^2*f + 280*(6*a^3*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sq
rt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - ((2*b*x - 5*a)*(b*x + a) + 9*a^2)*sq
rt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*C*a*b^3*e^3 + 280*(6*a^3*c*log(abs
(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - ((2*b*x
 - 5*a)*(b*x + a) + 9*a^2)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*B*b^4
*e^3 + 840*(6*a^3*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c +
2*a*c)))/sqrt(-c) - ((2*b*x - 5*a)*(b*x + a) + 9*a^2)*sqrt(-(b*x + a)*c +
2*a*c)*sqrt(b*x + a))*B*a*b^3*e^2*f + 840*(6*a^3*c*log(abs(-sqrt(b*x + a)*
sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - ((2*b*x - 5*a)*(b*x + a
) + 9*a^2)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*A*b^4*e^2*f + 840*(6*
a^3*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(
-c) - ((2*b*x - 5*a)*(b*x + a) + 9*a^2)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt...
```

3.20.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx = \text{Hanged}$$

input
```
int((e + f*x)^3*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)*(A + B*x + C*x^2),x)
```

output
```
\text{Hanged}
```

3.20. $\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx$

3.21 $\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx$

3.21.1 Optimal result

Integrand size = 40, antiderivative size = 451

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx$$
$$= \frac{(2A(4b^4e^2+a^2b^2f^2)+a^2(a^2Cf^2+2b^2e(Ce+2Bf)))\,x\sqrt{a+bx}\sqrt{ac-bcx}}{16b^4}$$
$$+ \frac{(Ce-2Bf)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\left(a^2-b^2x^2\right)}{10b^2f}$$
$$- \frac{C\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3\left(a^2-b^2x^2\right)}{6b^2f}$$
$$- \frac{\sqrt{a+bx}\sqrt{ac-bcx}(8(2a^2f^2(2Ce+Bf)-b^2e(Ce^2-2f(Be+5Af)))+3f(5a^2Cf^2-b^2(2Ce^2-2f(2}{120b^4f}$$
$$+ \frac{a^2\sqrt{c}(2A(4b^4e^2+a^2b^2f^2)+a^2(a^2Cf^2+2b^2e(Ce+2Bf)))\sqrt{a+bx}\sqrt{ac-bcx}\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{16b^5\sqrt{a^2c-b^2cx^2}}$$

output
```
1/16*(2*A*(a^2*b^2*f^2+4*b^4*e^2)+a^2*(a^2*C*f^2+2*b^2*e*(2*B*f+C*e)))*x*(
b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)/b^4+1/10*(-2*B*f+C*e)*(f*x+e)^2*(-b^2*x^2+
a^2)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)/b^2/f-1/6*C*(f*x+e)^3*(-b^2*x^2+a^2)
*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)/b^2/f-1/120*(16*a^2*f^2*(B*f+2*C*e)-8*b^
2*e*(C*e^2-2*f*(5*A*f+B*e))+3*f*(5*a^2*C*f^2-b^2*(2*C*e^2-2*f*(5*A*f+2*B*e
)))*x)*(-b^2*x^2+a^2)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)/b^4/f+1/16*a^2*(2*A
*(a^2*b^2*f^2+4*b^4*e^2)+a^2*(a^2*C*f^2+2*b^2*e*(2*B*f+C*e)))*arctan(b*x*c
^(1/2)/(-b^2*c*x^2+a^2*c)^(1/2))*c^(1/2)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)/
b^5/(-b^2*c*x^2+a^2*c)^(1/2)
```

3.21.2 Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.63

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx$$
$$= \frac{\sqrt{c(a-bx)}\left(b\sqrt{a-bx}\sqrt{a+bx}(-a^4f(64Ce+32Bf+15Cfx)-2a^2b^2(5Af(16e+3fx)+Cx(15e^2+16\right.}{}$$

input
```
Integrate[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^2*(A + B*x + C*x^2),x]
```

output
```
(Sqrt[c*(a - b*x)]*(b*Sqrt[a - b*x]*Sqrt[a + b*x]*(-(a^4*f*(64*C*e + 32*B*
f + 15*C*f*x)) - 2*a^2*b^2*(5*A*f*(16*e + 3*f*x) + C*x*(15*e^2 + 16*e*f*x
+ 5*f^2*x^2) + B*(40*e^2 + 30*e*f*x + 8*f^2*x^2)) + 4*b^4*x*(5*A*(6*e^2 +
8*e*f*x + 3*f^2*x^2) + x*(2*B*(10*e^2 + 15*e*f*x + 6*f^2*x^2) + C*x*(15*e^
2 + 24*e*f*x + 10*f^2*x^2)))) + 30*a^2*(a^4*C*f^2 + 2*a^2*b^2*e*(C*e + 2*B
*f) + 2*A*(4*b^4*e^2 + a^2*b^2*f^2))*ArcTan[Sqrt[a + b*x]/Sqrt[a - b*x]]))
/(240*b^5*Sqrt[a - b*x])
```

3.21.3 Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 398, normalized size of antiderivative = 0.88, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2113, 2185, 27, 687, 25, 27, 676, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx}(e+fx)^2\sqrt{ac-bcx}\left(A+Bx+Cx^2\right)\,dx$$
$$\Big\downarrow \text{2113}$$
$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\int (e+fx)^2\sqrt{a^2c-b^2cx^2}\left(Cx^2+Bx+A\right)dx}{\sqrt{a^2c-b^2cx^2}}$$
$$\Big\downarrow \text{2185}$$
$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(-\frac{\int -3cf(e+fx)^2\left(\left(Ca^2+2Ab^2\right)f-b^2(Ce-2Bf)x\right)\sqrt{a^2c-b^2cx^2}dx}{6b^2cf^2}-\frac{C(e+fx)^3\left(a^2c-b^2cx^2\right)^{3/2}}{6b^2cf}\right)}{\sqrt{a^2c-b^2cx^2}}$$

$$\Big\downarrow 27$$

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\int (e+fx)^2\left(\left(Ca^2+2Ab^2\right)f-b^2(Ce-2Bf)x\right)\sqrt{a^2c-b^2cx^2}dx}{2b^2f}-\frac{C(e+fx)^3\left(a^2c-b^2cx^2\right)^{3/2}}{6b^2cf}\right)}{\sqrt{a^2c-b^2cx^2}}$$

$$\Big\downarrow 687$$

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\frac{(e+fx)^2\left(a^2c-b^2cx^2\right)^{3/2}(Ce-2Bf)}{5c}-\frac{\int -b^2c(e+fx)\left(f\left((3Ce+4Bf)a^2+10Ab^2e\right)+\left(5a^2Cf^2-b^2\left(2Ce^2-2f(2Be+5Af)\right)\right)x\right)\sqrt{a^2c-\cdots}}{5b^2c}}{2b^2f}\cdots\right.}{\sqrt{a^2c-b^2cx^2}}$$

$$\Big\downarrow 25$$

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\frac{\int b^2c(e+fx)\left(f\left((3Ce+4Bf)a^2+10Ab^2e\right)+\left(5a^2Cf^2-b^2\left(2Ce^2-2f(2Be+5Af)\right)\right)x\right)\sqrt{a^2c-b^2cx^2}dx}{5b^2c}+\frac{(e+fx)^2\left(a^2c-b^2cx^2\right)^{3/2}(Ce\cdots}{5c}}{2b^2f}\cdots\right.}{\sqrt{a^2c-b^2cx^2}}$$

$$\Big\downarrow 27$$

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\frac{1}{5}\int (e+fx)\left(f\left((3Ce+4Bf)a^2+10Ab^2e\right)+\left(5a^2Cf^2-b^2\left(2Ce^2-2f(2Be+5Af)\right)\right)x\right)\sqrt{a^2c-b^2cx^2}dx+\frac{(e+fx)^2\left(a^2c-b^2cx^2\right)\cdots}{5c}}{2b^2f}\cdots\right.}{\sqrt{a^2c-b^2cx^2}}$$

$$\Big\downarrow 676$$

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\frac{1}{5}\left(\frac{5f\left(a^4Cf^2+2A\left(a^2b^2f^2+4b^4e^2\right)+2a^2b^2e(2Bf+Ce)\right)\int \sqrt{a^2c-b^2cx^2}dx}{4b^2}-\frac{2\left(a^2c-b^2cx^2\right)^{3/2}\left(2a^2f^2(Bf+2Ce)-b^2\left(Ce^3-2ef(5\cdots\right.\right.}{3b^2c}\right.}{2b^2f}\cdots\right.}{\sqrt{a^2c-b^2\cdots}}$$

$$\Big\downarrow 211$$

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\frac{1}{5}\left(\frac{5f\left(a^4Cf^2+2A\left(a^2b^2f^2+4b^4e^2\right)+2a^2b^2e(2Bf+Ce)\right)\left(\frac{1}{2}a^2c\int \frac{1}{\sqrt{a^2c-b^2cx^2}}dx+\frac{1}{2}x\sqrt{a^2c-b^2cx^2}\right)}{4b^2}-\frac{2\left(a^2c-b^2cx^2\right)^{3/2}\left(2a^2f^2\cdots\right.}{\cdots}\right.}{\cdots}\cdots\right.}{\cdots}$$

$$\Big\downarrow 224$$

3.21. $\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx$

$$\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\frac{1}{5}\left(\frac{5f\left(a^4Cf^2+2A\left(a^2b^2f^2+4b^4e^2\right)+2a^2b^2e(2Bf+Ce)\right)\left(\frac{1}{2}a^2c\int\frac{1}{\frac{b^2cx^2}{a^2c-b^2cx^2}+1}d\frac{x}{\sqrt{a^2c-b^2cx^2}}+\frac{1}{2}x\sqrt{a^2c-b^2cx^2}\right)}{4b^2}-\frac{2\left(a^2c-b^2c\cdots\right.}{\cdots}\right.}{\cdots}\right.$$

$\downarrow$ 216

$$\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\frac{(e+fx)^2\left(a^2c-b^2cx^2\right)^{3/2}(Ce-2Bf)}{5c}+\frac{1}{5}\left(-\frac{2\left(a^2c-b^2cx^2\right)^{3/2}\left(2a^2f^2(Bf+2Ce)-b^2\left(Ce^3-2ef(5Af+Be)\right)\right)}{3b^2c}-\frac{fx\left(a^2c-b^2cx^2\right)^{3/\cdots}}{\cdots}\right.}{\cdots}\right.$$

input
```
Int[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^2*(A + B*x + C*x^2),x]
```

output
```
(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(-1/6*(C*(e + f*x)^3*(a^2*c - b^2*c*x^2)^
(3/2))/(b^2*c*f) + (((C*e - 2*B*f)*(e + f*x)^2*(a^2*c - b^2*c*x^2)^(3/2))/
(5*c) + ((-2*(2*a^2*f^2*(2*C*e + B*f) - b^2*(C*e^3 - 2*e*f*(B*e + 5*A*f)))
*(a^2*c - b^2*c*x^2)^(3/2))/(3*b^2*c) - (f*(5*a^2*C*f^2 - b^2*(2*C*e^2 - 2
*f*(2*B*e + 5*A*f)))*x*(a^2*c - b^2*c*x^2)^(3/2))/(4*b^2*c) + (5*f*(a^4*C*
f^2 + 2*a^2*b^2*e*(C*e + 2*B*f) + 2*A*(4*b^4*e^2 + a^2*b^2*f^2))*((x*Sqrt[
a^2*c - b^2*c*x^2])/2 + (a^2*Sqrt[c]*ArcTan[(b*Sqrt[c]*x)/Sqrt[a^2*c - b^2
*c*x^2]])/(2*b)))/(4*b^2))/5)/(2*b^2*f)))/Sqrt[a^2*c - b^2*c*x^2]
```

3.21. $\int\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx$

3.21.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2113
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m])   Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a
*d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.21.4 Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 401, normalized size of antiderivative = 0.89

method	result
risch	$-\frac{\left(-40C f^2x^5b^4-48B b^4f^2x^4-96C b^4ef x^4-60A b^4f^2x^3-120B b^4ef x^3+10C a^2b^2f^2x^3-60C b^4e^2x^3-160A b^4ef x^2+16B a^2b^2f^2x\right.}{}$
default	$\frac{\sqrt{bx+a}\,\sqrt{c(-bx+a)}\left(60C b^4e^2x^3\sqrt{b^2c}\,\sqrt{c(-b^2x^2+a^2)}+80B b^4e^2x^2\sqrt{b^2c}\,\sqrt{c(-b^2x^2+a^2)}-80B a^2b^2e^2\sqrt{b^2c}\,\sqrt{c(-b^2x^2+a^2)}-64C\right.}{}$

input
```
int((f*x+e)^2*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/240/b^4*(-40*C*b^4*f^2*x^5-48*B*b^4*f^2*x^4-96*C*b^4*e*f*x^4-60*A*b^4*f
^2*x^3-120*B*b^4*e*f*x^3+10*C*a^2*b^2*f^2*x^3-60*C*b^4*e^2*x^3-160*A*b^4*e
*f*x^2+16*B*a^2*b^2*f^2*x^2-80*B*b^4*e^2*x^2+32*C*a^2*b^2*e*f*x^2+30*A*a^2
*b^2*f^2*x-120*A*b^4*e^2*x+60*B*a^2*b^2*e*f*x+15*C*a^4*f^2*x+30*C*a^2*b^2*
e^2*x+160*A*a^2*b^2*e*f+32*B*a^4*f^2+80*B*a^2*b^2*e^2+64*C*a^4*e*f)*(b*x+a
)^(1/2)*(-b*x+a)/(-c*(b*x-a))^(1/2)*c+1/16*a^2*(2*A*a^2*b^2*f^2+8*A*b^4*e^
2+4*B*a^2*b^2*e*f+C*a^4*f^2+2*C*a^2*b^2*e^2)/b^4/(b^2*c)^(1/2)*arctan((b^2
*c)^(1/2)*x/(-b^2*c*x^2+a^2*c)^(1/2))*(-(b*x+a)*c*(b*x-a))^(1/2)/(b*x+a)^(
1/2)/(-c*(b*x-a))^(1/2)*c
```

3.21.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 703, normalized size of antiderivative = 1.56

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx$$
$$=\left[\frac{15\left(4\,Ba^4b^2ef+2\left(Ca^4b^2+4\,Aa^2b^4\right)e^2+\left(Ca^6+2\,Aa^4b^2\right)f^2\right)\sqrt{-c}\log\left(2\,b^2cx^2+2\,\sqrt{-bcx+ac}\sqrt{bx+a}\right.}{}\right.$$
$$-\frac{15\left(4\,Ba^4b^2ef+2\left(Ca^4b^2+4\,Aa^2b^4\right)e^2+\left(Ca^6+2\,Aa^4b^2\right)f^2\right)\sqrt{c}\arctan\left(\frac{\sqrt{-bcx+ac}\sqrt{bx+a}b\sqrt{c}x}{b^2cx^2-a^2c}\right)-\left(40\,Cb^5\right.}{}$$

input
```
integrate((f*x+e)^2*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algo
rithm="fricas")
```

output
```
[1/480*(15*(4*B*a^4*b^2*e*f + 2*(C*a^4*b^2 + 4*A*a^2*b^4)*e^2 + (C*a^6 + 2
*A*a^4*b^2)*f^2)*sqrt(-c)*log(2*b^2*c*x^2 + 2*sqrt(-b*c*x + a*c)*sqrt(b*x
+ a)*b*sqrt(-c)*x - a^2*c) + 2*(40*C*b^5*f^2*x^5 - 80*B*a^2*b^3*e^2 - 32*B
*a^4*b*f^2 + 48*(2*C*b^5*e*f + B*b^5*f^2)*x^4 + 10*(6*C*b^5*e^2 + 12*B*b^5
*e*f - (C*a^2*b^3 - 6*A*b^5)*f^2)*x^3 - 32*(2*C*a^4*b + 5*A*a^2*b^3)*e*f +
 16*(5*B*b^5*e^2 - B*a^2*b^3*f^2 - 2*(C*a^2*b^3 - 5*A*b^5)*e*f)*x^2 - 15*(
4*B*a^2*b^3*e*f + 2*(C*a^2*b^3 - 4*A*b^5)*e^2 + (C*a^4*b + 2*A*a^2*b^3)*f^
2)*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/b^5, -1/240*(15*(4*B*a^4*b^2*e*f +
 2*(C*a^4*b^2 + 4*A*a^2*b^4)*e^2 + (C*a^6 + 2*A*a^4*b^2)*f^2)*sqrt(c)*arct
an(sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(c)*x/(b^2*c*x^2 - a^2*c)) - (40
*C*b^5*f^2*x^5 - 80*B*a^2*b^3*e^2 - 32*B*a^4*b*f^2 + 48*(2*C*b^5*e*f + B*b
^5*f^2)*x^4 + 10*(6*C*b^5*e^2 + 12*B*b^5*e*f - (C*a^2*b^3 - 6*A*b^5)*f^2)*
x^3 - 32*(2*C*a^4*b + 5*A*a^2*b^3)*e*f + 16*(5*B*b^5*e^2 - B*a^2*b^3*f^2 -
 2*(C*a^2*b^3 - 5*A*b^5)*e*f)*x^2 - 15*(4*B*a^2*b^3*e*f + 2*(C*a^2*b^3 - 4
*A*b^5)*e^2 + (C*a^4*b + 2*A*a^2*b^3)*f^2)*x)*sqrt(-b*c*x + a*c)*sqrt(b*x
+ a))/b^5]
```

3.21.6 Sympy [F]

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx = \int \sqrt{-c\,(-a+bx)}\sqrt{a+bx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx$$

input
```
integrate((f*x+e)**2*(C*x**2+B*x+A)*(b*x+a)**(1/2)*(-b*c*x+a*c)**(1/2),x)
```

output
```
Integral(sqrt(-c*(-a + b*x))*sqrt(a + b*x)*(e + f*x)**2*(A + B*x + C*x**2)
, x)
```

3.21.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 417, normalized size of antiderivative = 0.92

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx$$
$$= \frac{Aa^2\sqrt{c}e^2\arcsin\left(\frac{bx}{a}\right)}{2\,b} + \frac{Ca^6\sqrt{c}f^2\arcsin\left(\frac{bx}{a}\right)}{16\,b^5} + \frac{1}{2}\sqrt{-b^2cx^2+a^2c}Ae^2x$$
$$+ \frac{\sqrt{-b^2cx^2+a^2c}Ca^4f^2x}{16\,b^4} - \frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}Cf^2x^3}{6\,b^2c}$$
$$+ \frac{(Ce^2+2\,Bef+Af^2)a^4\sqrt{c}\arcsin\left(\frac{bx}{a}\right)}{8\,b^3} + \frac{\sqrt{-b^2cx^2+a^2c}(Ce^2+2\,Bef+Af^2)a^2x}{8\,b^2}$$
$$- \frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}Ca^2f^2x}{8\,b^4c} - \frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}Be^2}{3\,b^2c}$$
$$- \frac{2\,(-b^2cx^2+a^2c)^{\frac{3}{2}}Aef}{3\,b^2c} - \frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}(2\,Cef+Bf^2)x^2}{5\,b^2c}$$
$$- \frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}(Ce^2+2\,Bef+Af^2)x}{4\,b^2c} - \frac{2\,(-b^2cx^2+a^2c)^{\frac{3}{2}}(2\,Cef+Bf^2)a^2}{15\,b^4c}$$

input
```
integrate((f*x+e)^2*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algo
rithm="maxima")
```

output
```
1/2*A*a^2*sqrt(c)*e^2*arcsin(b*x/a)/b + 1/16*C*a^6*sqrt(c)*f^2*arcsin(b*x/
a)/b^5 + 1/2*sqrt(-b^2*c*x^2 + a^2*c)*A*e^2*x + 1/16*sqrt(-b^2*c*x^2 + a^2
*c)*C*a^4*f^2*x/b^4 - 1/6*(-b^2*c*x^2 + a^2*c)^(3/2)*C*f^2*x^3/(b^2*c) + 1
/8*(C*e^2 + 2*B*e*f + A*f^2)*a^4*sqrt(c)*arcsin(b*x/a)/b^3 + 1/8*sqrt(-b^2
*c*x^2 + a^2*c)*(C*e^2 + 2*B*e*f + A*f^2)*a^2*x/b^2 - 1/8*(-b^2*c*x^2 + a^
2*c)^(3/2)*C*a^2*f^2*x/(b^4*c) - 1/3*(-b^2*c*x^2 + a^2*c)^(3/2)*B*e^2/(b^2
*c) - 2/3*(-b^2*c*x^2 + a^2*c)^(3/2)*A*e*f/(b^2*c) - 1/5*(-b^2*c*x^2 + a^2
*c)^(3/2)*(2*C*e*f + B*f^2)*x^2/(b^2*c) - 1/4*(-b^2*c*x^2 + a^2*c)^(3/2)*(
C*e^2 + 2*B*e*f + A*f^2)*x/(b^2*c) - 2/15*(-b^2*c*x^2 + a^2*c)^(3/2)*(2*C*
e*f + B*f^2)*a^2/(b^4*c)
```

3.21.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1868 vs. 2(412) = 824.

Time = 0.85 (sec) , antiderivative size = 1868, normalized size of antiderivative = 4.14

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx=\text{Too large to display}$$

input
```
integrate((f*x+e)^2*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algo
rithm="giac")
```

output
```
-1/240*(240*(2*a*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2
*a*c)))/sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*A*a*b^4*e^2 -
 120*(2*a^2*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)
))/sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(b*x - 2*a))*B*a*b^
3*e^2 - 120*(2*a^2*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c +
 2*a*c)))/sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(b*x - 2*a))
*A*b^4*e^2 - 240*(2*a^2*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a
)*c + 2*a*c)))/sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(b*x -
2*a))*A*a*b^3*e*f + 40*(6*a^3*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b
*x + a)*c + 2*a*c)))/sqrt(-c) - ((2*b*x - 5*a)*(b*x + a) + 9*a^2)*sqrt(-(b
*x + a)*c + 2*a*c)*sqrt(b*x + a))*C*a*b^2*e^2 + 40*(6*a^3*c*log(abs(-sqrt(
b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - ((2*b*x - 5*a)
*(b*x + a) + 9*a^2)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*B*b^3*e^2 +
80*(6*a^3*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))
/sqrt(-c) - ((2*b*x - 5*a)*(b*x + a) + 9*a^2)*sqrt(-(b*x + a)*c + 2*a*c)*s
qrt(b*x + a))*B*a*b^2*e*f + 80*(6*a^3*c*log(abs(-sqrt(b*x + a)*sqrt(-c) +
sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - ((2*b*x - 5*a)*(b*x + a) + 9*a^2)*
sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*A*b^3*e*f + 40*(6*a^3*c*log(abs(
-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - ((2*b*x
- 5*a)*(b*x + a) + 9*a^2)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*A*a...
```

3.21.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx=\text{Hanged}$$

input
```
int((e + f*x)^2*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)*(A + B*x + C*x^2),x)
```

output
```
\text{Hanged}
```

3.21. $\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx$

3.22 $\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)\left(A+Bx+Cx^2\right)\,dx$

3.22.1 Optimal result

Integrand size = 38, antiderivative size = 300

$$\begin{aligned}&\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)\left(A+Bx+Cx^2\right)\,dx\\&=\frac{(4Ab^2e+a^2(Ce+Bf))\,x\sqrt{a+bx}\sqrt{ac-bcx}}{8b^2}-\frac{C\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\,(a^2-b^2x^2)}{5b^2f}\\&\quad-\frac{\sqrt{a+bx}\sqrt{ac-bcx}(4(2a^2Cf^2-b^2(3Ce^2-5f(Be+Af)))-3b^2f(3Ce-5Bf)x)\,(a^2-b^2x^2)}{60b^4f}\\&\quad+\frac{a^2\sqrt{c}(4Ab^2e+a^2(Ce+Bf))\,\sqrt{a+bx}\sqrt{ac-bcx}\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{8b^3\sqrt{a^2c-b^2cx^2}}\end{aligned}$$

output
```
1/8*(4*A*b^2*e+a^2*(B*f+C*e))*x*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)/b^2-1/5*C
*(f*x+e)^2*(-b^2*x^2+a^2)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)/b^2/f-1/60*(8*a
^2*C*f^2-4*b^2*(3*C*e^2-5*f*(A*f+B*e))-3*b^2*f*(-5*B*f+3*C*e)*x)*(-b^2*x^2
+a^2)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)/b^4/f+1/8*a^2*(4*A*b^2*e+a^2*(B*f+C
*e))*arctan(b*x*c^(1/2)/(-b^2*c*x^2+a^2*c)^(1/2))*c^(1/2)*(b*x+a)^(1/2)*(-
b*c*x+a*c)^(1/2)/b^3/(-b^2*c*x^2+a^2*c)^(1/2)
```

3.22.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.61

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)\left(A+Bx+Cx^2\right)\,dx$$
$$=\frac{\sqrt{c(a-bx)}\left(\sqrt{a-bx}\sqrt{a+bx}(-16a^4Cf-a^2b^2(40Af+5B(8e+3fx)+Cx(15e+8fx))+2b^4x(10A(3\right.}{120b^4\sqrt{a-}}$$

input
```
Integrate[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)*(A + B*x + C*x^2),x]
```

output
```
(Sqrt[c*(a - b*x)]*(Sqrt[a - b*x]*Sqrt[a + b*x]*(-16*a^4*C*f - a^2*b^2*(40
*A*f + 5*B*(8*e + 3*f*x) + C*x*(15*e + 8*f*x)) + 2*b^4*x*(10*A*(3*e + 2*f*
x) + x*(5*B*(4*e + 3*f*x) + 3*C*x*(5*e + 4*f*x)))) + 30*a^2*b*(4*A*b^2*e +
 a^2*(C*e + B*f))*ArcTan[Sqrt[a + b*x]/Sqrt[a - b*x]]))/(120*b^4*Sqrt[a -
b*x])
```

3.22.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2113, 2185, 25, 27, 676, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx}(e+fx)\sqrt{ac-bcx}\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{2113}$$

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\int(e+fx)\sqrt{a^2c-b^2cx^2}\left(Cx^2+Bx+A\right)dx}{\sqrt{a^2c-b^2cx^2}}$$

$$\Big\downarrow \text{2185}$$

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(-\frac{\int -cf(e+fx)\left(\left(2Ca^2+5Ab^2\right)f-b^2(3Ce-5Bf)x\right)\sqrt{a^2c-b^2cx^2}dx}{5b^2cf^2}-\frac{C(e+fx)^2\left(a^2c-b^2cx^2\right)^{3/2}}{5b^2cf}\right)}{\sqrt{a^2c-b^2cx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\int cf(e+fx)\left(\left(2Ca^2+5Ab^2\right)f-b^2(3Ce-5Bf)x\right)\sqrt{a^2c-b^2cx^2}dx}{5b^2cf^2}-\frac{C(e+fx)^2\left(a^2c-b^2cx^2\right)^{3/2}}{5b^2cf}\right)}{\sqrt{a^2c-b^2cx^2}}$$

$\downarrow$ 27

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\int (e+fx)\left(\left(2Ca^2+5Ab^2\right)f-b^2(3Ce-5Bf)x\right)\sqrt{a^2c-b^2cx^2}dx}{5b^2f}-\frac{C(e+fx)^2\left(a^2c-b^2cx^2\right)^{3/2}}{5b^2cf}\right)}{\sqrt{a^2c-b^2cx^2}}$$

$\downarrow$ 676

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\frac{5}{4}f\left(a^2(Bf+Ce)+4Ab^2e\right)\int\sqrt{a^2c-b^2cx^2}dx-\frac{\left(a^2c-b^2cx^2\right)^{3/2}\left(2a^2Cf^2-b^2\left(3Ce^2-5f(Af+Be)\right)\right)}{3b^2c}+\frac{fx\left(a^2c-b^2cx^2\right)^{3/2}(3Ce}{4c}}{5b^2f}\right.}{\sqrt{a^2c-b^2cx^2}}$$

$\downarrow$ 211

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\frac{5}{4}f\left(a^2(Bf+Ce)+4Ab^2e\right)\left(\frac{1}{2}a^2c\int\frac{1}{\sqrt{a^2c-b^2cx^2}}dx+\frac{1}{2}x\sqrt{a^2c-b^2cx^2}\right)-\frac{\left(a^2c-b^2cx^2\right)^{3/2}\left(2a^2Cf^2-b^2\left(3Ce^2-5f(Af+Be)\right)\right)}{3b^2c}}{5b^2f}\right.}{\sqrt{a^2c-b^2cx^2}}$$

$\downarrow$ 224

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\frac{5}{4}f\left(a^2(Bf+Ce)+4Ab^2e\right)\left(\frac{1}{2}a^2c\int\frac{1}{\frac{b^2cx^2}{a^2c-b^2cx^2}+1}d\frac{x}{\sqrt{a^2c-b^2cx^2}}+\frac{1}{2}x\sqrt{a^2c-b^2cx^2}\right)-\frac{\left(a^2c-b^2cx^2\right)^{3/2}\left(2a^2Cf^2-b^2\left(3Ce^2\right.\right.}{3b^2c}}{5b^2f}\right.}{\sqrt{a^2c-b^2cx^2}}$$

$\downarrow$ 216

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\frac{5}{4}f\left(\frac{a^2\sqrt{c}\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{2b}+\frac{1}{2}x\sqrt{a^2c-b^2cx^2}\right)\left(a^2(Bf+Ce)+4Ab^2e\right)-\frac{\left(a^2c-b^2cx^2\right)^{3/2}\left(2a^2Cf^2-b^2\left(3Ce^2-5f(Af+\right.\right.}{3b^2c}}{5b^2f}\right.}{\sqrt{a^2c-b^2cx^2}}$$

input
```
Int[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)*(A + B*x + C*x^2),x]
```

3.22. $\int\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)\left(A+Bx+Cx^2\right)dx$

output
```
(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(-1/5*(C*(e + f*x)^2*(a^2*c - b^2*c*x^2)^
(3/2))/(b^2*c*f) + (-1/3*((2*a^2*C*f^2 - b^2*(3*C*e^2 - 5*f*(B*e + A*f)))*
(a^2*c - b^2*c*x^2)^(3/2))/(b^2*c) + (f*(3*C*e - 5*B*f)*x*(a^2*c - b^2*c*x
^2)^(3/2))/(4*c) + (5*f*(4*A*b^2*e + a^2*(C*e + B*f))*((x*Sqrt[a^2*c - b^2
*c*x^2])/2 + (a^2*Sqrt[c]*ArcTan[(b*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(
2*b)))/4)/(5*b^2*f)))/Sqrt[a^2*c - b^2*c*x^2]
```

3.22.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 2113
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m])   Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a
*d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.22.4 Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.84

method	result
risch	$-\frac{\left(-24 f C\,x^{4} b^{4}-30 B\,b^{4} f\,x^{3}-30 C\,b^{4} e\,x^{3}-40 A\,b^{4} f\,x^{2}-40 B\,b^{4} e\,x^{2}+8 C\,a^{2} b^{2} f\,x^{2}-60 A\,b^{4} e x+15 B\,a^{2} b^{2} f x+15 C\,a^{2} b^{2} e x+40 A\,a^{2} f\,b^{2}}{120 b^{4} \sqrt{-c \left(b x-a\right)}}$
default	$\frac{\sqrt{b x+a}\,\sqrt{c \left(-b x+a\right)}\,\left(24 C\,b^{4} f\,x^{4} \sqrt{b^{2} c}\,\sqrt{c \left(-b^{2} x^{2}+a^{2}\right)}+30 B\,b^{4} f\,x^{3} \sqrt{b^{2} c}\,\sqrt{c \left(-b^{2} x^{2}+a^{2}\right)}+30 C\,b^{4} e\,x^{3} \sqrt{b^{2} c}\,\sqrt{c \left(-b^{2} x^{2}+a^{2}\right)}+60 A \text{ar}}{}$

input
```
int((f*x+e)*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x,method=_RETUR
NVERBOSE)
```

output
```
-1/120*(-24*C*b^4*f*x^4-30*B*b^4*f*x^3-30*C*b^4*e*x^3-40*A*b^4*f*x^2-40*B*
b^4*e*x^2+8*C*a^2*b^2*f*x^2-60*A*b^4*e*x+15*B*a^2*b^2*f*x+15*C*a^2*b^2*e*x
+40*A*a^2*b^2*f+40*B*a^2*b^2*e+16*C*a^4*f)*(b*x+a)^(1/2)/b^4*(-b*x+a)/(-c*
(b*x-a))^(1/2)*c+1/8*a^2/b^2*(4*A*b^2*e+B*a^2*f+C*a^2*e)/(b^2*c)^(1/2)*arc
tan((b^2*c)^(1/2)*x/(-b^2*c*x^2+a^2*c)^(1/2))*(-(b*x+a)*c*(b*x-a))^(1/2)/(
b*x+a)^(1/2)/(-c*(b*x-a))^(1/2)*c
```

3.22.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.47

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)\left(A+Bx+Cx^2\right)\,dx$$
$$= \left[\frac{15\left(Ba^4bf+(Ca^4b+4\,Aa^2b^3)e\right)\sqrt{-c}\log\left(2\,b^2cx^2+2\,\sqrt{-bcx+ac}\sqrt{bx+a}b\sqrt{-c}x-a^2c\right)+2\,(24\,Cb^4fx\right.$$
$$-\frac{15\left(Ba^4bf+(Ca^4b+4\,Aa^2b^3)e\right)\sqrt{c}\arctan\left(\frac{\sqrt{-bcx+ac}\sqrt{bx+a}b\sqrt{cx}}{b^2cx^2-a^2c}\right)-(24\,Cb^4fx^4-40\,Ba^2b^2e+30\,(Cb^4e}{}$$

input
```
integrate((f*x+e)*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algori
thm="fricas")
```

output
```
[1/240*(15*(B*a^4*b*f + (C*a^4*b + 4*A*a^2*b^3)*e)*sqrt(-c)*log(2*b^2*c*x^
2 + 2*sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(-c)*x - a^2*c) + 2*(24*C*b^4
*f*x^4 - 40*B*a^2*b^2*e + 30*(C*b^4*e + B*b^4*f)*x^3 + 8*(5*B*b^4*e - (C*a
^2*b^2 - 5*A*b^4)*f)*x^2 - 8*(2*C*a^4 + 5*A*a^2*b^2)*f - 15*(B*a^2*b^2*f +
 (C*a^2*b^2 - 4*A*b^4)*e)*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/b^4, -1/120
*(15*(B*a^4*b*f + (C*a^4*b + 4*A*a^2*b^3)*e)*sqrt(c)*arctan(sqrt(-b*c*x +
a*c)*sqrt(b*x + a)*b*sqrt(c)*x/(b^2*c*x^2 - a^2*c)) - (24*C*b^4*f*x^4 - 40
*B*a^2*b^2*e + 30*(C*b^4*e + B*b^4*f)*x^3 + 8*(5*B*b^4*e - (C*a^2*b^2 - 5*
A*b^4)*f)*x^2 - 8*(2*C*a^4 + 5*A*a^2*b^2)*f - 15*(B*a^2*b^2*f + (C*a^2*b^2
 - 4*A*b^4)*e)*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/b^4]
```

3.22.6 Sympy [F]

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)\left(A+Bx+Cx^2\right)\,dx$$
$$=\int \sqrt{-c\,(-a+bx)}\sqrt{a+bx}(e+fx)\left(A+Bx+Cx^2\right)\,dx$$

input
```
integrate((f*x+e)*(C*x**2+B*x+A)*(b*x+a)**(1/2)*(-b*c*x+a*c)**(1/2),x)
```

output
```
Integral(sqrt(-c*(-a + b*x))*sqrt(a + b*x)*(e + f*x)*(A + B*x + C*x**2), x
)
```

3.22.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.83

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)\left(A+Bx+Cx^2\right)\,dx$$
$$=\frac{Aa^2\sqrt{c}e\arcsin\left(\frac{bx}{a}\right)}{2\,b}+\frac{1}{2}\sqrt{-b^2cx^2+a^2c}Aex+\frac{(Ce+Bf)a^4\sqrt{c}\arcsin\left(\frac{bx}{a}\right)}{8\,b^3}+\frac{\sqrt{-b^2cx^2+a^2c}(Ce+Bf)a^2x}{8\,b^2}-\frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}Cfx^2}{5\,b^2c}-\frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}Be}{3\,b^2c}-\frac{2\,(-b^2cx^2+a^2c)^{\frac{3}{2}}Ca^2f}{15\,b^4c}-\frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}Af}{3\,b^2c}-\frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}(Ce+Bf)x}{4\,b^2c}$$

input
```
integrate((f*x+e)*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algori
thm="maxima")
```

output
```
1/2*A*a^2*sqrt(c)*e*arcsin(b*x/a)/b + 1/2*sqrt(-b^2*c*x^2 + a^2*c)*A*e*x +
 1/8*(C*e + B*f)*a^4*sqrt(c)*arcsin(b*x/a)/b^3 + 1/8*sqrt(-b^2*c*x^2 + a^2
*c)*(C*e + B*f)*a^2*x/b^2 - 1/5*(-b^2*c*x^2 + a^2*c)^(3/2)*C*f*x^2/(b^2*c)
 - 1/3*(-b^2*c*x^2 + a^2*c)^(3/2)*B*e/(b^2*c) - 2/15*(-b^2*c*x^2 + a^2*c)^
(3/2)*C*a^2*f/(b^4*c) - 1/3*(-b^2*c*x^2 + a^2*c)^(3/2)*A*f/(b^2*c) - 1/4*(
-b^2*c*x^2 + a^2*c)^(3/2)*(C*e + B*f)*x/(b^2*c)
```

3.22.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1142 vs. 2(267) = 534.

Time = 0.63 (sec) , antiderivative size = 1142, normalized size of antiderivative = 3.81

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)\left(A+Bx+Cx^2\right)\,dx=\text{Too large to display}$$

input
```
integrate((f*x+e)*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algori
thm="giac")
```

output
```
-1/120*(120*(2*a*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2
*a*c)))/sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*A*a*b^3*e - 6
0*(2*a^2*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/
sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(b*x - 2*a))*B*a*b^2*e
 - 60*(2*a^2*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c
)))/sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(b*x - 2*a))*A*b^3
*e - 60*(2*a^2*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a
*c)))/sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(b*x - 2*a))*A*a
*b^2*f + 20*(6*a^3*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c +
 2*a*c)))/sqrt(-c) - ((2*b*x - 5*a)*(b*x + a) + 9*a^2)*sqrt(-(b*x + a)*c +
 2*a*c)*sqrt(b*x + a))*C*a*b*e + 20*(6*a^3*c*log(abs(-sqrt(b*x + a)*sqrt(-
c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - ((2*b*x - 5*a)*(b*x + a) + 9*
a^2)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*B*b^2*e + 20*(6*a^3*c*log(a
bs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - ((2*b
*x - 5*a)*(b*x + a) + 9*a^2)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*B*a
*b*f + 20*(6*a^3*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2
*a*c)))/sqrt(-c) - ((2*b*x - 5*a)*(b*x + a) + 9*a^2)*sqrt(-(b*x + a)*c + 2
*a*c)*sqrt(b*x + a))*A*b^2*f - 5*(18*a^4*c*log(abs(-sqrt(b*x + a)*sqrt(-c)
 + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - (39*a^3 - (2*(3*b*x - 10*a)*(b*
x + a) + 43*a^2)*(b*x + a))*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*C...
```

3.22.9 Mupad [B] (verification not implemented)

Time = 37.00 (sec) , antiderivative size = 1765, normalized size of antiderivative = 5.88

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)\left(A+Bx+Cx^2\right)\,dx = \text{Too large to display}$$

input
```
int((e + f*x)*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)*(A + B*x + C*x^2),x)
```

output
```
((B*a^4*c^8*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(2*((a + b*x)^(1/2) - a
^(1/2))) - (B*a^4*c*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^15)/(2*((a + b*x
)^(1/2) - a^(1/2))^15) - (35*B*a^4*c^7*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2
))^3)/(2*((a + b*x)^(1/2) - a^(1/2))^3) + (273*B*a^4*c^6*f*((a*c - b*c*x)^
(1/2) - (a*c)^(1/2))^5)/(2*((a + b*x)^(1/2) - a^(1/2))^5) - (715*B*a^4*c^5
*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/(2*((a + b*x)^(1/2) - a^(1/2))^7
) + (715*B*a^4*c^4*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^9)/(2*((a + b*x)^
(1/2) - a^(1/2))^9) - (273*B*a^4*c^3*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))
^11)/(2*((a + b*x)^(1/2) - a^(1/2))^11) + (35*B*a^4*c^2*f*((a*c - b*c*x)^(
1/2) - (a*c)^(1/2))^13)/(2*((a + b*x)^(1/2) - a^(1/2))^13))/(b^3*c^8 + (b^
3*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^16)/((a + b*x)^(1/2) - a^(1/2))^16 +
 (8*b^3*c^7*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1
/2))^2 + (28*b^3*c^6*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4)/((a + b*x)^(1/
2) - a^(1/2))^4 + (56*b^3*c^5*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6)/((a +
 b*x)^(1/2) - a^(1/2))^6 + (70*b^3*c^4*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))
^8)/((a + b*x)^(1/2) - a^(1/2))^8 + (56*b^3*c^3*((a*c - b*c*x)^(1/2) - (a*
c)^(1/2))^10)/((a + b*x)^(1/2) - a^(1/2))^10 + (28*b^3*c^2*((a*c - b*c*x)^
(1/2) - (a*c)^(1/2))^12)/((a + b*x)^(1/2) - a^(1/2))^12 + (8*b^3*c*((a*c -
 b*c*x)^(1/2) - (a*c)^(1/2))^14)/((a + b*x)^(1/2) - a^(1/2))^14) - (a*c -
b*c*x)^(1/2)*((2*C*a^4*f*(a + b*x)^(1/2))/(15*b^4) - (C*f*x^4*(a + b*x)...
```

3.22. $\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)\left(A+Bx+Cx^2\right)\,dx$

3.23 $\int \sqrt{a+bx}\sqrt{ac-bcx}(A+Bx+Cx^2)\,dx$

3.23.1 Optimal result

Integrand size = 33, antiderivative size = 221

$$\begin{aligned}\int \sqrt{a+bx}\sqrt{ac-bcx}(A+Bx+Cx^2)\,dx &= \frac{1}{8}\left(4A+\frac{a^2C}{b^2}\right)x\sqrt{a+bx}\sqrt{ac-bcx} \\ &\quad -\frac{B\sqrt{a+bx}\sqrt{ac-bcx}(a^2-b^2x^2)}{3b^2}-\frac{Cx\sqrt{a+bx}\sqrt{ac-bcx}(a^2-b^2x^2)}{4b^2} \\ &\quad +\frac{a^2\sqrt{c}(4Ab^2+a^2C)\sqrt{a+bx}\sqrt{ac-bcx}\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{8b^3\sqrt{a^2c-b^2cx^2}}\end{aligned}$$

output
```
1/8*(4*A+a^2*C/b^2)*x*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)-1/3*B*(-b^2*x^2+a^2
)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)/b^2-1/4*C*x*(-b^2*x^2+a^2)*(b*x+a)^(1/2
)*(-b*c*x+a*c)^(1/2)/b^2+1/8*a^2*(4*A*b^2+C*a^2)*arctan(b*x*c^(1/2)/(-b^2*
c*x^2+a^2*c)^(1/2))*c^(1/2)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)/b^3/(-b^2*c*x
^2+a^2*c)^(1/2)
```

3.23.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.56

$$\int \sqrt{a+bx}\sqrt{ac-bcx}\left(A+Bx+Cx^2\right)\,dx$$
$$=\frac{\sqrt{c(a-bx)}\left(b\sqrt{a-bx}\sqrt{a+bx}(-a^2(8B+3Cx)+2b^2x(6A+x(4B+3Cx)))+6a^2(4Ab^2+a^2C)\arctan\right.}{24b^3\sqrt{a-bx}}$$

input
```
Integrate[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(A + B*x + C*x^2),x]
```

output
```
(Sqrt[c*(a - b*x)]*(b*Sqrt[a - b*x]*Sqrt[a + b*x]*(-(a^2*(8*B + 3*C*x)) +
2*b^2*x*(6*A + x*(4*B + 3*C*x))) + 6*a^2*(4*A*b^2 + a^2*C)*ArcTan[Sqrt[a +
 b*x]/Sqrt[a - b*x]]))/(24*b^3*Sqrt[a - b*x])
```

3.23.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.69, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1189, 83, 646, 40, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx}\sqrt{ac-bcx}\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{1189}$$

$$\int \sqrt{a+bx}\sqrt{ac-bcx}\left(Cx^2+A\right)dx+B\int x\sqrt{a+bx}\sqrt{ac-bcx}dx$$

$$\Big\downarrow \text{83}$$

$$\int \sqrt{a+bx}\sqrt{ac-bcx}\left(Cx^2+A\right)dx-\frac{B(a+bx)^{3/2}(ac-bcx)^{3/2}}{3b^2c}$$

$$\Big\downarrow \text{646}$$

$$\frac{1}{4}\left(\frac{a^2C}{b^2}+4A\right)\int \sqrt{a+bx}\sqrt{ac-bcx}dx-\frac{B(a+bx)^{3/2}(ac-bcx)^{3/2}}{3b^2c}-\frac{Cx(a+bx)^{3/2}(ac-bcx)^{3/2}}{4b^2c}$$

$$\Big\downarrow \text{40}$$

$$\frac{1}{4}\left(\frac{a^2C}{b^2}+4A\right)\left(\frac{1}{2}a^2c\int\frac{1}{\sqrt{a+bx}\sqrt{ac-bcx}}dx+\frac{1}{2}x\sqrt{a+bx}\sqrt{ac-bcx}\right)-\frac{B(a+bx)^{3/2}(ac-bcx)^{3/2}}{3b^2c}-\frac{Cx(a+bx)^{3/2}(ac-bcx)^{3/2}}{4b^2c}$$

$$\Big\downarrow \; 45$$

$$\frac{1}{4}\left(\frac{a^2C}{b^2}+4A\right)\left(a^2c\int\frac{1}{\frac{c(a+bx)b}{ac-bcx}+b}d\frac{\sqrt{a+bx}}{\sqrt{ac-bcx}}+\frac{1}{2}x\sqrt{a+bx}\sqrt{ac-bcx}\right)-\frac{B(a+bx)^{3/2}(ac-bcx)^{3/2}}{3b^2c}-\frac{Cx(a+bx)^{3/2}(ac-bcx)^{3/2}}{4b^2c}$$

$$\Big\downarrow \; 218$$

$$\frac{1}{4}\left(\frac{a^2C}{b^2}+4A\right)\left(\frac{a^2\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{ac-bcx}}\right)}{b}+\frac{1}{2}x\sqrt{a+bx}\sqrt{ac-bcx}\right)-\frac{B(a+bx)^{3/2}(ac-bcx)^{3/2}}{3b^2c}-\frac{Cx(a+bx)^{3/2}(ac-bcx)^{3/2}}{4b^2c}$$

input
```
Int[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(A + B*x + C*x^2),x]
```

output
```
-1/3*(B*(a + b*x)^(3/2)*(a*c - b*c*x)^(3/2))/(b^2*c) - (C*x*(a + b*x)^(3/2
)*(a*c - b*c*x)^(3/2))/(4*b^2*c) + ((4*A + (a^2*C)/b^2)*((x*Sqrt[a + b*x]*
Sqrt[a*c - b*c*x])/2 + (a^2*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[a + b*x])/Sqrt[a*
c - b*c*x]])/b))/4
```

3.23.3.1 Defintions of rubi rules used

rule 40
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Simp[2*a*c*(m/(2*m + 1))   Int[(a
 + b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
b*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 646
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2), x_Symbol] :> Simp[b*x*(c + d*x)^(m + 1)*((e + f*x)^(n + 1)/(d*f*(2*m +
 3))), x] - Simp[(b*c*e - a*d*f*(2*m + 3))/(d*f*(2*m + 3))   Int[(c + d*x)^
m*(e + f*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m, n] &&
 EqQ[d*e + c*f, 0] &&  !LtQ[m, -1]
```

rule 1189
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2), x_Symbol] :> Simp[b   Int[x*(d + e*x)^m*(f + g*x)^n, x],
x] + Int[(d + e*x)^m*(f + g*x)^n*(a + c*x^2), x] /; FreeQ[{a, b, c, d, e, f
, g, m, n}, x] && EqQ[m, n] && EqQ[e*f + d*g, 0]
```

3.23.4 Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.74

method	result
risch	$\frac{(6C\,b^2x^3+8b^2B\,x^2+12A\,b^2x-3C\,a^2x-8a^2B)\sqrt{bx+a}\,(-bx+a)c}{24b^2\sqrt{-c(bx-a)}}+\frac{a^2(4b^2A+C\,a^2)\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{-b^2c\,x^2+a^2c}}\right)\sqrt{-(bx+a)c(bx-a)}\;c}{8b^2\sqrt{b^2c}\,\sqrt{bx+a}\,\sqrt{-c(bx-a)}}$
default	$\frac{\sqrt{bx+a}\,\sqrt{c(-bx+a)}\left(6C\,b^2x^3\sqrt{b^2c}\,\sqrt{c(-b^2x^2+a^2)}+12A\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{c(-b^2x^2+a^2)}}\right)a^2b^2c+8B\,b^2x^2\sqrt{b^2c}\,\sqrt{c(-b^2x^2+a^2)}+3C\,\text{arcta}\cdots\right.}{24\sqrt{c(-b^2x^2+a^2)}\,b\cdots}$

input
```
int((C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
1/24*(6*C*b^2*x^3+8*B*b^2*x^2+12*A*b^2*x-3*C*a^2*x-8*B*a^2)*(b*x+a)^(1/2)/
b^2*(-b*x+a)/(-c*(b*x-a))^(1/2)*c+1/8*a^2*(4*A*b^2+C*a^2)/b^2/(b^2*c)^(1/2
)*arctan((b^2*c)^(1/2)*x/(-b^2*c*x^2+a^2*c)^(1/2))*(-(b*x+a)*c*(b*x-a))^(1
/2)/(b*x+a)^(1/2)/(-c*(b*x-a))^(1/2)*c
```

3.23.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.20

$$\int \sqrt{a+bx}\sqrt{ac-bcx}\left(A+Bx+Cx^2\right)\,dx$$
$$= \left[\frac{3\left(Ca^4+4\,Aa^2b^2\right)\sqrt{-c}\log\left(2\,b^2cx^2+2\,\sqrt{-bcx+ac}\sqrt{bx+a}b\sqrt{-c}x-a^2c\right)+2\left(6\,Cb^3x^3+8\,Bb^3x^2-8\,B\right.}{48\,b^3}\right.$$
$$-\frac{3\left(Ca^4+4\,Aa^2b^2\right)\sqrt{c}\arctan\left(\frac{\sqrt{-bcx+ac}\sqrt{bx+a}b\sqrt{c}x}{b^2cx^2-a^2c}\right)-\left(6\,Cb^3x^3+8\,Bb^3x^2-8\,Ba^2b-3\left(Ca^2b-4\,Ab^3\right)x\right)}{24\,b^3}$$

input
```
integrate((C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algorithm="fri
cas")
```

output
```
[1/48*(3*(C*a^4 + 4*A*a^2*b^2)*sqrt(-c)*log(2*b^2*c*x^2 + 2*sqrt(-b*c*x +
a*c)*sqrt(b*x + a)*b*sqrt(-c)*x - a^2*c) + 2*(6*C*b^3*x^3 + 8*B*b^3*x^2 -
8*B*a^2*b - 3*(C*a^2*b - 4*A*b^3)*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/b^3
, -1/24*(3*(C*a^4 + 4*A*a^2*b^2)*sqrt(c)*arctan(sqrt(-b*c*x + a*c)*sqrt(b*
x + a)*b*sqrt(c)*x/(b^2*c*x^2 - a^2*c)) - (6*C*b^3*x^3 + 8*B*b^3*x^2 - 8*B
*a^2*b - 3*(C*a^2*b - 4*A*b^3)*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/b^3]
```

3.23.6 Sympy [F]

$$\int \sqrt{a+bx}\sqrt{ac-bcx}\left(A+Bx+Cx^2\right)\,dx=\int \sqrt{-c\left(-a+bx\right)}\sqrt{a+bx}\left(A+Bx+Cx^2\right)\,dx$$

input
```
integrate((C*x**2+B*x+A)*(b*x+a)**(1/2)*(-b*c*x+a*c)**(1/2),x)
```

output
```
Integral(sqrt(-c*(-a + b*x))*sqrt(a + b*x)*(A + B*x + C*x**2), x)
```

3.23.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.63

$$\int \sqrt{a+bx}\sqrt{ac-bcx}\left(A+Bx+Cx^2\right)\,dx = \frac{Ca^4\sqrt{c}\arcsin\left(\frac{bx}{a}\right)}{8\,b^3}+\frac{Aa^2\sqrt{c}\arcsin\left(\frac{bx}{a}\right)}{2\,b}+\frac{1}{2}\sqrt{-b^2cx^2+a^2c}Ax+\frac{\sqrt{-b^2cx^2+a^2c}Ca^2x}{8\,b^2}-\frac{\left(-b^2cx^2+a^2c\right)^{\frac{3}{2}}Cx}{4\,b^2c}-\frac{\left(-b^2cx^2+a^2c\right)^{\frac{3}{2}}B}{3\,b^2c}$$

input
```
integrate((C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algorithm="max
ima")
```

output
```
1/8*C*a^4*sqrt(c)*arcsin(b*x/a)/b^3 + 1/2*A*a^2*sqrt(c)*arcsin(b*x/a)/b +
1/2*sqrt(-b^2*c*x^2 + a^2*c)*A*x + 1/8*sqrt(-b^2*c*x^2 + a^2*c)*C*a^2*x/b^
2 - 1/4*(-b^2*c*x^2 + a^2*c)^(3/2)*C*x/(b^2*c) - 1/3*(-b^2*c*x^2 + a^2*c)^
(3/2)*B/(b^2*c)
```

3.23.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. 2(191) = 382.

Time = 0.43 (sec) , antiderivative size = 527, normalized size of antiderivative = 2.38

$$\int \sqrt{a+bx}\sqrt{ac-bcx}\left(A+Bx+Cx^2\right)\,dx = -\frac{24\left(\frac{2\,ac\log\left(\left|-\sqrt{bx+a}\sqrt{-c}+\sqrt{-(bx+a)c+2\,ac}\right|\right)}{\sqrt{-c}}-\sqrt{-(bx+a)c+2\,ac}\sqrt{bx+a}\right)Aab^2-12\left(\frac{2\,a^2c\log\left(\left|-\sqrt{bx+a}\sqrt{-c}\cdots\right.\right.}{\sqrt{-c}}\cdots\right.}{\cdots}$$

input
```
integrate((C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algorithm="gia
c")
```

output
```
-1/24*(24*(2*a*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a
*c)))/sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*A*a*b^2 - 12*(2
*a^2*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt
(-c) + sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(b*x - 2*a))*B*a*b - 12*(2
*a^2*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt
(-c) + sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(b*x - 2*a))*A*b^2 + 4*(6*
a^3*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(
-c) - ((2*b*x - 5*a)*(b*x + a) + 9*a^2)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*
x + a))*C*a + 4*(6*a^3*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)
*c + 2*a*c)))/sqrt(-c) - ((2*b*x - 5*a)*(b*x + a) + 9*a^2)*sqrt(-(b*x + a)
*c + 2*a*c)*sqrt(b*x + a))*B*b - (18*a^4*c*log(abs(-sqrt(b*x + a)*sqrt(-c)
 + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - (39*a^3 - (2*(3*b*x - 10*a)*(b*
x + a) + 43*a^2)*(b*x + a))*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*C)/b
^3
```

3.23.9 Mupad [B] (verification not implemented)

Time = 19.44 (sec) , antiderivative size = 876, normalized size of antiderivative = 3.96

$$\int \sqrt{a+bx}\sqrt{ac-bcx}\left(A+Bx+Cx^2\right)\,dx$$

$$= \frac{\frac{C\,a^4\,c^8\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}{2\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}-\frac{C\,a^4\,c\,{\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}^{15}}{2\,{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^{15}}-\frac{35\,C\,a^4\,c^7\,{\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}^3}{2\,{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^3}+\frac{273\,C\,a^4\,c^6\,{\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}^5}{2\,{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^5}-\frac{715\,C\,a^4 \cdots}{2 \cdots}}{b^3\,c^8+\frac{b^3\,{\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}^{16}}{{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^{16}}+\frac{8\,b^3\,c^7\,{\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}^2}{{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^2}+\frac{28\,b^3\,c^6\,{\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}^4}{{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^4}+\frac{56\,b^3\,c^5\,{\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}^6}{{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^6}+\frac{7 \cdots}{\cdots}}$$

$$+\frac{A\,x\,\sqrt{a\,c-b\,c\,x}\,\sqrt{a+b\,x}}{2}-\frac{B\,\left(a^2-b^2\,x^2\right)\,\sqrt{a\,c-b\,c\,x}\,\sqrt{a+b\,x}}{3\,b^2}$$

$$-\frac{C\,a^4\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}}{\sqrt{c}\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}\right)}{2\,b^3}-\frac{A\,a^2\,\sqrt{b}\,c^2\,\ln\left(\sqrt{-b\,c}\,\sqrt{c\,\left(a-b\,x\right)}\,\sqrt{a+b\,x}-b^{3/2}\,c\,x\right)}{2\,{\left(-b\,c\right)}^{3/2}}$$

input
```
int((a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)*(A + B*x + C*x^2),x)
```

output
```
((C*a^4*c^8*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(2*((a + b*x)^(1/2) - a^(
1/2))) - (C*a^4*c*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^15)/(2*((a + b*x)^(1
/2) - a^(1/2))^15) - (35*C*a^4*c^7*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/
(2*((a + b*x)^(1/2) - a^(1/2))^3) + (273*C*a^4*c^6*((a*c - b*c*x)^(1/2) -
(a*c)^(1/2))^5)/(2*((a + b*x)^(1/2) - a^(1/2))^5) - (715*C*a^4*c^5*((a*c -
 b*c*x)^(1/2) - (a*c)^(1/2))^7)/(2*((a + b*x)^(1/2) - a^(1/2))^7) + (715*C
*a^4*c^4*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^9)/(2*((a + b*x)^(1/2) - a^(1
/2))^9) - (273*C*a^4*c^3*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^11)/(2*((a +
b*x)^(1/2) - a^(1/2))^11) + (35*C*a^4*c^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/
2))^13)/(2*((a + b*x)^(1/2) - a^(1/2))^13))/(b^3*c^8 + (b^3*((a*c - b*c*x)
^(1/2) - (a*c)^(1/2))^16)/((a + b*x)^(1/2) - a^(1/2))^16 + (8*b^3*c^7*((a*
c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 + (28*b^3
*c^6*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4)/((a + b*x)^(1/2) - a^(1/2))^4
+ (56*b^3*c^5*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6)/((a + b*x)^(1/2) - a^
(1/2))^6 + (70*b^3*c^4*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^8)/((a + b*x)^(
1/2) - a^(1/2))^8 + (56*b^3*c^3*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^10)/((
a + b*x)^(1/2) - a^(1/2))^10 + (28*b^3*c^2*((a*c - b*c*x)^(1/2) - (a*c)^(1
/2))^12)/((a + b*x)^(1/2) - a^(1/2))^12 + (8*b^3*c*((a*c - b*c*x)^(1/2) -
(a*c)^(1/2))^14)/((a + b*x)^(1/2) - a^(1/2))^14) + (A*x*(a*c - b*c*x)^(1/2
)*(a + b*x)^(1/2))/2 - (B*(a^2 - b^2*x^2)*(a*c - b*c*x)^(1/2)*(a + b*x)...
```

3.23. $\int \sqrt{a+bx}\sqrt{ac-bcx}(A+Bx+Cx^2)\,dx$

3.24 $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}\,dx$

3.24.1 Optimal result

Integrand size = 40, antiderivative size = 278

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}\,dx &= -\frac{C(a^2-b^2x^2)}{b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{(Ce-Bf)\sqrt{a^2c-b^2cx^2}\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}f^2\sqrt{a+bx}\sqrt{ac-bcx}} \\ &\quad + \frac{(Ce^2-Bef+Af^2)\sqrt{a^2c-b^2cx^2}\arctan\left(\frac{\sqrt{c}(a^2f+b^2ex)}{\sqrt{b^2e^2-a^2f^2}\sqrt{a^2c-b^2cx^2}}\right)}{\sqrt{c}f^2\sqrt{b^2e^2-a^2f^2}\sqrt{a+bx}\sqrt{ac-bcx}}\end{aligned}$$

output
```
-C*(-b^2*x^2+a^2)/b^2/f/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)-(-B*f+C*e)*arctan
(b*x*c^(1/2)/(-b^2*c*x^2+a^2*c)^(1/2))*(-b^2*c*x^2+a^2*c)^(1/2)/b/f^2/c^(1
/2)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)+(A*f^2-B*e*f+C*e^2)*arctan((b^2*e*x+a
^2*f)*c^(1/2)/(-a^2*f^2+b^2*e^2)^(1/2)/(-b^2*c*x^2+a^2*c)^(1/2))*(-b^2*c*x
^2+a^2*c)^(1/2)/f^2/c^(1/2)/(-a^2*f^2+b^2*e^2)^(1/2)/(b*x+a)^(1/2)/(-b*c*x
+a*c)^(1/2)
```

3.24.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.64

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}\,dx$$
$$=\frac{\frac{Cf(-a+bx)\sqrt{a+bx}}{b^2}-\frac{2(Ce-Bf)\sqrt{a-bx}\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{a-bx}}\right)}{b}+\frac{2(Ce^2+f(-Be+Af))\sqrt{a-bx}\arctan\left(\frac{\sqrt{be+af}\sqrt{a+bx}}{\sqrt{be-af}\sqrt{a-bx}}\right)}{\sqrt{be-af}\sqrt{be+af}}}{f^2\sqrt{c(a-bx)}}$$

input
```
Integrate[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)),x]
```

output
```
((C*f*(-a + b*x)*Sqrt[a + b*x])/b^2 - (2*(C*e - B*f)*Sqrt[a - b*x]*ArcTan[
Sqrt[a + b*x]/Sqrt[a - b*x]])/b + (2*(C*e^2 + f*(-(B*e) + A*f))*Sqrt[a - b
*x]*ArcTan[(Sqrt[b*e + a*f]*Sqrt[a + b*x])/(Sqrt[b*e - a*f]*Sqrt[a - b*x])
])/(Sqrt[b*e - a*f]*Sqrt[b*e + a*f]))/(f^2*Sqrt[c*(a - b*x)])
```

3.24.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.84, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.225, Rules used = {2113, 2185, 25, 27, 719, 224, 216, 488, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}(e+fx)\sqrt{ac-bcx}}\,dx$$
$$\Big\downarrow \text{2113}$$
$$\frac{\sqrt{a^2c-b^2cx^2}\int \frac{Cx^2+Bx+A}{(e+fx)\sqrt{a^2c-b^2cx^2}}dx}{\sqrt{a+bx}\sqrt{ac-bcx}}$$
$$\Big\downarrow \text{2185}$$
$$\frac{\sqrt{a^2c-b^2cx^2}\left(-\frac{\int -\frac{b^2cf(Af-(Ce-Bf)x)}{(e+fx)\sqrt{a^2c-b^2cx^2}}dx}{b^2cf^2}-\frac{C\sqrt{a^2c-b^2cx^2}}{b^2cf}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$
$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\int \frac{b^2cf(Af-(Ce-Bf)x)}{(e+fx)\sqrt{a^2c-b^2cx^2}}dx}{b^2cf^2}-\frac{C\sqrt{a^2c-b^2cx^2}}{b^2cf}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\int \frac{Af-(Ce-Bf)x}{(e+fx)\sqrt{a^2c-b^2cx^2}}dx}{f}-\frac{C\sqrt{a^2c-b^2cx^2}}{b^2cf}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \; 719$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{\left(Af^2-Bef+Ce^2\right)\int \frac{1}{(e+fx)\sqrt{a^2c-b^2cx^2}}dx}{f}-\frac{(Ce-Bf)\int \frac{1}{\sqrt{a^2c-b^2cx^2}}dx}{f}}{f}-\frac{C\sqrt{a^2c-b^2cx^2}}{b^2cf}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \; 224$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{\left(Af^2-Bef+Ce^2\right)\int \frac{1}{(e+fx)\sqrt{a^2c-b^2cx^2}}dx}{f}-\frac{(Ce-Bf)\int \frac{1}{\frac{b^2cx^2}{a^2c-b^2cx^2}+1}d\frac{x}{\sqrt{a^2c-b^2cx^2}}}{f}}{f}-\frac{C\sqrt{a^2c-b^2cx^2}}{b^2cf}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \; 216$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{\left(Af^2-Bef+Ce^2\right)\int \frac{1}{(e+fx)\sqrt{a^2c-b^2cx^2}}dx}{f}-\frac{(Ce-Bf)\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}f}}{f}-\frac{C\sqrt{a^2c-b^2cx^2}}{b^2cf}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \; 488$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{-\frac{\left(Af^2-Bef+Ce^2\right)\int \frac{1}{-b^2ce^2+a^2cf^2-\frac{\left(cfa^2+b^2cex\right)^2}{a^2c-b^2cx^2}}d\frac{cfa^2+b^2cex}{\sqrt{a^2c-b^2cx^2}}}{f}-\frac{(Ce-Bf)\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}f}}{f}-\frac{C\sqrt{a^2c-b^2cx^2}}{b^2cf}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \; 217$$

3.24. $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}\,dx$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{\left(Af^2-Bef+Ce^2\right)\arctan\left(\frac{a^2cf+b^2cex}{\sqrt{c}\sqrt{a^2c-b^2cx^2}\sqrt{b^2e^2-a^2f^2}}\right)}{\sqrt{c}f\sqrt{b^2e^2-a^2f^2}}-\frac{(Ce-Bf)\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}f}}{f}-\frac{C\sqrt{a^2c-b^2cx^2}}{b^2cf}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

input
```
Int[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)),x]
```

output
```
(Sqrt[a^2*c - b^2*c*x^2]*(-((C*Sqrt[a^2*c - b^2*c*x^2])/(b^2*c*f)) + (-(((
C*e - B*f)*ArcTan[(b*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(b*Sqrt[c]*f)) +
 ((C*e^2 - B*e*f + A*f^2)*ArcTan[(a^2*c*f + b^2*c*e*x)/(Sqrt[c]*Sqrt[b^2*e
^2 - a^2*f^2]*Sqrt[a^2*c - b^2*c*x^2])])/(Sqrt[c]*f*Sqrt[b^2*e^2 - a^2*f^2
]))/f))/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
```

3.24.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2113
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m])   Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a
*d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.24.4 Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.08

method	result
risch	$-\frac{C\sqrt{bx+a}\,(-bx+a)}{f\,b^2\sqrt{-c(bx-a)}}+\frac{\left(\frac{(Bf-Ce)\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{-b^2cx^2+a^2c}}\right)}{f\sqrt{b^2c}}-\frac{\left(Af^2-Bef+Ce^2\right)\ln\left(\frac{\frac{2c\left(a^2f^2-b^2e^2\right)}{f^2}+\frac{2b^2ce\left(x+\frac{e}{f}\right)}{f}+2\sqrt{\frac{c\left(a^2f^2-b^2e^2\right)}{f^2}}}{x+}\right.}{f^2\sqrt{\frac{c\left(a^2f^2-b^2e^2\right)}{f^2}}}\right.}{f\sqrt{bx+a}\,\sqrt{-c(bx-a)}}$
default	$\frac{\left(-A\ln\left(\frac{2b^2cex+2a^2cf+2\sqrt{\frac{c\left(a^2f^2-b^2e^2\right)}{f^2}}\sqrt{c\left(-b^2x^2+a^2\right)}\,f}{fx+e}\right)b^2cf^2\sqrt{b^2c}+B\ln\left(\frac{2b^2cex+2a^2cf+2\sqrt{\frac{c\left(a^2f^2-b^2e^2\right)}{f^2}}\sqrt{c\left(-b^2x^2+a^2\right)}\,f}{fx+e}\right)\right.}{}$

input
```
int((C*x^2+B*x+A)/(f*x+e)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x,method=_RETUR
NVERBOSE)
```

output
```
-C*(b*x+a)^(1/2)*(-b*x+a)/f/b^2/(-c*(b*x-a))^(1/2)+1/f*((B*f-C*e)/f/(b^2*c
)^(1/2)*arctan((b^2*c)^(1/2)*x/(-b^2*c*x^2+a^2*c)^(1/2))-(A*f^2-B*e*f+C*e^
2)/f^2/(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*ln((2*c*(a^2*f^2-b^2*e^2)/f^2+2*b^2
*c*e/f*(x+e/f)+2*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-b^2*c*(x+e/f)^2+2*b^2*c
*e/f*(x+e/f)+c*(a^2*f^2-b^2*e^2)/f^2)^(1/2))/(x+e/f)))*(-(b*x+a)*c*(b*x-a)
)^(1/2)/(b*x+a)^(1/2)/(-c*(b*x-a))^(1/2)
```

3.24.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}\,dx=\text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/(f*x+e)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algori
thm="fricas")
```

output
```
Timed out
```

3.24.6 Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}\,dx = \int \frac{A+Bx+Cx^2}{\sqrt{-c\left(-a+bx\right)}\sqrt{a+bx}\left(e+fx\right)}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(f*x+e)/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(sqrt(-c*(-a + b*x))*sqrt(a + b*x)*(e + f*x)),
 x)
```

3.24.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+B*x+A)/(f*x+e)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algori
thm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((4*b^2*c>0)', see `assume?` for
more detai
```

3.24.8 Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((C*x^2+B*x+A)/(f*x+e)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.24.9 Mupad [B] (verification not implemented)

Time = 56.99 (sec) , antiderivative size = 9298, normalized size of antiderivative = 33.45

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/((e + f*x)*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),x)
```

output
```
(B*a*e*atan(((B*a*e*((4096*(32*B^3*a^(17/2)*c^3*e*f^2*(a*c)^(5/2) + 24*B^3
*a^(15/2)*b^2*c^4*e^3*(a*c)^(3/2)))/(a^6*b^8*e^6) - (4096*(32*B^3*a^(17/2)
*c^2*e*f^2*(a*c)^(5/2) - 96*B^3*a^(15/2)*b^2*c^3*e^3*(a*c)^(3/2))*((a*c -
b*c*x)^(1/2) - (a*c)^(1/2))^2)/(a^6*b^8*e^6*((a + b*x)^(1/2) - a^(1/2))^2)
 - (B*a*e*((4096*(16*B^2*a^12*c^6*f^4 + 9*B^2*a^8*b^4*c^6*e^4))/(a^6*b^8*e
^6) + (B*a*e*((4096*(24*B*a^(17/2)*b^2*c^4*e*f^4*(a*c)^(5/2) - 30*B*a^(15/
2)*b^4*c^5*e^3*f^2*(a*c)^(3/2)))/(a^6*b^8*e^6) + (16384*(20*B*a^12*c^6*f^5
 - 22*B*a^10*b^2*c^6*e^2*f^3)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^
7*e^6*((a + b*x)^(1/2) - a^(1/2))) + (B*a*e*((4096*(9*a^8*b^6*c^7*e^4*f^2
- 7*a^10*b^4*c^7*e^2*f^4))/(a^6*b^8*e^6) + (4096*(9*a^8*b^6*c^6*e^4*f^2 -
11*a^10*b^4*c^6*e^2*f^4)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(a^6*b^8*e
^6*((a + b*x)^(1/2) - a^(1/2))^2) - (16384*(5*a^(17/2)*b^2*c^4*e*f^5*(a*c)
^(5/2) - 6*a^(15/2)*b^4*c^5*e^3*f^3*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a
*c)^(1/2)))/(a^6*b^7*e^6*((a + b*x)^(1/2) - a^(1/2)))))/(f*(a^4*c*f^2 - a^
2*b^2*c*e^2)^(1/2)) + (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(96*B*a^
(17/2)*b^2*c^3*e*f^4*(a*c)^(5/2) - 90*B*a^(15/2)*b^4*c^4*e^3*f^2*(a*c)^(3/
2)))/(a^6*b^8*e^6*((a + b*x)^(1/2) - a^(1/2))^2)))/(f*(a^4*c*f^2 - a^2*b^2
*c*e^2)^(1/2)) + (16384*(8*B^2*a^(17/2)*c^3*e*f^3*(a*c)^(5/2) + 3*B^2*a^(1
5/2)*b^2*c^4*e^3*f*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*
b^7*e^6*((a + b*x)^(1/2) - a^(1/2))) + (4096*((a*c - b*c*x)^(1/2) - (a*...
```

3.25 $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2}\,dx$

3.25.1 Optimal result

Integrand size = 40, antiderivative size = 322

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2}\,dx = \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)(a^2-b^2x^2)}{(b^2e^2-a^2f^2)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}+\frac{C\sqrt{a^2c-b^2cx^2}\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}f^2\sqrt{a+bx}\sqrt{ac-bcx}} + \frac{(a^2f^2(2Ce-Bf)-b^2(Ce^3-Aef^2))\sqrt{a^2c-b^2cx^2}\arctan\left(\frac{\sqrt{c}(a^2f+b^2ex)}{\sqrt{b^2e^2-a^2f^2}\sqrt{a^2c-b^2cx^2}}\right)}{\sqrt{c}f^2(b^2e^2-a^2f^2)^{3/2}\sqrt{a+bx}\sqrt{ac-bcx}}$$

output
```
f*(A+e*(-B*f+C*e)/f^2)*(-b^2*x^2+a^2)/(-a^2*f^2+b^2*e^2)/(f*x+e)/(b*x+a)^(
1/2)/(-b*c*x+a*c)^(1/2)+C*arctan(b*x*c^(1/2)/(-b^2*c*x^2+a^2*c)^(1/2))*(-b
^2*c*x^2+a^2*c)^(1/2)/b/f^2/c^(1/2)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)+(a^2*
f^2*(-B*f+2*C*e)-b^2*(-A*e*f^2+C*e^3))*arctan((b^2*e*x+a^2*f)*c^(1/2)/(-a^
2*f^2+b^2*e^2)^(1/2)/(-b^2*c*x^2+a^2*c)^(1/2))*(-b^2*c*x^2+a^2*c)^(1/2)/f^
2/(-a^2*f^2+b^2*e^2)^(3/2)/c^(1/2)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)
```

3.25.2 Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.71

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2}\,dx$$
$$= \frac{2\left(\frac{f(Ce^2+f(-Be+Af))(-a+bx)\sqrt{a+bx}}{2(-be+af)(be+af)(e+fx)} + \frac{C\sqrt{a-bx}\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{a-bx}}\right)}{b} - \frac{(a^2f^2(-2Ce+Bf)+b^2(Ce^3-Aef^2))\sqrt{a-bx}\arctan\left(\frac{\sqrt{be+af}\sqrt{a+bx}}{\sqrt{be-af}\sqrt{a-bx}}\right)}{(be-af)^{3/2}(be+af)^{3/2}}\right)}{f^2\sqrt{c(a-bx)}}$$

input
```
Integrate[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^2),
x]
```

output
```
(2*((f*(C*e^2 + f*(-(B*e) + A*f))*(-a + b*x)*Sqrt[a + b*x])/(2*(-(b*e) + a
*f)*(b*e + a*f)*(e + f*x)) + (C*Sqrt[a - b*x]*ArcTan[Sqrt[a + b*x]/Sqrt[a
- b*x]])/b - ((a^2*f^2*(-2*C*e + B*f) + b^2*(C*e^3 - A*e*f^2))*Sqrt[a - b*
x]*ArcTan[(Sqrt[b*e + a*f]*Sqrt[a + b*x])/(Sqrt[b*e - a*f]*Sqrt[a - b*x])]
)/((b*e - a*f)^(3/2)*(b*e + a*f)^(3/2))))/(f^2*Sqrt[c*(a - b*x)])
```

3.25.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2113, 2182, 27, 719, 224, 216, 488, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}(e+fx)^2\sqrt{ac-bcx}}\,dx$$
$$\Big\downarrow \text{2113}$$
$$\frac{\sqrt{a^2c-b^2cx^2}\int \frac{Cx^2+Bx+A}{(e+fx)^2\sqrt{a^2c-b^2cx^2}}dx}{\sqrt{a+bx}\sqrt{ac-bcx}}$$
$$\Big\downarrow \text{2182}$$
$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\int \frac{c\left((Ce-Bf)a^2+Ab^2e+C\left(\frac{b^2e^2}{f}-a^2f\right)x\right)}{(e+fx)\sqrt{a^2c-b^2cx^2}}dx}{c(b^2e^2-a^2f^2)} + \frac{f\sqrt{a^2c-b^2cx^2}\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{c(e+fx)(b^2e^2-a^2f^2)}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\int \frac{(Ce-Bf)a^2+Ab^2e+C\left(\frac{b^2e^2}{f}-a^2f\right)x}{(e+fx)\sqrt{a^2c-b^2cx^2}}dx}{b^2e^2-a^2f^2}+\frac{f\sqrt{a^2c-b^2cx^2}\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{c(e+fx)(b^2e^2-a^2f^2)}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$\Big\downarrow$ 719

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{\left(a^2f^2(2Ce-Bf)-b^2\left(Ce^3-Aef^2\right)\right)\int\frac{1}{(e+fx)\sqrt{a^2c-b^2cx^2}}dx}{f^2}+\frac{C\left(b^2e^2-a^2f^2\right)\int\frac{1}{\sqrt{a^2c-b^2cx^2}}dx}{f^2}}{b^2e^2-a^2f^2}+\frac{f\sqrt{a^2c-b^2cx^2}\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{c(e+fx)(b^2e^2-a^2f^2)}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$\Big\downarrow$ 224

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{\left(a^2f^2(2Ce-Bf)-b^2\left(Ce^3-Aef^2\right)\right)\int\frac{1}{(e+fx)\sqrt{a^2c-b^2cx^2}}dx}{f^2}+\frac{C\left(b^2e^2-a^2f^2\right)\int\frac{1}{\frac{b^2cx^2}{a^2c-b^2cx^2}+1}d\frac{x}{\sqrt{a^2c-b^2cx^2}}}{f^2}}{b^2e^2-a^2f^2}+\frac{f\sqrt{a^2c-b^2cx^2}\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{c(e+fx)(b^2e^2-a^2f^2)}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$\Big\downarrow$ 216

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{\left(a^2f^2(2Ce-Bf)-b^2\left(Ce^3-Aef^2\right)\right)\int\frac{1}{(e+fx)\sqrt{a^2c-b^2cx^2}}dx}{f^2}+\frac{C\left(b^2e^2-a^2f^2\right)\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}f^2}}{b^2e^2-a^2f^2}+\frac{f\sqrt{a^2c-b^2cx^2}\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{c(e+fx)(b^2e^2-a^2f^2)}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$\Big\downarrow$ 488

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{C\left(b^2e^2-a^2f^2\right)\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}f^2}-\frac{\left(a^2f^2(2Ce-Bf)-b^2\left(Ce^3-Aef^2\right)\right)\int\frac{1}{-b^2ce^2+a^2cf^2-\frac{\left(cfa^2+b^2cex\right)^2}{a^2c-b^2cx^2}}d\frac{cfa^2+b^2cex}{\sqrt{a^2c-b^2cx^2}}}{f^2}}{b^2e^2-a^2f^2}+\frac{f\sqrt{a^2c-b^2cx^2}\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{c(e+fx)(b^2e^2-a^2f^2)}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$\Big\downarrow$ 217

3.25. $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2}\,dx$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{\left(a^2f^2(2Ce-Bf)-b^2\left(Ce^3-Aef^2\right)\right)\arctan\left(\frac{a^2cf+b^2cex}{\sqrt{c}\sqrt{a^2c-b^2cx^2}\sqrt{b^2e^2-a^2f^2}}\right)}{\sqrt{c}f^2\sqrt{b^2e^2-a^2f^2}}+\frac{C\left(b^2e^2-a^2f^2\right)\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}f^2}}{b^2e^2-a^2f^2}+\frac{f\sqrt{a^2c-b^2cx}}{c(e+fx)}\right.}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

input
```
Int[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^2),x]
```

output
```
(Sqrt[a^2*c - b^2*c*x^2]*((f*(A + (e*(C*e - B*f))/f^2)*Sqrt[a^2*c - b^2*c*
x^2])/(c*(b^2*e^2 - a^2*f^2)*(e + f*x)) + ((C*(b^2*e^2 - a^2*f^2)*ArcTan[(
b*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(b*Sqrt[c]*f^2) + ((a^2*f^2*(2*C*e
- B*f) - b^2*(C*e^3 - A*e*f^2))*ArcTan[(a^2*c*f + b^2*c*e*x)/(Sqrt[c]*Sqrt
[b^2*e^2 - a^2*f^2]*Sqrt[a^2*c - b^2*c*x^2])])/(Sqrt[c]*f^2*Sqrt[b^2*e^2 -
 a^2*f^2]))/(b^2*e^2 - a^2*f^2)))/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
```

3.25.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2113
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m])   Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a
*d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

3.25.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1165 vs. $2(290) = 580$.

Time = 1.70 (sec) , antiderivative size = 1166, normalized size of antiderivative = 3.62

method	result	size
default	Expression too large to display	1166

input
```
int((C*x^2+B*x+A)/(f*x+e)^2/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
(A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^
2))^(1/2)*f)/(f*x+e))*b^2*c*e*f^3*x*(b^2*c)^(1/2)-B*ln(2*(b^2*c*e*x+a^2*c*
f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^2
*c*f^4*x*(b^2*c)^(1/2)+C*arctan((b^2*c)^(1/2)*x/(c*(-b^2*x^2+a^2))^(1/2))*
a^2*c*f^4*x*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)-C*arctan((b^2*c)^(1/2)*x/(c*(-
b^2*x^2+a^2))^(1/2))*b^2*c*e^2*f^2*x*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)+2*C*l
n(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^
(1/2)*f)/(f*x+e))*a^2*c*e*f^3*x*(b^2*c)^(1/2)-C*ln(2*(b^2*c*e*x+a^2*c*f+(c
*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*b^2*c*e
^3*f*x*(b^2*c)^(1/2)+A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(
1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*b^2*c*e^2*f^2*(b^2*c)^(1/2)-B*ln
(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(
1/2)*f)/(f*x+e))*a^2*c*e*f^3*(b^2*c)^(1/2)+C*arctan((b^2*c)^(1/2)*x/(c*(-b
^2*x^2+a^2))^(1/2))*a^2*c*e*f^3*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)-C*arctan((
b^2*c)^(1/2)*x/(c*(-b^2*x^2+a^2))^(1/2))*b^2*c*e^3*f*(c*(a^2*f^2-b^2*e^2)/
f^2)^(1/2)+2*C*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*
(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^2*c*e^2*f^2*(b^2*c)^(1/2)-C*ln(2*(b^2*
c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/
(f*x+e))*b^2*c*e^4*(b^2*c)^(1/2)-A*f^4*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*
(-b^2*x^2+a^2))^(1/2)*(b^2*c)^(1/2)+B*e*f^3*(c*(a^2*f^2-b^2*e^2)/f^2)^(...
```

3.25.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/(f*x+e)^2/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algo
rithm="fricas")
```

output
```
Timed out
```

3.25.6 Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2}\,dx = \int \frac{A+Bx+Cx^2}{\sqrt{-c\left(-a+bx\right)}\sqrt{a+bx}\left(e+fx\right)^2}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(f*x+e)**2/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(sqrt(-c*(-a + b*x))*sqrt(a + b*x)*(e + f*x)**
2), x)
```

3.25.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+B*x+A)/(f*x+e)^2/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((4*b^2*c>0)', see `assume?` for
more detai
```

3.25.8 Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 526, normalized size of antiderivative = 1.63

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2}\,dx = \frac{\frac{2\,(Cb^3\sqrt{-c}e^3-2\,Ca^2b\sqrt{-c}ef^2-Ab^3\sqrt{-c}ef^2+Ba^2b\sqrt{-c}f^3)\arctan\left(-\frac{2\,bce-\left(\sqrt{bx+a}\sqrt{-c}-\sqrt{-(bx+a)c+2\,ac}\right)^2f}{2\,\sqrt{-b^2e^2+a^2f^2}c}\right)}{(b^2e^2f^2-a^2f^4)\sqrt{-b^2e^2+a^2f^2}c} - \frac{C\log\left(\left(\sqrt{bx+a}\sqrt{-c}-\sqrt{-(\cdots}\right.\right.}{\sqrt{-c}f^2}}{}$$

input
```
integrate((C*x^2+B*x+A)/(f*x+e)^2/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algo
rithm="giac")
```

output
```
(2*(C*b^3*sqrt(-c)*e^3 - 2*C*a^2*b*sqrt(-c)*e*f^2 - A*b^3*sqrt(-c)*e*f^2 +
 B*a^2*b*sqrt(-c)*f^3)*arctan(-1/2*(2*b*c*e - (sqrt(b*x + a)*sqrt(-c) - sq
rt(-(b*x + a)*c + 2*a*c))^2*f)/(sqrt(-b^2*e^2 + a^2*f^2)*c))/((b^2*e^2*f^2
 - a^2*f^4)*sqrt(-b^2*e^2 + a^2*f^2)*c) - C*log((sqrt(b*x + a)*sqrt(-c) -
sqrt(-(b*x + a)*c + 2*a*c))^2)/(sqrt(-c)*f^2) + 4*(C*b^3*(sqrt(b*x + a)*sq
rt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^2*sqrt(-c)*e^3 - B*b^3*(sqrt(b*x + a)
*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^2*sqrt(-c)*e^2*f - 2*C*a^2*b^2*sqr
t(-c)*c*e^2*f + A*b^3*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c)
)^2*sqrt(-c)*e*f^2 + 2*B*a^2*b^2*sqrt(-c)*c*e*f^2 - 2*A*a^2*b^2*sqrt(-c)*c
*f^3)/((b^2*e^2*f^2 - a^2*f^4)*(4*b*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x +
 a)*c + 2*a*c))^2*c*e - (sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*
c))^4*f - 4*a^2*c^2*f)))/b
```

3.25.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx + Cx^2}{\sqrt{a + bx}\sqrt{ac - bcx}(e + fx)^2} \, dx = \text{Hanged}$$

input
```
int((A + B*x + C*x^2)/((e + f*x)^2*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),x)
```

output
```
\text{Hanged}
```

3.26 $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3}\,dx$

3.26.1 Optimal result

Integrand size = 40, antiderivative size = 363

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3}\,dx = \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)(a^2-b^2x^2)}{2(b^2e^2-a^2f^2)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2} + \frac{(2a^2f^2(2Ce-Bf)-b^2e(Ce^2+f(Be-3Af)))(a^2-b^2x^2)}{2f(b^2e^2-a^2f^2)^2\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)} + \frac{(A(2b^4e^2+a^2b^2f^2)+a^2(2a^2Cf^2+b^2e(Ce-3Bf)))\sqrt{a^2c-b^2cx^2}\arctan\left(\frac{\sqrt{c}(a^2f+b^2ex)}{\sqrt{b^2e^2-a^2f^2}\sqrt{a^2c-b^2cx^2}}\right)}{2\sqrt{c}(b^2e^2-a^2f^2)^{5/2}\sqrt{a+bx}\sqrt{ac-bcx}}$$

output
```
1/2*f*(A+e*(-B*f+C*e)/f^2)*(-b^2*x^2+a^2)/(-a^2*f^2+b^2*e^2)/(f*x+e)^2/(b*
x+a)^(1/2)/(-b*c*x+a*c)^(1/2)+1/2*(2*a^2*f^2*(-B*f+2*C*e)-b^2*e*(C*e^2+f*(
-3*A*f+B*e)))*(-b^2*x^2+a^2)/f/(-a^2*f^2+b^2*e^2)^2/(f*x+e)/(b*x+a)^(1/2)/
(-b*c*x+a*c)^(1/2)+1/2*(A*(a^2*b^2*f^2+2*b^4*e^2)+a^2*(2*a^2*C*f^2+b^2*e*(
-3*B*f+C*e)))*arctan((b^2*e*x+a^2*f)*c^(1/2)/(-a^2*f^2+b^2*e^2)^(1/2)/(-b^
2*c*x^2+a^2*c)^(1/2))*(-b^2*c*x^2+a^2*c)^(1/2)/(-a^2*f^2+b^2*e^2)^(5/2)/c^
(1/2)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)
```

3.26.2 Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.69

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3}\,dx$$
$$= \frac{\frac{(-a+bx)\sqrt{a+bx}\left(b^2e\left(Ce^2x+Be(2e+fx)-Af(4e+3fx)\right)+a^2f\left(-Ce(3e+4fx)+f(Af+B(e+2fx))\right)\right)}{2(be-af)^2(be+af)^2(e+fx)^2}+\frac{\left(2a^4Cf^2+a^2b^2e(Ce-3Bf)+A\left(2b^4e^2+\right.\right.}{(be-af)^{5/2}}}{\sqrt{c(a-bx)}}$$

input
```
Integrate[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^3),
x]
```

output
```
(((-a + b*x)*Sqrt[a + b*x]*(b^2*e*(C*e^2*x + B*e*(2*e + f*x) - A*f*(4*e +
3*f*x)) + a^2*f*(-(C*e*(3*e + 4*f*x)) + f*(A*f + B*(e + 2*f*x)))))/(2*(b*e
 - a*f)^2*(b*e + a*f)^2*(e + f*x)^2) + ((2*a^4*C*f^2 + a^2*b^2*e*(C*e - 3*
B*f) + A*(2*b^4*e^2 + a^2*b^2*f^2))*Sqrt[a - b*x]*ArcTan[(Sqrt[b*e + a*f]*
Sqrt[a + b*x])/(Sqrt[b*e - a*f]*Sqrt[a - b*x])])/((b*e - a*f)^(5/2)*(b*e +
 a*f)^(5/2)))/Sqrt[c*(a - b*x)]
```

3.26.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 357, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2113, 2182, 27, 679, 488, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}(e+fx)^3\sqrt{ac-bcx}}\,dx$$
$$\Big\downarrow \text{ 2113}$$
$$\frac{\sqrt{a^2c-b^2cx^2}\int \frac{Cx^2+Bx+A}{(e+fx)^3\sqrt{a^2c-b^2cx^2}}dx}{\sqrt{a+bx}\sqrt{ac-bcx}}$$
$$\Big\downarrow \text{ 2182}$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\int \frac{c\left(2\left((Ce-Bf)a^2+Ab^2e\right)-\left(2a^2Cf-b^2\left(\frac{Ce^2}{f}+Be-Af\right)\right)x\right)}{(e+fx)^2\sqrt{a^2c-b^2cx^2}}dx}{2c(b^2e^2-a^2f^2)}+\frac{f\sqrt{a^2c-b^2cx^2}\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{2c(e+fx)^2(b^2e^2-a^2f^2)}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\int \frac{2\left((Ce-Bf)a^2+Ab^2e\right)-\left(2a^2Cf-b^2\left(\frac{Ce^2}{f}+Be-Af\right)\right)x}{(e+fx)^2\sqrt{a^2c-b^2cx^2}}dx}{2(b^2e^2-a^2f^2)}+\frac{f\sqrt{a^2c-b^2cx^2}\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{2c(e+fx)^2(b^2e^2-a^2f^2)}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \text{679}$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{\left(2a^4Cf^2+A\left(a^2b^2f^2+2b^4e^2\right)+a^2b^2e(Ce-3Bf)\right)\int\frac{1}{(e+fx)\sqrt{a^2c-b^2cx^2}}dx}{b^2e^2-a^2f^2}+\frac{\sqrt{a^2c-b^2cx^2}\left(2a^2f^2(2Ce-Bf)-b^2\left(ef(Be-3Af)+Ce^3\right)\right)}{cf(e+fx)\left(b^2e^2-a^2f^2\right)}}{2(b^2e^2-a^2f^2)} \cdots\right.}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \text{488}$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{\sqrt{a^2c-b^2cx^2}\left(2a^2f^2(2Ce-Bf)-b^2\left(ef(Be-3Af)+Ce^3\right)\right)}{cf(e+fx)\left(b^2e^2-a^2f^2\right)}-\frac{\left(2a^4Cf^2+A\left(a^2b^2f^2+2b^4e^2\right)+a^2b^2e(Ce-3Bf)\right)\int\frac{1}{-b^2ce^2+a^2cf^2-\frac{(cfa^2+\cdots}{a^2c-\cdots}}\cdots}{b^2e^2-a^2f^2}}{2(b^2e^2-a^2f^2)} \cdots\right.}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{f\sqrt{a^2c-b^2cx^2}\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{2c(e+fx)^2(b^2e^2-a^2f^2)}+\frac{\frac{\sqrt{a^2c-b^2cx^2}\left(2a^2f^2(2Ce-Bf)-b^2\left(ef(Be-3Af)+Ce^3\right)\right)}{cf(e+fx)\left(b^2e^2-a^2f^2\right)}+\frac{\left(2a^4Cf^2+A\left(a^2b^2f^2+2b^4e^2\right)+a^2b^2e(\cdots\right.}{\sqrt{c}(\cdots}}{2(b^2e^2-a^2f^2)} \cdots\right.}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

input
```
Int[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^3),x]
```

3.26. $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3}\,dx$

output
```
(Sqrt[a^2*c - b^2*c*x^2]*((f*(A + (e*(C*e - B*f))/f^2)*Sqrt[a^2*c - b^2*c*
x^2])/(2*c*(b^2*e^2 - a^2*f^2)*(e + f*x)^2) + (((2*a^2*f^2*(2*C*e - B*f) -
 b^2*(C*e^3 + e*f*(B*e - 3*A*f)))*Sqrt[a^2*c - b^2*c*x^2])/(c*f*(b^2*e^2 -
 a^2*f^2)*(e + f*x)) + ((2*a^4*C*f^2 + a^2*b^2*e*(C*e - 3*B*f) + A*(2*b^4*
e^2 + a^2*b^2*f^2))*ArcTan[(a^2*c*f + b^2*c*e*x)/(Sqrt[c]*Sqrt[b^2*e^2 - a
^2*f^2]*Sqrt[a^2*c - b^2*c*x^2])])/(Sqrt[c]*(b^2*e^2 - a^2*f^2)^(3/2)))/(2
*(b^2*e^2 - a^2*f^2))))/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
```

3.26.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2113
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m])   Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a
*d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

3.26.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1793 vs. $2(333) = 666$.

Time = 1.67 (sec) , antiderivative size = 1794, normalized size of antiderivative = 4.94

method	result	size
default	Expression too large to display	1794

input
```
int((C*x^2+B*x+A)/(f*x+e)^3/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/2*(A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x
^2+a^2))^(1/2)*f)/(f*x+e))*a^2*b^2*c*f^4*x^2+2*A*ln(2*(b^2*c*e*x+a^2*c*f+(
c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*b^4*c*
e^2*f^2*x^2+4*A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c
*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*b^4*c*e^3*f*x+4*C*ln(2*(b^2*c*e*x+a^2*c
*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^
4*c*e*f^3*x+A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(
-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^2*b^2*c*e^2*f^2-3*B*ln(2*(b^2*c*e*x+a^2
*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*
a^2*b^2*c*e^3*f+2*C*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2
)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^4*c*f^4*x^2+2*B*a^2*f^4*x*(c*(-b^
2*x^2+a^2))^(1/2)*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)-4*A*b^2*e^2*f^2*(c*(-b^2
*x^2+a^2))^(1/2)*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)+A*a^2*f^4*(c*(-b^2*x^2+a^
2))^(1/2)*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)+2*A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(
a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*b^4*c*e^4
+2*C*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+
a^2))^(1/2)*f)/(f*x+e))*a^4*c*e^2*f^2+C*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^
2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^2*b^2*c*e^4+B
*a^2*e*f^3*(c*(-b^2*x^2+a^2))^(1/2)*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)+2*B*b^
2*e^3*f*(c*(-b^2*x^2+a^2))^(1/2)*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)-3*C*a^...
```

3.26.5 Fricas [A] (verification not implemented)

Time = 41.19 (sec) , antiderivative size = 1355, normalized size of antiderivative = 3.73

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(f*x+e)^3/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algo
rithm="fricas")
```

output
```
[1/4*((3*B*a^2*b^2*e^3*f - (C*a^2*b^2 + 2*A*b^4)*e^4 - (2*C*a^4 + A*a^2*b^
2)*e^2*f^2 + (3*B*a^2*b^2*e*f^3 - (C*a^2*b^2 + 2*A*b^4)*e^2*f^2 - (2*C*a^4
 + A*a^2*b^2)*f^4)*x^2 + 2*(3*B*a^2*b^2*e^2*f^2 - (C*a^2*b^2 + 2*A*b^4)*e^
3*f - (2*C*a^4 + A*a^2*b^2)*e*f^3)*x)*sqrt(-b^2*c*e^2 + a^2*c*f^2)*log((2*
a^2*b^2*c*e*f*x - a^2*b^2*c*e^2 + 2*a^4*c*f^2 + (2*b^4*c*e^2 - a^2*b^2*c*f
^2)*x^2 - 2*sqrt(-b^2*c*e^2 + a^2*c*f^2)*(b^2*e*x + a^2*f)*sqrt(-b*c*x + a
*c)*sqrt(b*x + a))/(f^2*x^2 + 2*e*f*x + e^2)) - 2*(2*B*b^4*e^5 - B*a^2*b^2
*e^3*f^2 - B*a^4*e*f^4 - A*a^4*f^5 - (3*C*a^2*b^2 + 4*A*b^4)*e^4*f + (3*C*
a^4 + 5*A*a^2*b^2)*e^2*f^3 + (C*b^4*e^5 + B*b^4*e^4*f + B*a^2*b^2*e^2*f^3
- 2*B*a^4*f^5 - (5*C*a^2*b^2 + 3*A*b^4)*e^3*f^2 + (4*C*a^4 + 3*A*a^2*b^2)*
e*f^4)*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/(b^6*c*e^8 - 3*a^2*b^4*c*e^6*f
^2 + 3*a^4*b^2*c*e^4*f^4 - a^6*c*e^2*f^6 + (b^6*c*e^6*f^2 - 3*a^2*b^4*c*e^
4*f^4 + 3*a^4*b^2*c*e^2*f^6 - a^6*c*f^8)*x^2 + 2*(b^6*c*e^7*f - 3*a^2*b^4*
c*e^5*f^3 + 3*a^4*b^2*c*e^3*f^5 - a^6*c*e*f^7)*x), -1/2*((3*B*a^2*b^2*e^3*
f - (C*a^2*b^2 + 2*A*b^4)*e^4 - (2*C*a^4 + A*a^2*b^2)*e^2*f^2 + (3*B*a^2*b
^2*e*f^3 - (C*a^2*b^2 + 2*A*b^4)*e^2*f^2 - (2*C*a^4 + A*a^2*b^2)*f^4)*x^2
+ 2*(3*B*a^2*b^2*e^2*f^2 - (C*a^2*b^2 + 2*A*b^4)*e^3*f - (2*C*a^4 + A*a^2*
b^2)*e*f^3)*x)*sqrt(b^2*c*e^2 - a^2*c*f^2)*arctan(sqrt(b^2*c*e^2 - a^2*c*f
^2)*(b^2*e*x + a^2*f)*sqrt(-b*c*x + a*c)*sqrt(b*x + a)/(a^2*b^2*c*e^2 - a^
4*c*f^2 - (b^4*c*e^2 - a^2*b^2*c*f^2)*x^2)) + (2*B*b^4*e^5 - B*a^2*b^2*...
```

3.26.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/(f*x+e)**3/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

output
```
Timed out
```

3.26.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A + Bx + Cx^2}{\sqrt{a + bx}\sqrt{ac - bcx}(e + fx)^3} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+B*x+A)/(f*x+e)^3/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((a*f-b*e)>0)', see `assume?` for
 more deta
```

3.26.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1425 vs. $2(335) = 670$.

Time = 0.71 (sec) , antiderivative size = 1425, normalized size of antiderivative = 3.93

$$\int \frac{A + Bx + Cx^2}{\sqrt{a + bx}\sqrt{ac - bcx}(e + fx)^3} \, dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(f*x+e)^3/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algo
rithm="giac")
```

output
```
-((C*a^2*b^3*sqrt(-c)*e^2 + 2*A*b^5*sqrt(-c)*e^2 - 3*B*a^2*b^3*sqrt(-c)*e*
f + 2*C*a^4*b*sqrt(-c)*f^2 + A*a^2*b^3*sqrt(-c)*f^2)*arctan(-1/2*(2*b*c*e
- (sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^2*f)/(sqrt(-b^2*e^
2 + a^2*f^2)*c))/((b^4*e^4 - 2*a^2*b^2*e^2*f^2 + a^4*f^4)*sqrt(-b^2*e^2 +
a^2*f^2)*c) + 2*(4*C*b^6*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a
*c))^4*sqrt(-c)*c*e^5 - 2*C*b^5*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*
c + 2*a*c))^6*sqrt(-c)*e^4*f + 4*B*b^6*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*
x + a)*c + 2*a*c))^4*sqrt(-c)*c*e^4*f - 8*C*a^2*b^5*(sqrt(b*x + a)*sqrt(-c
) - sqrt(-(b*x + a)*c + 2*a*c))^2*sqrt(-c)*c^2*e^4*f - 14*C*a^2*b^4*(sqrt(
b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^4*sqrt(-c)*c*e^3*f^2 - 12*
A*b^6*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^4*sqrt(-c)*c*e
^3*f^2 - 16*B*a^2*b^5*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c)
)^2*sqrt(-c)*c^2*e^3*f^2 + 8*C*a^4*b^4*sqrt(-c)*c^3*e^3*f^2 + 5*C*a^2*b^3*
(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^6*sqrt(-c)*e^2*f^3 +
 2*A*b^5*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^6*sqrt(-c)*
e^2*f^3 + 10*B*a^2*b^4*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c
))^4*sqrt(-c)*c*e^2*f^3 + 44*C*a^4*b^3*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*
x + a)*c + 2*a*c))^2*sqrt(-c)*c^2*e^2*f^3 + 40*A*a^2*b^5*(sqrt(b*x + a)*sq
rt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^2*sqrt(-c)*c^2*e^2*f^3 + 8*B*a^4*b^4*
sqrt(-c)*c^3*e^2*f^3 - 3*B*a^2*b^3*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x...
```

3.26.9 Mupad [B] (verification not implemented)

Time = 108.41 (sec) , antiderivative size = 9344, normalized size of antiderivative = 25.74

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/((e + f*x)^3*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),x)
```

output
```
((((a*c - b*c*x)^(1/2) - (a*c)^(1/2))*(4*C*a^4*c^3*f^2 + 2*C*a^2*b^2*c^3*e
^2))/(((a + b*x)^(1/2) - a^(1/2))*(b^5*e^5 - 2*a^2*b^3*e^3*f^2 + a^4*b*e*f
^4)) + (((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3*(68*C*a^4*c^2*f^2 - 14*C*a^2
*b^2*c^2*e^2))/(((a + b*x)^(1/2) - a^(1/2))^3*(b^5*e^5 - 2*a^2*b^3*e^3*f^2
 + a^4*b*e*f^4)) - ((68*C*a^4*c*f^2 - 14*C*a^2*b^2*c*e^2)*((a*c - b*c*x)^(
1/2) - (a*c)^(1/2))^5)/(((a + b*x)^(1/2) - a^(1/2))^5*(b^5*e^5 - 2*a^2*b^3
*e^3*f^2 + a^4*b*e*f^4)) - ((4*C*a^4*f^2 + 2*C*a^2*b^2*e^2)*((a*c - b*c*x)
^(1/2) - (a*c)^(1/2))^7)/(((a + b*x)^(1/2) - a^(1/2))^7*(b^5*e^5 - 2*a^2*b
^3*e^3*f^2 + a^4*b*e*f^4)) - (a^(1/2)*(a*c)^(1/2)*(48*C*a^4*c*f^3 - 24*C*a
^2*b^2*c*e^2*f)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4)/(((a + b*x)^(1/2) -
 a^(1/2))^4*(b^6*e^6 - 2*a^2*b^4*e^4*f^2 + a^4*b^2*e^2*f^4)) + (a^(1/2)*(a
*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6*(24*C*a^4*f^3 + 12*C*a^2*b
^2*e^2*f))/(((a + b*x)^(1/2) - a^(1/2))^6*(b^6*e^6 - 2*a^2*b^4*e^4*f^2 + a
^4*b^2*e^2*f^4)) + (a^(1/2)*(a*c)^(1/2)*(24*C*a^4*c^2*f^3 + 12*C*a^2*b^2*c
^2*e^2*f)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(((a + b*x)^(1/2) - a^(1/
2))^2*(b^6*e^6 - 2*a^2*b^4*e^4*f^2 + a^4*b^2*e^2*f^4)))/(((a*c - b*c*x)^(1
/2) - (a*c)^(1/2))^8/((a + b*x)^(1/2) - a^(1/2))^8 + c^4 + (((a*c - b*c*x)
^(1/2) - (a*c)^(1/2))^6*(16*a^2*c*f^2 + 4*b^2*c*e^2))/(b^2*e^2*((a + b*x)^
(1/2) - a^(1/2))^6) + ((16*a^2*c^3*f^2 + 4*b^2*c^3*e^2)*((a*c - b*c*x)^(1/
2) - (a*c)^(1/2))^2)/(b^2*e^2*((a + b*x)^(1/2) - a^(1/2))^2) - ((32*a^2...
```

3.26. $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3}\,dx$

3.27 $\int \frac{(e+fx)^3(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$

3.27.1 Optimal result

Integrand size = 40, antiderivative size = 501

$$\int \frac{(e+fx)^3\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$$
$$= -\frac{(16a^2Cf^2-b^2(3Ce^2-5f(3Be+4Af)))\,(e+fx)^2\left(a^2-b^2x^2\right)}{60b^4f\sqrt{a+bx}\sqrt{ac-bcx}}$$
$$+\frac{(Ce-5Bf)(e+fx)^3\left(a^2-b^2x^2\right)}{20b^2f\sqrt{a+bx}\sqrt{ac-bcx}}-\frac{C(e+fx)^4\left(a^2-b^2x^2\right)}{5b^2f\sqrt{a+bx}\sqrt{ac-bcx}}$$
$$-\frac{(4(16a^4Cf^4+4a^2b^2f^2(13Ce^2+5f(3Be+Af))-b^4e^2(3Ce^2-5f(3Be+16Af)))+b^2f(a^2f^2(71Ce+}{120b^6f\sqrt{a+bx}\sqrt{ac-bcx}}$$
$$+\frac{(4A(2b^4e^3+3a^2b^2ef^2)+a^2(3a^2f^2(3Ce+Bf)+4b^2e^2(Ce+3Bf)))\sqrt{a^2c-b^2cx^2}\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{8b^5\sqrt{c}\sqrt{a+bx}\sqrt{ac-bcx}}$$

output
```
-1/60*(16*a^2*C*f^2-b^2*(3*C*e^2-5*f*(4*A*f+3*B*e)))*(f*x+e)^2*(-b^2*x^2+a
^2)/b^4/f/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)+1/20*(-5*B*f+C*e)*(f*x+e)^3*(-b
^2*x^2+a^2)/b^2/f/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)-1/5*C*(f*x+e)^4*(-b^2*x
^2+a^2)/b^2/f/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)-1/120*(64*a^4*C*f^4+16*a^2*
b^2*f^2*(13*C*e^2+5*f*(A*f+3*B*e))-4*b^4*e^2*(3*C*e^2-5*f*(16*A*f+3*B*e))+
b^2*f*(a^2*f^2*(45*B*f+71*C*e)-2*b^2*e*(3*C*e^2-5*f*(10*A*f+3*B*e)))*x)*(-
b^2*x^2+a^2)/b^6/f/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)+1/8*(4*A*(3*a^2*b^2*e*
f^2+2*b^4*e^3)+a^2*(3*a^2*f^2*(B*f+3*C*e)+4*b^2*e^2*(3*B*f+C*e)))*arctan(b
*x*c^(1/2)/(-b^2*c*x^2+a^2*c)^(1/2))*(-b^2*c*x^2+a^2*c)^(1/2)/b^5/c^(1/2)/
(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)
```

3.27.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.56

$$\int \frac{(e+fx)^3 (A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$$
$$= \frac{-\left((a-bx)\sqrt{a+bx}(64a^4Cf^3+a^2b^2f(5f(48Be+16Af+9Bfx)+C(240e^2+135efx+32f^2x^2))+2b^4\right.}{}$$

input
```
Integrate[((e + f*x)^3*(A + B*x + C*x^2))/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]
),x]
```

output
```
(-((a - b*x)*Sqrt[a + b*x]*(64*a^4*C*f^3 + a^2*b^2*f*(5*f*(48*B*e + 16*A*f
 + 9*B*f*x) + C*(240*e^2 + 135*e*f*x + 32*f^2*x^2)) + 2*b^4*(10*A*f*(18*e^
2 + 9*e*f*x + 2*f^2*x^2) + 15*B*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3
) + 3*C*x*(10*e^3 + 20*e^2*f*x + 15*e*f^2*x^2 + 4*f^3*x^3)))) + 30*b*(3*a^
4*f^2*(3*C*e + B*f) + 4*a^2*b^2*e^2*(C*e + 3*B*f) + 4*A*(2*b^4*e^3 + 3*a^2
*b^2*e*f^2))*Sqrt[a - b*x]*ArcTan[Sqrt[a + b*x]/Sqrt[a - b*x]])/(120*b^6*S
qrt[c*(a - b*x)])
```

3.27.3 Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 482, normalized size of antiderivative = 0.96, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.325$, Rules used = {2113, 2185, 25, 27, 687, 25, 27, 687, 25, 27, 676, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3 (A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$$
$$\Big\downarrow \ 2113$$
$$\frac{\sqrt{a^2c-b^2cx^2}\int \frac{(e+fx)^3(Cx^2+Bx+A)}{\sqrt{a^2c-b^2cx^2}}dx}{\sqrt{a+bx}\sqrt{ac-bcx}}$$
$$\Big\downarrow \ 2185$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(-\frac{\int -\frac{cf(e+fx)^3\left(\left(4Ca^2+5Ab^2\right)f-b^2(Ce-5Bf)x\right)}{\sqrt{a^2c-b^2cx^2}}dx}{5b^2cf^2}-\frac{C(e+fx)^4\sqrt{a^2c-b^2cx^2}}{5b^2cf}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$\Big\downarrow$ 25

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\int \frac{cf(e+fx)^3\left(\left(4Ca^2+5Ab^2\right)f-b^2(Ce-5Bf)x\right)}{\sqrt{a^2c-b^2cx^2}}dx}{5b^2cf^2}-\frac{C(e+fx)^4\sqrt{a^2c-b^2cx^2}}{5b^2cf}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\int \frac{(e+fx)^3\left(\left(4Ca^2+5Ab^2\right)f-b^2(Ce-5Bf)x\right)}{\sqrt{a^2c-b^2cx^2}}dx}{5b^2f}-\frac{C(e+fx)^4\sqrt{a^2c-b^2cx^2}}{5b^2cf}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$\Big\downarrow$ 687

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{(e+fx)^3\sqrt{a^2c-b^2cx^2}(Ce-5Bf)}{4c}-\frac{\int -\frac{b^2c(e+fx)^2\left(f\left((13Ce+15Bf)a^2+20Ab^2e\right)+\left(16a^2Cf^2-b^2\left(3Ce^2-5f(3Be+4Af)\right)\right)x\right)}{\sqrt{a^2c-b^2cx^2}}dx}{4b^2c}}{5b^2f}-C(e+\right.}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$\Big\downarrow$ 25

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{\int \frac{b^2c(e+fx)^2\left(f\left((13Ce+15Bf)a^2+20Ab^2e\right)+\left(16a^2Cf^2-b^2\left(3Ce^2-5f(3Be+4Af)\right)\right)x\right)}{\sqrt{a^2c-b^2cx^2}}dx}{4b^2c}+\frac{(e+fx)^3\sqrt{a^2c-b^2cx^2}(Ce-5Bf)}{4c}}{5b^2f}-C(e+f\right.}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{1}{4}\int \frac{(e+fx)^2\left(f\left((13Ce+15Bf)a^2+20Ab^2e\right)+\left(16a^2Cf^2-b^2\left(3Ce^2-5f(3Be+4Af)\right)\right)x\right)}{\sqrt{a^2c-b^2cx^2}}dx+\frac{(e+fx)^3\sqrt{a^2c-b^2cx^2}(Ce-5Bf)}{4c}}{5b^2f}-C(e+fx\right.}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$\Big\downarrow$ 687

3.27. $\int \frac{(e+fx)^3\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$

$$\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{1}{4}\left(-\frac{\int -\frac{c(e+fx)\left(\left(a^2f^2(71Ce+45Bf)-b^2\left(6Ce^3-10ef(3Be+10Af)\right)\right)xb^2+f\left(32Cf^2a^4+3b^2e(11Ce+25Bf)a^2+20A\left(3e^2b^4+2a^2f^2b^2\right)\right)\right)}{\sqrt{a^2c-b^2cx^2}}}{3b^2c}\right.}{5b^2f}\right.$$

$$\sqrt{a+bx}\sqrt{ac-bc$$

$\downarrow$ 25

$$\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{1}{4}\left(\frac{\int \frac{c(e+fx)\left(\left(a^2f^2(71Ce+45Bf)-b^2\left(6Ce^3-10ef(3Be+10Af)\right)\right)xb^2+f\left(32Cf^2a^4+3b^2e(11Ce+25Bf)a^2+20A\left(3e^2b^4+2a^2f^2b^2\right)\right)\right)}{\sqrt{a^2c-b^2cx^2}}dx}{3b^2c}-\right.}{5b^2f}\right.$$

$$\sqrt{a+bx}\sqrt{ac-bcx}$$

$\downarrow$ 27

$$\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{1}{4}\left(\frac{\int \frac{(e+fx)\left(\left(a^2f^2(71Ce+45Bf)-b^2\left(6Ce^3-10ef(3Be+10Af)\right)\right)xb^2+f\left(32Cf^2a^4+3b^2e(11Ce+25Bf)a^2+20A\left(3e^2b^4+2a^2f^2b^2\right)\right)\right)}{\sqrt{a^2c-b^2cx^2}}dx}{3b^2}-\right.}{5b^2f}\right.$$

$$\sqrt{a+bx}\sqrt{ac-bcx}$$

$\downarrow$ 676

$$\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{1}{4}\left(\frac{\frac{15}{2}f\left(3a^4f^2(Bf+3Ce)+4A\left(3a^2b^2ef^2+2b^4e^3\right)+4a^2b^2e^2(3Bf+Ce)\right)\int\frac{1}{\sqrt{a^2c-b^2cx^2}}dx-\frac{fx\sqrt{a^2c-b^2cx^2}\left(a^2f^2(45Bf+71Ce)-b^2\left(6Ce\right.\right.}{2c}}{3b^2}\right.}{}\right.$$

$\downarrow$ 224

3.27. $\int \frac{(e+fx)^3\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$

$$\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{1}{4}\left(\frac{\frac{15}{2}f\left(3a^4f^2(Bf+3Ce)+4A\left(3a^2b^2ef^2+2b^4e^3\right)+4a^2b^2e^2(3Bf+Ce)\right)\int\frac{1}{\frac{b^2cx^2}{a^2c-b^2cx^2}+1}d\frac{x}{\sqrt{a^2c-b^2cx^2}}-\frac{fx\sqrt{a^2c-b^2cx^2}\left(a^2f^2(45Bf+7\cdots\right.}{\cdots}}{3b\cdots}\right.}{\cdots}\right.$$

$$\Big\downarrow \textbf{216}$$

$$\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{(e+fx)^3\sqrt{a^2c-b^2cx^2}(Ce-5Bf)}{4c}+\frac{1}{4}\left(-\frac{fx\sqrt{a^2c-b^2cx^2}\left(a^2f^2(45Bf+71Ce)-b^2\left(6Ce^3-10ef(10Af+3Be)\right)\right)}{2c}+\frac{15f\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)\cdots}{\cdots}\right.}{\cdots}\right.$$

input
```
Int[((e + f*x)^3*(A + B*x + C*x^2))/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]),x]
```

output
```
(Sqrt[a^2*c - b^2*c*x^2]*(-1/5*(C*(e + f*x)^4*Sqrt[a^2*c - b^2*c*x^2])/(b^
2*c*f) + (((C*e - 5*B*f)*(e + f*x)^3*Sqrt[a^2*c - b^2*c*x^2])/(4*c) + (-1/
3*((16*a^2*C*f^2 - b^2*(3*C*e^2 - 5*f*(3*B*e + 4*A*f)))*(e + f*x)^2*Sqrt[a
^2*c - b^2*c*x^2])/(b^2*c) + ((-2*(16*a^4*C*f^4 + 4*a^2*b^2*f^2*(13*C*e^2
+ 5*f*(3*B*e + A*f)) - b^4*(3*C*e^4 - 5*e^2*f*(3*B*e + 16*A*f)))*Sqrt[a^2*
c - b^2*c*x^2])/(b^2*c) - (f*(a^2*f^2*(71*C*e + 45*B*f) - b^2*(6*C*e^3 - 1
0*e*f*(3*B*e + 10*A*f)))*x*Sqrt[a^2*c - b^2*c*x^2])/(2*c) + (15*f*(3*a^4*f
^2*(3*C*e + B*f) + 4*a^2*b^2*e^2*(C*e + 3*B*f) + 4*A*(2*b^4*e^3 + 3*a^2*b^
2*e*f^2))*ArcTan[(b*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(2*b*Sqrt[c]))/(3
*b^2))/4)/(5*b^2*f)))/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
```

3.27. $\int \frac{(e+fx)^3(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$

3.27.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2113
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m])   Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a
*d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.27.4 Maple [A] (verified)

Time = 1.65 (sec) , antiderivative size = 390, normalized size of antiderivative = 0.78

method	result
risch	$-\frac{(24C f^3x^4b^4+30B b^4f^3x^3+90C b^4e f^2x^3+40A b^4f^3x^2+120B b^4e f^2x^2+32C a^2b^2f^3x^2+120C b^4e^2f x^2+180A b^4e f^2x+45B a^2b^2}{}$
default	$\frac{\sqrt{bx+a}\,\sqrt{c(-bx+a)}\left(-24C b^4f^3x^4\sqrt{c(-b^2x^2+a^2)}\,\sqrt{b^2c}-30B b^4f^3x^3\sqrt{c(-b^2x^2+a^2)}\,\sqrt{b^2c}-90C b^4e f^2x^3\sqrt{c(-b^2x^2+a^2)}\,\sqrt{b^2c}+1\right.}{}$

input
```
int((f*x+e)^3*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/120*(24*C*b^4*f^3*x^4+30*B*b^4*f^3*x^3+90*C*b^4*e*f^2*x^3+40*A*b^4*f^3*
x^2+120*B*b^4*e*f^2*x^2+32*C*a^2*b^2*f^3*x^2+120*C*b^4*e^2*f*x^2+180*A*b^4
*e*f^2*x+45*B*a^2*b^2*f^3*x+180*B*b^4*e^2*f*x+135*C*a^2*b^2*e*f^2*x+60*C*b
^4*e^3*x+80*A*a^2*b^2*f^3+360*A*b^4*e^2*f+240*B*a^2*b^2*e*f^2+120*B*b^4*e^
3+64*C*a^4*f^3+240*C*a^2*b^2*e^2*f)*(b*x+a)^(1/2)/b^6*(-b*x+a)/(-c*(b*x-a)
)^(1/2)+1/8*(12*A*a^2*b^2*e*f^2+8*A*b^4*e^3+3*B*a^4*f^3+12*B*a^2*b^2*e^2*f
+9*C*a^4*e*f^2+4*C*a^2*b^2*e^3)/b^4/(b^2*c)^(1/2)*arctan((b^2*c)^(1/2)*x/(
-b^2*c*x^2+a^2*c)^(1/2))*(-(b*x+a)*c*(b*x-a))^(1/2)/(b*x+a)^(1/2)/(-c*(b*x
-a))^(1/2)
```

3.27.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 700, normalized size of antiderivative = 1.40

$$\int \frac{(e+fx)^3(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$$
$$= \left[-\frac{15\,(12\,Ba^2b^3e^2f+3\,Ba^4bf^3+4\,(Ca^2b^3+2\,Ab^5)e^3+3\,(3\,Ca^4b+4\,Aa^2b^3)ef^2)\sqrt{-c}\log\left(2\,b^2cx^2-2\,\sqrt{}\right.}{}\right.$$
$$-\frac{15\,(12\,Ba^2b^3e^2f+3\,Ba^4bf^3+4\,(Ca^2b^3+2\,Ab^5)e^3+3\,(3\,Ca^4b+4\,Aa^2b^3)ef^2)\sqrt{c}\arctan\left(\frac{\sqrt{-bcx+ac}\sqrt{bx}}{b^2cx^2-a^2}\right.}{}$$

input
```
integrate((f*x+e)^3*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algo
rithm="fricas")
```

output
```
[-1/240*(15*(12*B*a^2*b^3*e^2*f + 3*B*a^4*b*f^3 + 4*(C*a^2*b^3 + 2*A*b^5)*
e^3 + 3*(3*C*a^4*b + 4*A*a^2*b^3)*e*f^2)*sqrt(-c)*log(2*b^2*c*x^2 - 2*sqrt
(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(-c)*x - a^2*c) + 2*(24*C*b^4*f^3*x^4 +
 120*B*b^4*e^3 + 240*B*a^2*b^2*e*f^2 + 120*(2*C*a^2*b^2 + 3*A*b^4)*e^2*f +
 16*(4*C*a^4 + 5*A*a^2*b^2)*f^3 + 30*(3*C*b^4*e*f^2 + B*b^4*f^3)*x^3 + 8*(
15*C*b^4*e^2*f + 15*B*b^4*e*f^2 + (4*C*a^2*b^2 + 5*A*b^4)*f^3)*x^2 + 15*(4
*C*b^4*e^3 + 12*B*b^4*e^2*f + 3*B*a^2*b^2*f^3 + 3*(3*C*a^2*b^2 + 4*A*b^4)*
e*f^2)*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/(b^6*c), -1/120*(15*(12*B*a^2*
b^3*e^2*f + 3*B*a^4*b*f^3 + 4*(C*a^2*b^3 + 2*A*b^5)*e^3 + 3*(3*C*a^4*b + 4
*A*a^2*b^3)*e*f^2)*sqrt(c)*arctan(sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(
c)*x/(b^2*c*x^2 - a^2*c)) + (24*C*b^4*f^3*x^4 + 120*B*b^4*e^3 + 240*B*a^2*
b^2*e*f^2 + 120*(2*C*a^2*b^2 + 3*A*b^4)*e^2*f + 16*(4*C*a^4 + 5*A*a^2*b^2)
*f^3 + 30*(3*C*b^4*e*f^2 + B*b^4*f^3)*x^3 + 8*(15*C*b^4*e^2*f + 15*B*b^4*e
*f^2 + (4*C*a^2*b^2 + 5*A*b^4)*f^3)*x^2 + 15*(4*C*b^4*e^3 + 12*B*b^4*e^2*f
 + 3*B*a^2*b^2*f^3 + 3*(3*C*a^2*b^2 + 4*A*b^4)*e*f^2)*x)*sqrt(-b*c*x + a*c
)*sqrt(b*x + a))/(b^6*c)]
```

3.27.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*(C*x**2+B*x+A)/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

output
```
Timed out
```

3.27.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 471, normalized size of antiderivative = 0.94

$$\begin{aligned}\int \frac{(e+fx)^3(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = &-\frac{\sqrt{-b^2cx^2+a^2c}Cf^3x^4}{5\,b^2c} - \frac{4\sqrt{-b^2cx^2+a^2c}Ca^2f^3x^2}{15\,b^4c} \\ &+ \frac{Ae^3\arcsin\left(\frac{bx}{a}\right)}{b\sqrt{c}} - \frac{\sqrt{-b^2cx^2+a^2c}Be^3}{b^2c} \\ &- \frac{3\sqrt{-b^2cx^2+a^2c}Ae^2f}{b^2c} - \frac{8\sqrt{-b^2cx^2+a^2c}Ca^4f^3}{15\,b^6c} \\ &- \frac{\sqrt{-b^2cx^2+a^2c}(3\,Cef^2+Bf^3)x^3}{4\,b^2c} \\ &- \frac{\sqrt{-b^2cx^2+a^2c}(3\,Ce^2f+3\,Bef^2+Af^3)x^2}{3\,b^2c} \\ &+ \frac{3\,(3\,Cef^2+Bf^3)a^4\arcsin\left(\frac{bx}{a}\right)}{8\,b^5\sqrt{c}} \\ &+ \frac{(Ce^3+3\,Be^2f+3\,Aef^2)a^2\arcsin\left(\frac{bx}{a}\right)}{2\,b^3\sqrt{c}} \\ &- \frac{3\sqrt{-b^2cx^2+a^2c}(3\,Cef^2+Bf^3)a^2x}{8\,b^4c} \\ &- \frac{\sqrt{-b^2cx^2+a^2c}(Ce^3+3\,Be^2f+3\,Aef^2)x}{2\,b^2c} \\ &- \frac{2\sqrt{-b^2cx^2+a^2c}(3\,Ce^2f+3\,Bef^2+Af^3)a^2}{3\,b^4c}\end{aligned}$$

input
```
integrate((f*x+e)^3*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algo
rithm="maxima")
```

output
```
-1/5*sqrt(-b^2*c*x^2 + a^2*c)*C*f^3*x^4/(b^2*c) - 4/15*sqrt(-b^2*c*x^2 + a
^2*c)*C*a^2*f^3*x^2/(b^4*c) + A*e^3*arcsin(b*x/a)/(b*sqrt(c)) - sqrt(-b^2*
c*x^2 + a^2*c)*B*e^3/(b^2*c) - 3*sqrt(-b^2*c*x^2 + a^2*c)*A*e^2*f/(b^2*c)
- 8/15*sqrt(-b^2*c*x^2 + a^2*c)*C*a^4*f^3/(b^6*c) - 1/4*sqrt(-b^2*c*x^2 +
a^2*c)*(3*C*e*f^2 + B*f^3)*x^3/(b^2*c) - 1/3*sqrt(-b^2*c*x^2 + a^2*c)*(3*C
*e^2*f + 3*B*e*f^2 + A*f^3)*x^2/(b^2*c) + 3/8*(3*C*e*f^2 + B*f^3)*a^4*arcs
in(b*x/a)/(b^5*sqrt(c)) + 1/2*(C*e^3 + 3*B*e^2*f + 3*A*e*f^2)*a^2*arcsin(b
*x/a)/(b^3*sqrt(c)) - 3/8*sqrt(-b^2*c*x^2 + a^2*c)*(3*C*e*f^2 + B*f^3)*a^2
*x/(b^4*c) - 1/2*sqrt(-b^2*c*x^2 + a^2*c)*(C*e^3 + 3*B*e^2*f + 3*A*e*f^2)*
x/(b^2*c) - 2/3*sqrt(-b^2*c*x^2 + a^2*c)*(3*C*e^2*f + 3*B*e*f^2 + A*f^3)*a
^2/(b^4*c)
```

3.27.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 571, normalized size of antiderivative = 1.14

$$\int \frac{(e+fx)^3 (A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx =$$

$$-\frac{\left(\left(2\left(3\left(\frac{4\,(bx+a)Cf^3}{c}+\frac{15\,Cbc^4ef^2-16\,Cac^4f^3+5\,Bbc^4f^3}{c^5}\right)(bx+a)+\frac{60\,Cb^2c^4e^2f-135\,Cabc^4ef^2+60\,Bb^2c^4ef^2+88\,Ca^2c^4f^3-}{c^5}\right.\right.\right.}{}$$

input
```
integrate((f*x+e)^3*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algo
rithm="giac")
```

output
```
-1/120*(((2*(3*(4*(b*x + a)*C*f^3/c + (15*C*b*c^4*e*f^2 - 16*C*a*c^4*f^3 +
 5*B*b*c^4*f^3)/c^5)*(b*x + a) + (60*C*b^2*c^4*e^2*f - 135*C*a*b*c^4*e*f^2
 + 60*B*b^2*c^4*e*f^2 + 88*C*a^2*c^4*f^3 - 45*B*a*b*c^4*f^3 + 20*A*b^2*c^4
*f^3)/c^5)*(b*x + a) + 5*(12*C*b^3*c^4*e^3 - 48*C*a*b^2*c^4*e^2*f + 36*B*b
^3*c^4*e^2*f + 81*C*a^2*b*c^4*e*f^2 - 48*B*a*b^2*c^4*e*f^2 + 36*A*b^3*c^4*
e*f^2 - 32*C*a^3*c^4*f^3 + 27*B*a^2*b*c^4*f^3 - 16*A*a*b^2*c^4*f^3)/c^5)*(
b*x + a) - 15*(4*C*a*b^3*c^4*e^3 - 8*B*b^4*c^4*e^3 - 24*C*a^2*b^2*c^4*e^2*
f + 12*B*a*b^3*c^4*e^2*f - 24*A*b^4*c^4*e^2*f + 15*C*a^3*b*c^4*e*f^2 - 24*
B*a^2*b^2*c^4*e*f^2 + 12*A*a*b^3*c^4*e*f^2 - 8*C*a^4*c^4*f^3 + 5*B*a^3*b*c
^4*f^3 - 8*A*a^2*b^2*c^4*f^3)/c^5)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a
) + 30*(4*C*a^2*b^3*e^3 + 8*A*b^5*e^3 + 12*B*a^2*b^3*e^2*f + 9*C*a^4*b*e*f
^2 + 12*A*a^2*b^3*e*f^2 + 3*B*a^4*b*f^3)*log(abs(-sqrt(b*x + a)*sqrt(-c) +
 sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c))/b^6
```

3.27.9 Mupad [B] (verification not implemented)

Time = 151.65 (sec) , antiderivative size = 4167, normalized size of antiderivative = 8.32

$$\int \frac{(e+fx)^3 (A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)^3*(A + B*x + C*x^2))/((a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),
x)
```

output
```
- ((((23*B*a^4*c*f^3)/2 - 18*B*a^2*b^2*c*e^2*f)*((a*c - b*c*x)^(1/2) - (a*
c)^(1/2))^13)/(b^5*((a + b*x)^(1/2) - a^(1/2))^13) + (((a*c - b*c*x)^(1/2)
 - (a*c)^(1/2))^15*((3*B*a^4*f^3)/2 + 6*B*a^2*b^2*e^2*f))/(b^5*((a + b*x)^
(1/2) - a^(1/2))^15) - (((3*B*a^4*c^7*f^3)/2 + 6*B*a^2*b^2*c^7*e^2*f)*((a*
c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b^5*((a + b*x)^(1/2) - a^(1/2))) - (((23
*B*a^4*c^6*f^3)/2 - 18*B*a^2*b^2*c^6*e^2*f)*((a*c - b*c*x)^(1/2) - (a*c)^(
1/2))^3)/(b^5*((a + b*x)^(1/2) - a^(1/2))^3) + (((333*B*a^4*c^5*f^3)/2 + 9
0*B*a^2*b^2*c^5*e^2*f)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5)/(b^5*((a + b
*x)^(1/2) - a^(1/2))^5) - (((333*B*a^4*c^2*f^3)/2 + 90*B*a^2*b^2*c^2*e^2*f
)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^11)/(b^5*((a + b*x)^(1/2) - a^(1/2))
^11) - (((671*B*a^4*c^4*f^3)/2 - 66*B*a^2*b^2*c^4*e^2*f)*((a*c - b*c*x)^(1
/2) - (a*c)^(1/2))^7)/(b^5*((a + b*x)^(1/2) - a^(1/2))^7) + (((671*B*a^4*c
^3*f^3)/2 - 66*B*a^2*b^2*c^3*e^2*f)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^9)
/(b^5*((a + b*x)^(1/2) - a^(1/2))^9) + (a^(1/2)*(a*c)^(1/2)*(48*B*b^2*c^5*
e^3 + 192*B*a^2*c^5*e*f^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4)/(b^4*((a
 + b*x)^(1/2) - a^(1/2))^4) + (a^(1/2)*(a*c)^(1/2)*(160*B*b^2*c^3*e^3 + 12
8*B*a^2*c^3*e*f^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^8)/(b^4*((a + b*x)^
(1/2) - a^(1/2))^8) + (a^(1/2)*(a*c)^(1/2)*(120*B*b^2*c^4*e^3 + 256*B*a^2*
c^4*e*f^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6)/(b^4*((a + b*x)^(1/2) -
a^(1/2))^6) + (a^(1/2)*(a*c)^(1/2)*(120*B*b^2*c^2*e^3 + 256*B*a^2*c^2*e...
```

3.28 $\int \frac{(e+fx)^2(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$

3.28.1 Optimal result

Integrand size = 40, antiderivative size = 368

$$\int \frac{(e+fx)^2(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$$
$$= \frac{(Ce-4Bf)(e+fx)^2(a^2-b^2x^2)}{12b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{C(e+fx)^3(a^2-b^2x^2)}{4b^2f\sqrt{a+bx}\sqrt{ac-bcx}}$$
$$- \frac{(4(4a^2f^2(2Ce+Bf)-b^2e(Ce^2-4f(Be+3Af)))+f(9a^2Cf^2-b^2(2Ce^2-4f(2Be+3Af)))x)(a^2}{24b^4f\sqrt{a+bx}\sqrt{ac-bcx}}$$
$$+ \frac{(4A(2b^4e^2+a^2b^2f^2)+a^2(3a^2Cf^2+4b^2e(Ce+2Bf)))\sqrt{a^2c-b^2cx^2}\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{8b^5\sqrt{c}\sqrt{a+bx}\sqrt{ac-bcx}}$$

output
```
1/12*(-4*B*f+C*e)*(f*x+e)^2*(-b^2*x^2+a^2)/b^2/f/(b*x+a)^(1/2)/(-b*c*x+a*c
)^(1/2)-1/4*C*(f*x+e)^3*(-b^2*x^2+a^2)/b^2/f/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1
/2)-1/24*(16*a^2*f^2*(B*f+2*C*e)-4*b^2*e*(C*e^2-4*f*(3*A*f+B*e))+f*(9*a^2*
C*f^2-b^2*(2*C*e^2-4*f*(3*A*f+2*B*e)))*x)*(-b^2*x^2+a^2)/b^4/f/(b*x+a)^(1/
2)/(-b*c*x+a*c)^(1/2)+1/8*(4*A*(a^2*b^2*f^2+2*b^4*e^2)+a^2*(3*a^2*C*f^2+4*
b^2*e*(2*B*f+C*e)))*arctan(b*x*c^(1/2)/(-b^2*c*x^2+a^2*c)^(1/2))*(-b^2*c*x
^2+a^2*c)^(1/2)/b^5/c^(1/2)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)
```

3.28.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.54

$$\int \frac{(e+fx)^2 (A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$$
$$= \frac{-b(a-bx)\sqrt{a+bx}(a^2f(32Ce+16Bf+9Cfx)+2b^2(6Af(4e+fx)+4B(3e^2+3efx+f^2x^2)+Cx(6e}{24b^5\sqrt{c(}}$$

input
```
Integrate[((e + f*x)^2*(A + B*x + C*x^2))/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]
),x]
```

output
```
(-(b*(a - b*x)*Sqrt[a + b*x]*(a^2*f*(32*C*e + 16*B*f + 9*C*f*x) + 2*b^2*(6
*A*f*(4*e + f*x) + 4*B*(3*e^2 + 3*e*f*x + f^2*x^2) + C*x*(6*e^2 + 8*e*f*x
+ 3*f^2*x^2)))) + 6*(3*a^4*C*f^2 + 4*a^2*b^2*e*(C*e + 2*B*f) + 4*A*(2*b^4*
e^2 + a^2*b^2*f^2))*Sqrt[a - b*x]*ArcTan[Sqrt[a + b*x]/Sqrt[a - b*x]])/(24
*b^5*Sqrt[c*(a - b*x)])
```

3.28.3 Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 362, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2113, 2185, 25, 27, 687, 25, 27, 676, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2 (A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$$
$$\Big\downarrow \text{2113}$$
$$\frac{\sqrt{a^2c-b^2cx^2}\int \frac{(e+fx)^2(Cx^2+Bx+A)}{\sqrt{a^2c-b^2cx^2}}dx}{\sqrt{a+bx}\sqrt{ac-bcx}}$$
$$\Big\downarrow \text{2185}$$
$$\frac{\sqrt{a^2c-b^2cx^2}\left(-\frac{\int -\frac{cf(e+fx)^2\left(\left(3Ca^2+4Ab^2\right)f-b^2(Ce-4Bf)x\right)}{\sqrt{a^2c-b^2cx^2}}dx}{4b^2cf^2}-\frac{C(e+fx)^3\sqrt{a^2c-b^2cx^2}}{4b^2cf}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\downarrow 25$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\int \frac{cf(e+fx)^2\left(\left(3Ca^2+4Ab^2\right)f-b^2(Ce-4Bf)x\right)}{\sqrt{a^2c-b^2cx^2}}dx}{4b^2cf^2}-\frac{C(e+fx)^3\sqrt{a^2c-b^2cx^2}}{4b^2cf}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\downarrow 27$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\int \frac{(e+fx)^2\left(\left(3Ca^2+4Ab^2\right)f-b^2(Ce-4Bf)x\right)}{\sqrt{a^2c-b^2cx^2}}dx}{4b^2f}-\frac{C(e+fx)^3\sqrt{a^2c-b^2cx^2}}{4b^2cf}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\downarrow 687$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{(e+fx)^2\sqrt{a^2c-b^2cx^2}(Ce-4Bf)}{3c}-\frac{\int -\frac{b^2c(e+fx)\left(f\left((7Ce+8Bf)a^2+12Ab^2e\right)+\left(9a^2Cf^2-b^2\left(2Ce^2-4f(2Be+3Af)\right)\right)x\right)}{\sqrt{a^2c-b^2cx^2}}dx}{3b^2c}}{4b^2f}-\frac{C(e+fx)}{4}\right.}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\downarrow 25$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{\int \frac{b^2c(e+fx)\left(f\left((7Ce+8Bf)a^2+12Ab^2e\right)+\left(9a^2Cf^2-b^2\left(2Ce^2-4f(2Be+3Af)\right)\right)x\right)}{\sqrt{a^2c-b^2cx^2}}dx}{3b^2c}+\frac{(e+fx)^2\sqrt{a^2c-b^2cx^2}(Ce-4Bf)}{3c}}{4b^2f}-\frac{C(e+fx)^3}{4b}\right.}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\downarrow 27$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{1}{3}\int \frac{(e+fx)\left(f\left((7Ce+8Bf)a^2+12Ab^2e\right)+\left(9a^2Cf^2-b^2\left(2Ce^2-4f(2Be+3Af)\right)\right)x\right)}{\sqrt{a^2c-b^2cx^2}}dx+\frac{(e+fx)^2\sqrt{a^2c-b^2cx^2}(Ce-4Bf)}{3c}}{4b^2f}-\frac{C(e+fx)^3\sqrt{}}{4b^2}\right.}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\downarrow 676$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{1}{3}\left(\frac{3f\left(3a^4Cf^2+4A\left(a^2b^2f^2+2b^4e^2\right)+4a^2b^2e(2Bf+Ce)\right)\int \frac{1}{\sqrt{a^2c-b^2cx^2}}dx}{2b^2}-\frac{2\sqrt{a^2c-b^2cx^2}\left(4a^2f^2(Bf+2Ce)-b^2\left(Ce^3-4ef(3Af+Be)\right)\right.}{b^2c}\right.}{4b^2f}\right.}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\downarrow 224$$

3.28. $\int \frac{(e+fx)^2\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{1}{3}\left(\frac{3f\left(3a^4Cf^2+4A\left(a^2b^2f^2+2b^4e^2\right)+4a^2b^2e(2Bf+Ce)\right)\int\frac{1}{\frac{b^2cx^2}{a^2c-b^2cx^2}+1}d\frac{x}{\sqrt{a^2c-b^2cx^2}}}{2b^2}-\frac{2\sqrt{a^2c-b^2cx^2}\left(4a^2f^2(Bf+2Ce)-b^2\left(Ce^3-4\right.\right.}{b^2c}\right.}{4b^2f}\right.}{\sqrt{a+bx}\sqrt{ac-}}$$

$\Big\downarrow$ 216

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{(e+fx)^2\sqrt{a^2c-b^2cx^2}(Ce-4Bf)}{3c}+\frac{1}{3}\left(-\frac{2\sqrt{a^2c-b^2cx^2}\left(4a^2f^2(Bf+2Ce)-b^2\left(Ce^3-4ef(3Af+Be)\right)\right)}{b^2c}-\frac{fx\sqrt{a^2c-b^2cx^2}\left(9a^2Cf^2-b^2\left(2Ce\right.\right.}{2b^2c}\right.}{4b^2f}\right.}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

input
```
Int[((e + f*x)^2*(A + B*x + C*x^2))/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]),x]
```

output
```
(Sqrt[a^2*c - b^2*c*x^2]*(-1/4*(C*(e + f*x)^3*Sqrt[a^2*c - b^2*c*x^2])/(b^
2*c*f) + (((C*e - 4*B*f)*(e + f*x)^2*Sqrt[a^2*c - b^2*c*x^2])/(3*c) + ((-2
*(4*a^2*f^2*(2*C*e + B*f) - b^2*(C*e^3 - 4*e*f*(B*e + 3*A*f)))*Sqrt[a^2*c
- b^2*c*x^2])/(b^2*c) - (f*(9*a^2*C*f^2 - b^2*(2*C*e^2 - 4*f*(2*B*e + 3*A*
f)))*x*Sqrt[a^2*c - b^2*c*x^2])/(2*b^2*c) + (3*f*(3*a^4*C*f^2 + 4*a^2*b^2*
e*(C*e + 2*B*f) + 4*A*(2*b^4*e^2 + a^2*b^2*f^2))*ArcTan[(b*Sqrt[c]*x)/Sqrt
[a^2*c - b^2*c*x^2]])/(2*b^3*Sqrt[c]))/3)/(4*b^2*f)))/(Sqrt[a + b*x]*Sqrt[
a*c - b*c*x])
```

3.28.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] && !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) && !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2113
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m])   Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a
*d, 0] && EqQ[m, n] && !IntegerQ[m]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] && !(EqQ[d, 0] &&
True) && !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.28. $\int \frac{(e+fx)^2(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$

3.28.4 Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{(6C f^2x^3b^2+8B b^2f^2x^2+16C b^2ef x^2+12A b^2f^2x+24B b^2efx+9C a^2f^2x+12C b^2e^2x+48A b^2ef+16B a^2f^2+24B b^2e^2+32C a^2e$
default	$\sqrt{bx+a}\,\sqrt{c(-bx+a)}\left(-6C b^2f^2x^3\sqrt{b^2c}\,\sqrt{c(-b^2x^2+a^2)}+12A\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{c\left(-b^2x^2+a^2\right)}}\right)a^2b^2c f^2+24A\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{c\left(-b^2x^2+a^2\right)}}\right)b^4$

input
```
int((f*x+e)^2*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/24*(6*C*b^2*f^2*x^3+8*B*b^2*f^2*x^2+16*C*b^2*e*f*x^2+12*A*b^2*f^2*x+24*
B*b^2*e*f*x+9*C*a^2*f^2*x+12*C*b^2*e^2*x+48*A*b^2*e*f+16*B*a^2*f^2+24*B*b^
2*e^2+32*C*a^2*e*f)*(b*x+a)^(1/2)/b^4*(-b*x+a)/(-c*(b*x-a))^(1/2)+1/8*(4*A
*a^2*b^2*f^2+8*A*b^4*e^2+8*B*a^2*b^2*e*f+3*C*a^4*f^2+4*C*a^2*b^2*e^2)/b^4/
(b^2*c)^(1/2)*arctan((b^2*c)^(1/2)*x/(-b^2*c*x^2+a^2*c)^(1/2))*(-(b*x+a)*c
*(b*x-a))^(1/2)/(b*x+a)^(1/2)/(-c*(b*x-a))^(1/2)
```

3.28.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.31

$$\int \frac{(e+fx)^2\,(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$$

$$= \left[-\frac{3\,(8\,Ba^2b^2ef+4\,(Ca^2b^2+2\,Ab^4)e^2+(3\,Ca^4+4\,Aa^2b^2)f^2)\sqrt{-c}\log\left(2\,b^2cx^2-2\,\sqrt{-bcx+ac}\sqrt{bx+a}\right.}{}\right.$$

$$-\frac{3\,(8\,Ba^2b^2ef+4\,(Ca^2b^2+2\,Ab^4)e^2+(3\,Ca^4+4\,Aa^2b^2)f^2)\sqrt{c}\arctan\left(\frac{\sqrt{-bcx+ac}\sqrt{bx+a}b\sqrt{c}x}{b^2cx^2-a^2c}\right)+(6\,Cb^3f^2}{}$$

input
```
integrate((f*x+e)^2*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algo
rithm="fricas")
```

output
```
[-1/48*(3*(8*B*a^2*b^2*e*f + 4*(C*a^2*b^2 + 2*A*b^4)*e^2 + (3*C*a^4 + 4*A*
a^2*b^2)*f^2)*sqrt(-c)*log(2*b^2*c*x^2 - 2*sqrt(-b*c*x + a*c)*sqrt(b*x + a
)*b*sqrt(-c)*x - a^2*c) + 2*(6*C*b^3*f^2*x^3 + 24*B*b^3*e^2 + 16*B*a^2*b*f
^2 + 16*(2*C*a^2*b + 3*A*b^3)*e*f + 8*(2*C*b^3*e*f + B*b^3*f^2)*x^2 + 3*(4
*C*b^3*e^2 + 8*B*b^3*e*f + (3*C*a^2*b + 4*A*b^3)*f^2)*x)*sqrt(-b*c*x + a*c
)*sqrt(b*x + a))/(b^5*c), -1/24*(3*(8*B*a^2*b^2*e*f + 4*(C*a^2*b^2 + 2*A*b
^4)*e^2 + (3*C*a^4 + 4*A*a^2*b^2)*f^2)*sqrt(c)*arctan(sqrt(-b*c*x + a*c)*s
qrt(b*x + a)*b*sqrt(c)*x/(b^2*c*x^2 - a^2*c)) + (6*C*b^3*f^2*x^3 + 24*B*b^
3*e^2 + 16*B*a^2*b*f^2 + 16*(2*C*a^2*b + 3*A*b^3)*e*f + 8*(2*C*b^3*e*f + B
*b^3*f^2)*x^2 + 3*(4*C*b^3*e^2 + 8*B*b^3*e*f + (3*C*a^2*b + 4*A*b^3)*f^2)*
x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/(b^5*c)]
```

3.28.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*(C*x**2+B*x+A)/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

output
```
Timed out
```

3.28.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 317, normalized size of antiderivative = 0.86

$$\begin{aligned}\int \frac{(e+fx)^2(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = &-\frac{\sqrt{-b^2cx^2+a^2c}\,Cf^2x^3}{4\,b^2c} + \frac{Ae^2\arcsin\left(\frac{bx}{a}\right)}{b\sqrt{c}} \\ &+ \frac{3\,Ca^4f^2\arcsin\left(\frac{bx}{a}\right)}{8\,b^5\sqrt{c}} - \frac{3\,\sqrt{-b^2cx^2+a^2c}\,Ca^2f^2x}{8\,b^4c} \\ &- \frac{\sqrt{-b^2cx^2+a^2c}\,Be^2}{b^2c} - \frac{2\,\sqrt{-b^2cx^2+a^2c}\,Aef}{b^2c} \\ &- \frac{\sqrt{-b^2cx^2+a^2c}\,(2\,Cef+Bf^2)x^2}{3\,b^2c} \\ &+ \frac{(Ce^2+2\,Bef+Af^2)a^2\arcsin\left(\frac{bx}{a}\right)}{2\,b^3\sqrt{c}} \\ &- \frac{\sqrt{-b^2cx^2+a^2c}\,(Ce^2+2\,Bef+Af^2)x}{2\,b^2c} \\ &- \frac{2\,\sqrt{-b^2cx^2+a^2c}\,(2\,Cef+Bf^2)a^2}{3\,b^4c}\end{aligned}$$

input
```
integrate((f*x+e)^2*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algo
rithm="maxima")
```

output
```
-1/4*sqrt(-b^2*c*x^2 + a^2*c)*C*f^2*x^3/(b^2*c) + A*e^2*arcsin(b*x/a)/(b*s
qrt(c)) + 3/8*C*a^4*f^2*arcsin(b*x/a)/(b^5*sqrt(c)) - 3/8*sqrt(-b^2*c*x^2
+ a^2*c)*C*a^2*f^2*x/(b^4*c) - sqrt(-b^2*c*x^2 + a^2*c)*B*e^2/(b^2*c) - 2*
sqrt(-b^2*c*x^2 + a^2*c)*A*e*f/(b^2*c) - 1/3*sqrt(-b^2*c*x^2 + a^2*c)*(2*C
*e*f + B*f^2)*x^2/(b^2*c) + 1/2*(C*e^2 + 2*B*e*f + A*f^2)*a^2*arcsin(b*x/a
)/(b^3*sqrt(c)) - 1/2*sqrt(-b^2*c*x^2 + a^2*c)*(C*e^2 + 2*B*e*f + A*f^2)*x
/(b^2*c) - 2/3*sqrt(-b^2*c*x^2 + a^2*c)*(2*C*e*f + B*f^2)*a^2/(b^4*c)
```

3.28.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 363, normalized size of antiderivative = 0.99

$$\int \frac{(e+fx)^2\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx =$$

$$-\frac{\left(\left(2\left(\frac{3\,(bx+a)Cf^2}{c}+\frac{8\,Cbc^3ef-9\,Cac^3f^2+4\,Bbc^3f^2}{c^4}\right)(bx+a)+\frac{12\,Cb^2c^3e^2-32\,Cabc^3ef+24\,Bb^2c^3ef+27\,Ca^2c^3f^2-16\,Babc^3f^2+\cdots}{c^4}\right.\right.}{}$$

input
```
integrate((f*x+e)^2*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algo
rithm="giac")
```

output
```
-1/24*(((2*(3*(b*x + a)*C*f^2/c + (8*C*b*c^3*e*f - 9*C*a*c^3*f^2 + 4*B*b*c
^3*f^2)/c^4)*(b*x + a) + (12*C*b^2*c^3*e^2 - 32*C*a*b*c^3*e*f + 24*B*b^2*c
^3*e*f + 27*C*a^2*c^3*f^2 - 16*B*a*b*c^3*f^2 + 12*A*b^2*c^3*f^2)/c^4)*(b*x
 + a) - 3*(4*C*a*b^2*c^3*e^2 - 8*B*b^3*c^3*e^2 - 16*C*a^2*b*c^3*e*f + 8*B*
a*b^2*c^3*e*f - 16*A*b^3*c^3*e*f + 5*C*a^3*c^3*f^2 - 8*B*a^2*b*c^3*f^2 + 4
*A*a*b^2*c^3*f^2)/c^4)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a) + 6*(4*C*a
^2*b^2*e^2 + 8*A*b^4*e^2 + 8*B*a^2*b^2*e*f + 3*C*a^4*f^2 + 4*A*a^2*b^2*f^2
)*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c))
/b^5
```

3.28.9 Mupad [B] (verification not implemented)

Time = 81.28 (sec) , antiderivative size = 2799, normalized size of antiderivative = 7.61

$$\int \frac{(e+fx)^2 (A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)^2*(A + B*x + C*x^2))/((a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),
x)
```

output
```
- ((a^(1/2)*(a*c)^(1/2)*(64*B*a^2*c*f^2 + 32*B*b^2*c*e^2)*((a*c - b*c*x)^(
1/2) - (a*c)^(1/2))^8)/(b^4*((a + b*x)^(1/2) - a^(1/2))^8) + (a^(1/2)*(a*c
)^(1/2)*(64*B*a^2*c^3*f^2 + 32*B*b^2*c^3*e^2)*((a*c - b*c*x)^(1/2) - (a*c)
^(1/2))^4)/(b^4*((a + b*x)^(1/2) - a^(1/2))^4) - (a^(1/2)*(a*c)^(1/2)*((12
8*B*a^2*c^2*f^2)/3 - 48*B*b^2*c^2*e^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))
^6)/(b^4*((a + b*x)^(1/2) - a^(1/2))^6) + (4*B*a^2*e*f*((a*c - b*c*x)^(1/2
) - (a*c)^(1/2))^11)/(b^3*((a + b*x)^(1/2) - a^(1/2))^11) + (8*B*a^(1/2)*e
^2*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^10)/(b^2*((a + b*x)^(1/
2) - a^(1/2))^10) + (20*B*a^2*c^4*e*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^
3)/(b^3*((a + b*x)^(1/2) - a^(1/2))^3) + (24*B*a^2*c^3*e*f*((a*c - b*c*x)^
(1/2) - (a*c)^(1/2))^5)/(b^3*((a + b*x)^(1/2) - a^(1/2))^5) - (24*B*a^2*c^
2*e*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/(b^3*((a + b*x)^(1/2) - a^(1/
2))^7) + (8*B*a^(1/2)*c^4*e^2*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/
2))^2)/(b^2*((a + b*x)^(1/2) - a^(1/2))^2) - (4*B*a^2*c^5*e*f*((a*c - b*c*
x)^(1/2) - (a*c)^(1/2)))/(b^3*((a + b*x)^(1/2) - a^(1/2))) - (20*B*a^2*c*e
*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^9)/(b^3*((a + b*x)^(1/2) - a^(1/2))
^9))/(((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^12/((a + b*x)^(1/2) - a^(1/2))^1
2 + c^6 + (6*c*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^10)/((a + b*x)^(1/2) -
a^(1/2))^10 + (6*c^5*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a + b*x)^(1/
2) - a^(1/2))^2 + (15*c^4*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4)/((a + ...
```

3.29 $\int \frac{(e+fx)(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$

3.29.1 Optimal result

Integrand size = 38, antiderivative size = 246

$$\begin{aligned}\int \frac{(e+fx)(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx &= -\frac{C(e+fx)^2(a^2-b^2x^2)}{3b^2f\sqrt{a+bx}\sqrt{ac-bcx}} \\ &\quad -\frac{(2(2a^2Cf^2-b^2(Ce^2-3f(Be+Af)))-b^2f(Ce-3Bf)x)(a^2-b^2x^2)}{6b^4f\sqrt{a+bx}\sqrt{ac-bcx}} \\ &\quad +\frac{(2Ab^2e+a^2(Ce+Bf))\sqrt{a^2c-b^2cx^2}\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{2b^3\sqrt{c}\sqrt{a+bx}\sqrt{ac-bcx}}\end{aligned}$$

output
```
-1/3*C*(f*x+e)^2*(-b^2*x^2+a^2)/b^2/f/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)-1/6
*(4*a^2*C*f^2-2*b^2*(C*e^2-3*f*(A*f+B*e))-b^2*f*(-3*B*f+C*e)*x)*(-b^2*x^2+
a^2)/b^4/f/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)+1/2*(2*A*b^2*e+a^2*(B*f+C*e))*
arctan(b*x*c^(1/2)/(-b^2*c*x^2+a^2*c)^(1/2))*(-b^2*c*x^2+a^2*c)^(1/2)/b^3/
c^(1/2)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)
```

3.29.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.52

$$\int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = \frac{-\left((a-bx)\sqrt{a+bx}\left(4a^2Cf+b^2\left(6Be+6Af+3Cex+3Bfx+2Cfx^2\right)\right)\right)+6b\left(2Ab^2e+a^2(Ce+Bf)\right)\surd}{6b^4\sqrt{c(a-bx)}}$$

input
```
Integrate[((e + f*x)*(A + B*x + C*x^2))/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]),
x]
```

output
```
(-((a - b*x)*Sqrt[a + b*x]*(4*a^2*C*f + b^2*(6*B*e + 6*A*f + 3*C*e*x + 3*B
*f*x + 2*C*f*x^2))) + 6*b*(2*A*b^2*e + a^2*(C*e + B*f))*Sqrt[a - b*x]*ArcT
an[Sqrt[a + b*x]/Sqrt[a - b*x]])/(6*b^4*Sqrt[c*(a - b*x)])
```

3.29.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {2113, 2185, 25, 27, 676, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$$

$$\Big\downarrow \; 2113$$

$$\frac{\sqrt{a^2c-b^2cx^2}\int \frac{(e+fx)(Cx^2+Bx+A)}{\sqrt{a^2c-b^2cx^2}}dx}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \; 2185$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(-\frac{\int -\frac{cf(e+fx)\left(\left(2Ca^2+3Ab^2\right)f-b^2(Ce-3Bf)x\right)}{\sqrt{a^2c-b^2cx^2}}dx}{3b^2cf^2}-\frac{C(e+fx)^2\sqrt{a^2c-b^2cx^2}}{3b^2cf}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\int \frac{cf(e+fx)\left(\left(2Ca^2+3Ab^2\right)f-b^2(Ce-3Bf)x\right)}{\sqrt{a^2c-b^2cx^2}}dx}{3b^2cf^2}-\frac{C(e+fx)^2\sqrt{a^2c-b^2cx^2}}{3b^2cf}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\int \frac{(e+fx)\left(\left(2Ca^2+3Ab^2\right)f-b^2(Ce-3Bf)x\right)}{\sqrt{a^2c-b^2cx^2}}dx}{3b^2f}-\frac{C(e+fx)^2\sqrt{a^2c-b^2cx^2}}{3b^2cf}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \text{676}$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{3}{2}f\left(a^2(Bf+Ce)+2Ab^2e\right)\int \frac{1}{\sqrt{a^2c-b^2cx^2}}dx-\frac{\sqrt{a^2c-b^2cx^2}\left(2a^2Cf^2-b^2\left(Ce^2-3f(Af+Be)\right)\right)}{b^2c}+\frac{fx\sqrt{a^2c-b^2cx^2}(Ce-3Bf)}{2c}}{3b^2f}-\frac{C(e+}{}\right.}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{3}{2}f\left(a^2(Bf+Ce)+2Ab^2e\right)\int \frac{1}{\frac{b^2cx^2}{a^2c-b^2cx^2}+1}d\frac{x}{\sqrt{a^2c-b^2cx^2}}-\frac{\sqrt{a^2c-b^2cx^2}\left(2a^2Cf^2-b^2\left(Ce^2-3f(Af+Be)\right)\right)}{b^2c}+\frac{fx\sqrt{a^2c-b^2cx^2}(Ce-3B}{2c}}{3b^2f}\right.}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{3f\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)\left(a^2(Bf+Ce)+2Ab^2e\right)}{2b\sqrt{c}}-\frac{\sqrt{a^2c-b^2cx^2}\left(2a^2Cf^2-b^2\left(Ce^2-3f(Af+Be)\right)\right)}{b^2c}+\frac{fx\sqrt{a^2c-b^2cx^2}(Ce-3Bf)}{2c}}{3b^2f}-\frac{C(e}{}\right.}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

input
```
Int[((e + f*x)*(A + B*x + C*x^2))/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]),x]
```

output
```
(Sqrt[a^2*c - b^2*c*x^2]*(-1/3*(C*(e + f*x)^2*Sqrt[a^2*c - b^2*c*x^2])/(b^
2*c*f) + (-(((2*a^2*C*f^2 - b^2*(C*e^2 - 3*f*(B*e + A*f)))*Sqrt[a^2*c - b^
2*c*x^2])/(b^2*c)) + (f*(C*e - 3*B*f)*x*Sqrt[a^2*c - b^2*c*x^2])/(2*c) + (
3*f*(2*A*b^2*e + a^2*(C*e + B*f))*ArcTan[(b*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*
x^2]])/(2*b*Sqrt[c]))/(3*b^2*f)))/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
```

3.29. $\int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$

3.29.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 2113
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m])   Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a
*d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.29. $\int \frac{(e+fx)(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$

3.29.4 Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.71

method	result
risch	$-\frac{(2Cf\,x^2b^2+3B\,b^2fx+3C\,b^2ex+6A\,b^2f+6B\,b^2e+4a^2Cf)\sqrt{bx+a}\,(-bx+a)}{6b^4\sqrt{-c(bx-a)}}+\frac{(2A\,b^2e+B\,a^2f+C\,a^2e)\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{-b^2c\,x^2+a^2c}}\right)\sqrt{}}{2b^2\sqrt{b^2c}\,\sqrt{bx+a}\,\sqrt{-c(bx-a)}}$
default	$\frac{\sqrt{bx+a}\,\sqrt{c(-bx+a)}\left(6A\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{c\left(-b^2x^2+a^2\right)}}\right)b^4ce+3B\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{c\left(-b^2x^2+a^2\right)}}\right)a^2b^2cf+3C\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{c\left(-b^2x^2+a^2\right)}}\right)a^2b^2c\right.}{}$

input
```
int((f*x+e)*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x,method=_RETUR
NVERBOSE)
```

output
```
-1/6*(2*C*b^2*f*x^2+3*B*b^2*f*x+3*C*b^2*e*x+6*A*b^2*f+6*B*b^2*e+4*C*a^2*f)
*(b*x+a)^(1/2)/b^4*(-b*x+a)/(-c*(b*x-a))^(1/2)+1/2*(2*A*b^2*e+B*a^2*f+C*a^
2*e)/b^2/(b^2*c)^(1/2)*arctan((b^2*c)^(1/2)*x/(-b^2*c*x^2+a^2*c)^(1/2))*(-
(b*x+a)*c*(b*x-a))^(1/2)/(b*x+a)^(1/2)/(-c*(b*x-a))^(1/2)
```

3.29.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.23

$$\int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$$
$$=\left[-\frac{3\left(Ba^2bf+(Ca^2b+2\,Ab^3)e\right)\sqrt{-c}\log\left(2\,b^2cx^2-2\,\sqrt{-bcx+ac}\sqrt{bx+a}b\sqrt{-c}x-a^2c\right)+2\left(2\,Cb^2fx^2\right.}{12\,b^4c}\right.$$
$$-\frac{3\left(Ba^2bf+(Ca^2b+2\,Ab^3)e\right)\sqrt{c}\arctan\left(\frac{\sqrt{-bcx+ac}\sqrt{bx+a}b\sqrt{c}x}{b^2cx^2-a^2c}\right)+\left(2\,Cb^2fx^2+6\,Bb^2e+2\left(2\,Ca^2+3\,Ab^2\right)\right.}{6\,b^4c}$$

input
```
integrate((f*x+e)*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algori
thm="fricas")
```

output
```
[-1/12*(3*(B*a^2*b*f + (C*a^2*b + 2*A*b^3)*e)*sqrt(-c)*log(2*b^2*c*x^2 - 2
*sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(-c)*x - a^2*c) + 2*(2*C*b^2*f*x^2
 + 6*B*b^2*e + 2*(2*C*a^2 + 3*A*b^2)*f + 3*(C*b^2*e + B*b^2*f)*x)*sqrt(-b*
c*x + a*c)*sqrt(b*x + a))/(b^4*c), -1/6*(3*(B*a^2*b*f + (C*a^2*b + 2*A*b^3
)*e)*sqrt(c)*arctan(sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(c)*x/(b^2*c*x^
2 - a^2*c)) + (2*C*b^2*f*x^2 + 6*B*b^2*e + 2*(2*C*a^2 + 3*A*b^2)*f + 3*(C*
b^2*e + B*b^2*f)*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/(b^4*c)]
```

3.29.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*(C*x**2+B*x+A)/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

output
```
Timed out
```

3.29.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.77

$$\begin{aligned}\int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = &-\frac{\sqrt{-b^2cx^2+a^2c}Cfx^2}{3\,b^2c} + \frac{Ae\arcsin\left(\frac{bx}{a}\right)}{b\sqrt{c}} \\ &+ \frac{(Ce+Bf)a^2\arcsin\left(\frac{bx}{a}\right)}{2\,b^3\sqrt{c}} - \frac{\sqrt{-b^2cx^2+a^2c}Be}{b^2c} \\ &- \frac{2\sqrt{-b^2cx^2+a^2c}Ca^2f}{3\,b^4c} - \frac{\sqrt{-b^2cx^2+a^2c}Af}{b^2c} \\ &- \frac{\sqrt{-b^2cx^2+a^2c}(Ce+Bf)x}{2\,b^2c}\end{aligned}$$

input
```
integrate((f*x+e)*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algori
thm="maxima")
```

output
```
-1/3*sqrt(-b^2*c*x^2 + a^2*c)*C*f*x^2/(b^2*c) + A*e*arcsin(b*x/a)/(b*sqrt(
c)) + 1/2*(C*e + B*f)*a^2*arcsin(b*x/a)/(b^3*sqrt(c)) - sqrt(-b^2*c*x^2 +
a^2*c)*B*e/(b^2*c) - 2/3*sqrt(-b^2*c*x^2 + a^2*c)*C*a^2*f/(b^4*c) - sqrt(-
b^2*c*x^2 + a^2*c)*A*f/(b^2*c) - 1/2*sqrt(-b^2*c*x^2 + a^2*c)*(C*e + B*f)*
x/(b^2*c)
```

3.29.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.78

$$\int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx =$$
$$-\frac{\left(\left(\frac{2\,(bx+a)Cf}{c}+\frac{3\,Cbc^2e-4\,Cac^2f+3\,Bbc^2f}{c^3}\right)(bx+a)-\frac{3\,(Cabc^2e-2\,Bb^2c^2e-2\,Ca^2c^2f+Babc^2f-2\,Ab^2c^2f)}{c^3}\right)\sqrt{-(bx+a)c}}{6\,b^4}$$

input
```
integrate((f*x+e)*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algori
thm="giac")
```

output
```
-1/6*(((2*(b*x + a)*C*f/c + (3*C*b*c^2*e - 4*C*a*c^2*f + 3*B*b*c^2*f)/c^3)
*(b*x + a) - 3*(C*a*b*c^2*e - 2*B*b^2*c^2*e - 2*C*a^2*c^2*f + B*a*b*c^2*f
- 2*A*b^2*c^2*f)/c^3)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a) + 6*(C*a^2*
b*e + 2*A*b^3*e + B*a^2*b*f)*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x
+ a)*c + 2*a*c)))/sqrt(-c))/b^4
```

3.29.9 Mupad [B] (verification not implemented)

Time = 37.95 (sec) , antiderivative size = 1011, normalized size of antiderivative = 4.11

$$\int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = -\frac{\frac{2\,B\,a^2\,f\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^7}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^7}-\frac{2\,B\,a^2\,c^3\,f\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}{\sqrt{a+b\,x}-\sqrt{a}}-\frac{14\,B\,a^2\,c\,f\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^5}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^5}+\frac{14\,B\,a^2\,c^2\,f\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^3}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^3}}{b^3\,c^4+\frac{b^3\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^8}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^8}+\frac{4\,b^3\,c^3\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^2}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}+\frac{6\,b^3\,c^2\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^4}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^4}+\frac{4\,b^3\,c\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^6}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^6}} -\frac{\frac{2\,C\,a^2\,e\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^7}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^7}-\frac{2\,C\,a^2\,c^3\,e\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}{\sqrt{a+b\,x}-\sqrt{a}}-\frac{14\,C\,a^2\,c\,e\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^5}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^5}+\frac{14\,C\,a^2\,c^2\,e\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^3}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^3}}{b^3\,c^4+\frac{b^3\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^8}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^8}+\frac{4\,b^3\,c^3\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^2}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}+\frac{6\,b^3\,c^2\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^4}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^4}+\frac{4\,b^3\,c\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^6}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^6}} -\frac{\sqrt{a\,c-b\,c\,x}\,\left(\frac{2\,C\,a^3\,f}{3\,b^4\,c}+\frac{C\,f\,x^3}{3\,b\,c}+\frac{C\,a\,f\,x^2}{3\,b^2\,c}+\frac{2\,C\,a^2\,f\,x}{3\,b^3\,c}\right)}{\sqrt{a+b\,x}}-\frac{4\,A\,e\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}{\sqrt{b^2\,c}\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}\right)}{\sqrt{b^2\,c}} -\frac{A\,f\,\sqrt{a\,c-b\,c\,x}\,\sqrt{a+b\,x}}{b^2\,c}-\frac{B\,e\,\sqrt{a\,c-b\,c\,x}\,\sqrt{a+b\,x}}{b^2\,c} -\frac{2\,B\,a^2\,f\,\mathrm{atan}\left(\frac{\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}}{\sqrt{c}\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}\right)}{b^3\,\sqrt{c}}-\frac{2\,C\,a^2\,e\,\mathrm{atan}\left(\frac{\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}}{\sqrt{c}\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}\right)}{b^3\,\sqrt{c}}$$

input
```
int(((e + f*x)*(A + B*x + C*x^2))/((a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),x)
```

output
```
- ((2*B*a^2*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/((a + b*x)^(1/2) - a^
(1/2))^7 - (2*B*a^2*c^3*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/((a + b*x)^
(1/2) - a^(1/2)) - (14*B*a^2*c*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5)/((
a + b*x)^(1/2) - a^(1/2))^5 + (14*B*a^2*c^2*f*((a*c - b*c*x)^(1/2) - (a*c)
^(1/2))^3)/((a + b*x)^(1/2) - a^(1/2))^3)/(b^3*c^4 + (b^3*((a*c - b*c*x)^(
1/2) - (a*c)^(1/2))^8)/((a + b*x)^(1/2) - a^(1/2))^8 + (4*b^3*c^3*((a*c -
b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 + (6*b^3*c^2*
((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4)/((a + b*x)^(1/2) - a^(1/2))^4 + (4*
b^3*c*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6)/((a + b*x)^(1/2) - a^(1/2))^6
) - ((2*C*a^2*e*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/((a + b*x)^(1/2) -
a^(1/2))^7 - (2*C*a^2*c^3*e*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/((a + b*x
)^(1/2) - a^(1/2)) - (14*C*a^2*c*e*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5)/
((a + b*x)^(1/2) - a^(1/2))^5 + (14*C*a^2*c^2*e*((a*c - b*c*x)^(1/2) - (a*
c)^(1/2))^3)/((a + b*x)^(1/2) - a^(1/2))^3)/(b^3*c^4 + (b^3*((a*c - b*c*x)
^(1/2) - (a*c)^(1/2))^8)/((a + b*x)^(1/2) - a^(1/2))^8 + (4*b^3*c^3*((a*c
- b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 + (6*b^3*c^
2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4)/((a + b*x)^(1/2) - a^(1/2))^4 + (
4*b^3*c*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6)/((a + b*x)^(1/2) - a^(1/2))
^6) - ((a*c - b*c*x)^(1/2)*((2*C*a^3*f)/(3*b^4*c) + (C*f*x^3)/(3*b*c) + (C
*a*f*x^2)/(3*b^2*c) + (2*C*a^2*f*x)/(3*b^3*c)))/(a + b*x)^(1/2) - (4*A*...
```

3.29. $\int \frac{(e+fx)(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$

3.30 $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$

3.30.1 Optimal result

Integrand size = 33, antiderivative size = 177

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = -\frac{B(a^2-b^2x^2)}{b^2\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{Cx(a^2-b^2x^2)}{2b^2\sqrt{a+bx}\sqrt{ac-bcx}} + \frac{(2Ab^2+a^2C)\sqrt{a^2c-b^2cx^2}\arctan\left(\frac{b\sqrt{cx}}{\sqrt{a^2c-b^2cx^2}}\right)}{2b^3\sqrt{c}\sqrt{a+bx}\sqrt{ac-bcx}}$$

output
```
-B*(-b^2*x^2+a^2)/b^2/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)-1/2*C*x*(-b^2*x^2+a
^2)/b^2/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)+1/2*(2*A*b^2+C*a^2)*arctan(b*x*c^
(1/2)/(-b^2*c*x^2+a^2*c)^(1/2))*(-b^2*c*x^2+a^2*c)^(1/2)/b^3/c^(1/2)/(b*x+
a)^(1/2)/(-b*c*x+a*c)^(1/2)
```

3.30.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.51

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = \frac{b(-a+bx)\sqrt{a+bx}(2B+Cx)+2(2Ab^2+a^2C)\sqrt{a-bx}\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{a-bx}}\right)}{2b^3\sqrt{c(a-bx)}}$$

input
```
Integrate[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]),x]
```

output
```
(b*(-a + b*x)*Sqrt[a + b*x]*(2*B + C*x) + 2*(2*A*b^2 + a^2*C)*Sqrt[a - b*x
]*ArcTan[Sqrt[a + b*x]/Sqrt[a - b*x]])/(2*b^3*Sqrt[c*(a - b*x)])
```

3.30.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.66, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1189, 83, 646, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}}dx$$

$$\Big\downarrow \text{1189}$$

$$\int \frac{Cx^2+A}{\sqrt{a+bx}\sqrt{ac-bcx}}dx+B\int \frac{x}{\sqrt{a+bx}\sqrt{ac-bcx}}dx$$

$$\Big\downarrow \text{83}$$

$$\int \frac{Cx^2+A}{\sqrt{a+bx}\sqrt{ac-bcx}}dx-\frac{B\sqrt{a+bx}\sqrt{ac-bcx}}{b^2c}$$

$$\Big\downarrow \text{646}$$

$$\frac{1}{2}\left(\frac{a^2C}{b^2}+2A\right)\int \frac{1}{\sqrt{a+bx}\sqrt{ac-bcx}}dx-\frac{B\sqrt{a+bx}\sqrt{ac-bcx}}{b^2c}-\frac{Cx\sqrt{a+bx}\sqrt{ac-bcx}}{2b^2c}$$

$$\Big\downarrow \text{45}$$

$$\left(\frac{a^2C}{b^2}+2A\right)\int \frac{1}{\frac{c(a+bx)b}{ac-bcx}+b}d\frac{\sqrt{a+bx}}{\sqrt{ac-bcx}}-\frac{B\sqrt{a+bx}\sqrt{ac-bcx}}{b^2c}-\frac{Cx\sqrt{a+bx}\sqrt{ac-bcx}}{2b^2c}$$

$$\Big\downarrow \text{218}$$

$$\frac{\left(\frac{a^2C}{b^2}+2A\right)\arctan\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{ac-bcx}}\right)}{b\sqrt{c}}-\frac{B\sqrt{a+bx}\sqrt{ac-bcx}}{b^2c}-\frac{Cx\sqrt{a+bx}\sqrt{ac-bcx}}{2b^2c}$$

input
```
Int[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]),x]
```

output
```
-((B*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])/(b^2*c)) - (C*x*Sqrt[a + b*x]*Sqrt[a
*c - b*c*x])/(2*b^2*c) + ((2*A + (a^2*C)/b^2)*ArcTan[(Sqrt[c]*Sqrt[a + b*x
])/Sqrt[a*c - b*c*x]])/(b*Sqrt[c])
```

3.30.3.1 Defintions of rubi rules used

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 646
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2), x_Symbol] :> Simp[b*x*(c + d*x)^(m + 1)*((e + f*x)^(n + 1)/(d*f*(2*m +
 3))), x] - Simp[(b*c*e - a*d*f*(2*m + 3))/(d*f*(2*m + 3))   Int[(c + d*x)^
m*(e + f*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m, n] &&
 EqQ[d*e + c*f, 0] &&  !LtQ[m, -1]
```

rule 1189
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2), x_Symbol] :> Simp[b   Int[x*(d + e*x)^m*(f + g*x)^n, x],
x] + Int[(d + e*x)^m*(f + g*x)^n*(a + c*x^2), x] /; FreeQ[{a, b, c, d, e, f
, g, m, n}, x] && EqQ[m, n] && EqQ[e*f + d*g, 0]
```

3.30.4 Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.71

method	result
risch	$-\frac{(Cx+2B)\sqrt{bx+a}\,(-bx+a)}{2b^{2}\sqrt{-c(bx-a)}}+\frac{(2b^{2}A+C\,a^{2})\arctan\left(\frac{\sqrt{b^{2}c}\,x}{\sqrt{-b^{2}c\,x^{2}+a^{2}c}}\right)\sqrt{-(bx+a)c(bx-a)}}{2b^{2}\sqrt{b^{2}c}\,\sqrt{bx+a}\,\sqrt{-c(bx-a)}}$
default	$\frac{\sqrt{bx+a}\,\sqrt{c(-bx+a)}\,\left(2A\arctan\left(\frac{\sqrt{b^{2}c}\,x}{\sqrt{c\left(-b^{2}x^{2}+a^{2}\right)}}\right)b^{2}c+C\arctan\left(\frac{\sqrt{b^{2}c}\,x}{\sqrt{c\left(-b^{2}x^{2}+a^{2}\right)}}\right)a^{2}c-C\sqrt{c(-b^{2}x^{2}+a^{2})}\,\sqrt{b^{2}c}\,x-2B\sqrt{b^{2}c}\,\sqrt{c(-}{2b^{2}\sqrt{c(-b^{2}x^{2}+a^{2})}\,c\sqrt{b^{2}c}}$

input
```
int((C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
-1/2*(C*x+2*B)*(b*x+a)^(1/2)/b^2*(-b*x+a)/(-c*(b*x-a))^(1/2)+1/2*(2*A*b^2+
C*a^2)/b^2/(b^2*c)^(1/2)*arctan((b^2*c)^(1/2)*x/(-b^2*c*x^2+a^2*c)^(1/2))*
(-(b*x+a)*c*(b*x-a))^(1/2)/(b*x+a)^(1/2)/(-c*(b*x-a))^(1/2)
```

3.30.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$$
$$= \left[-\frac{(Ca^2+2\,Ab^2)\sqrt{-c}\log\left(2\,b^2cx^2-2\,\sqrt{-bcx+ac}\sqrt{bx+a}b\sqrt{-c}x-a^2c\right)+2\,(Cbx+2\,Bb)\sqrt{-bcx+ac}}{4\,b^3c}\right.$$
$$\left.-\frac{(Ca^2+2\,Ab^2)\sqrt{c}\arctan\left(\frac{\sqrt{-bcx+ac}\sqrt{bx+a}b\sqrt{c}x}{b^2cx^2-a^2c}\right)+(Cbx+2\,Bb)\sqrt{-bcx+ac}\sqrt{bx+a}}{2\,b^3c}\right]$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorithm="fri
cas")
```

output
```
[-1/4*((C*a^2 + 2*A*b^2)*sqrt(-c)*log(2*b^2*c*x^2 - 2*sqrt(-b*c*x + a*c)*s
qrt(b*x + a)*b*sqrt(-c)*x - a^2*c) + 2*(C*b*x + 2*B*b)*sqrt(-b*c*x + a*c)*
sqrt(b*x + a))/(b^3*c), -1/2*((C*a^2 + 2*A*b^2)*sqrt(c)*arctan(sqrt(-b*c*x
 + a*c)*sqrt(b*x + a)*b*sqrt(c)*x/(b^2*c*x^2 - a^2*c)) + (C*b*x + 2*B*b)*s
qrt(-b*c*x + a*c)*sqrt(b*x + a))/(b^3*c)]
```

3.30.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

output
```
Timed out
```

3.30.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.50

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = \frac{Ca^2\arcsin\left(\frac{bx}{a}\right)}{2\,b^3\sqrt{c}} + \frac{A\arcsin\left(\frac{bx}{a}\right)}{b\sqrt{c}} - \frac{\sqrt{-b^2cx^2+a^2c}Cx}{2\,b^2c} - \frac{\sqrt{-b^2cx^2+a^2c}B}{b^2c}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorithm="max
ima")
```

output
```
1/2*C*a^2*arcsin(b*x/a)/(b^3*sqrt(c)) + A*arcsin(b*x/a)/(b*sqrt(c)) - 1/2*
sqrt(-b^2*c*x^2 + a^2*c)*C*x/(b^2*c) - sqrt(-b^2*c*x^2 + a^2*c)*B/(b^2*c)
```

3.30.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.60

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = -\frac{\sqrt{-(bx+a)c+2\,ac}\sqrt{bx+a}\left(\frac{(bx+a)C}{c}-\frac{Cac-2\,Bbc}{c^2}\right)+\frac{2\,(Ca^2+2\,Ab^2)\log\left(\left|-\sqrt{bx+a}\sqrt{-c}+\sqrt{-(bx+a)c+2\,ac}\right|\right)}{\sqrt{-c}}}{2\,b^3}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorithm="gia
c")
```

output
```
-1/2*(sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*((b*x + a)*C/c - (C*a*c - 2
*B*b*c)/c^2) + 2*(C*a^2 + 2*A*b^2)*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(
-(b*x + a)*c + 2*a*c)))/sqrt(-c))/b^3
```

3.30.9 Mupad [B] (verification not implemented)

Time = 20.55 (sec) , antiderivative size = 489, normalized size of antiderivative = 2.76

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = -\frac{\frac{2\,C\,a^2\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^7}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^7}-\frac{2\,C\,a^2\,c^3\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}{\sqrt{a+b\,x}-\sqrt{a}}-\frac{14\,C\,a^2\,c\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^5}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^5}+\frac{14\,C\,a^2\,c^2\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^3}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^3}}{b^3\,c^4+\frac{b^3\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^8}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^8}+\frac{4\,b^3\,c^3\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^2}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}+\frac{6\,b^3\,c^2\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^4}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^4}+\frac{4\,b^3\,c\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^6}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^6}} - \frac{4\,A\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}{\sqrt{b^2\,c}\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}\right)}{\sqrt{b^2\,c}} - \frac{2\,C\,a^2\,\mathrm{atan}\left(\frac{\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}}{\sqrt{c}\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}\right)}{b^3\,\sqrt{c}} - \frac{B\,\sqrt{a\,c-b\,c\,x}\,\sqrt{a+b\,x}}{b^2\,c}$$

input
```
int((A + B*x + C*x^2)/((a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),x)
```

output
```
- ((2*C*a^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/((a + b*x)^(1/2) - a^(1
/2))^7 - (2*C*a^2*c^3*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/((a + b*x)^(1/2
) - a^(1/2)) - (14*C*a^2*c*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5)/((a + b*
x)^(1/2) - a^(1/2))^5 + (14*C*a^2*c^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^
3)/((a + b*x)^(1/2) - a^(1/2))^3)/(b^3*c^4 + (b^3*((a*c - b*c*x)^(1/2) - (
a*c)^(1/2))^8)/((a + b*x)^(1/2) - a^(1/2))^8 + (4*b^3*c^3*((a*c - b*c*x)^(
1/2) - (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 + (6*b^3*c^2*((a*c -
b*c*x)^(1/2) - (a*c)^(1/2))^4)/((a + b*x)^(1/2) - a^(1/2))^4 + (4*b^3*c*((
a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6)/((a + b*x)^(1/2) - a^(1/2))^6) - (4*A
*atan((b*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/((b^2*c)^(1/2)*((a + b*x)^(1
/2) - a^(1/2)))))/(b^2*c)^(1/2) - (2*C*a^2*atan(((a*c - b*c*x)^(1/2) - (a*
c)^(1/2))/(c^(1/2)*((a + b*x)^(1/2) - a^(1/2)))))/(b^3*c^(1/2)) - (B*(a*c
- b*c*x)^(1/2)*(a + b*x)^(1/2))/(b^2*c)
```

3.31 $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}\,dx$

3.31.1 Optimal result

Integrand size = 40, antiderivative size = 278

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}\,dx &= -\frac{C(a^2-b^2x^2)}{b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{(Ce-Bf)\sqrt{a^2c-b^2cx^2}\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}f^2\sqrt{a+bx}\sqrt{ac-bcx}} \\ &\quad + \frac{(Ce^2-Bef+Af^2)\sqrt{a^2c-b^2cx^2}\arctan\left(\frac{\sqrt{c}(a^2f+b^2ex)}{\sqrt{b^2e^2-a^2f^2}\sqrt{a^2c-b^2cx^2}}\right)}{\sqrt{c}f^2\sqrt{b^2e^2-a^2f^2}\sqrt{a+bx}\sqrt{ac-bcx}}\end{aligned}$$

output
```
-C*(-b^2*x^2+a^2)/b^2/f/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)-(-B*f+C*e)*arctan
(b*x*c^(1/2)/(-b^2*c*x^2+a^2*c)^(1/2))*(-b^2*c*x^2+a^2*c)^(1/2)/b/f^2/c^(1
/2)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)+(A*f^2-B*e*f+C*e^2)*arctan((b^2*e*x+a
^2*f)*c^(1/2)/(-a^2*f^2+b^2*e^2)^(1/2)/(-b^2*c*x^2+a^2*c)^(1/2))*(-b^2*c*x
^2+a^2*c)^(1/2)/f^2/c^(1/2)/(-a^2*f^2+b^2*e^2)^(1/2)/(b*x+a)^(1/2)/(-b*c*x
+a*c)^(1/2)
```

3.31.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.64

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}\,dx$$
$$=\frac{\frac{Cf(-a+bx)\sqrt{a+bx}}{b^2}-\frac{2(Ce-Bf)\sqrt{a-bx}\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{a-bx}}\right)}{b}+\frac{2(Ce^2+f(-Be+Af))\sqrt{a-bx}\arctan\left(\frac{\sqrt{be+af}\sqrt{a+bx}}{\sqrt{be-af}\sqrt{a-bx}}\right)}{\sqrt{be-af}\sqrt{be+af}}}{f^2\sqrt{c(a-bx)}}$$

input
```
Integrate[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)),x]
```

output
```
((C*f*(-a + b*x)*Sqrt[a + b*x])/b^2 - (2*(C*e - B*f)*Sqrt[a - b*x]*ArcTan[
Sqrt[a + b*x]/Sqrt[a - b*x]])/b + (2*(C*e^2 + f*(-(B*e) + A*f))*Sqrt[a - b
*x]*ArcTan[(Sqrt[b*e + a*f]*Sqrt[a + b*x])/(Sqrt[b*e - a*f]*Sqrt[a - b*x])
])/(Sqrt[b*e - a*f]*Sqrt[b*e + a*f]))/(f^2*Sqrt[c*(a - b*x)])
```

3.31.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.84, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.225, Rules used = {2113, 2185, 25, 27, 719, 224, 216, 488, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}(e+fx)\sqrt{ac-bcx}}\,dx$$

$$\Big\downarrow \text{ 2113}$$

$$\frac{\sqrt{a^2c-b^2cx^2}\int \frac{Cx^2+Bx+A}{(e+fx)\sqrt{a^2c-b^2cx^2}}dx}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \text{ 2185}$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(-\frac{\int -\frac{b^2cf(Af-(Ce-Bf)x)}{(e+fx)\sqrt{a^2c-b^2cx^2}}dx}{b^2cf^2}-\frac{C\sqrt{a^2c-b^2cx^2}}{b^2cf}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\int \frac{b^2cf(Af-(Ce-Bf)x)}{(e+fx)\sqrt{a^2c-b^2cx^2}}dx}{b^2cf^2}-\frac{C\sqrt{a^2c-b^2cx^2}}{b^2cf}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\int \frac{Af-(Ce-Bf)x}{(e+fx)\sqrt{a^2c-b^2cx^2}}dx}{f}-\frac{C\sqrt{a^2c-b^2cx^2}}{b^2cf}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \text{719}$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{\left(Af^2-Bef+Ce^2\right)\int \frac{1}{(e+fx)\sqrt{a^2c-b^2cx^2}}dx}{f}-\frac{(Ce-Bf)\int \frac{1}{\sqrt{a^2c-b^2cx^2}}dx}{f}}{f}-\frac{C\sqrt{a^2c-b^2cx^2}}{b^2cf}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{\left(Af^2-Bef+Ce^2\right)\int \frac{1}{(e+fx)\sqrt{a^2c-b^2cx^2}}dx}{f}-\frac{(Ce-Bf)\int \frac{1}{\frac{b^2cx^2}{a^2c-b^2cx^2}+1}d\frac{x}{\sqrt{a^2c-b^2cx^2}}}{f}}{f}-\frac{C\sqrt{a^2c-b^2cx^2}}{b^2cf}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{\left(Af^2-Bef+Ce^2\right)\int \frac{1}{(e+fx)\sqrt{a^2c-b^2cx^2}}dx}{f}-\frac{(Ce-Bf)\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}f}}{f}-\frac{C\sqrt{a^2c-b^2cx^2}}{b^2cf}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \text{488}$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{-\frac{\left(Af^2-Bef+Ce^2\right)\int \frac{1}{-b^2ce^2+a^2cf^2-\frac{\left(cfa^2+b^2cex\right)^2}{a^2c-b^2cx^2}}d\frac{cfa^2+b^2cex}{\sqrt{a^2c-b^2cx^2}}}{f}-\frac{(Ce-Bf)\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}f}}{f}-\frac{C\sqrt{a^2c-b^2cx^2}}{b^2cf}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \text{217}$$

3.31. $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}dx$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{\left(Af^2-Bef+Ce^2\right)\arctan\left(\frac{a^2cf+b^2cex}{\sqrt{c}\sqrt{a^2c-b^2cx^2}\sqrt{b^2e^2-a^2f^2}}\right)}{\sqrt{c}f\sqrt{b^2e^2-a^2f^2}}-\frac{(Ce-Bf)\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}f}}{f}-\frac{C\sqrt{a^2c-b^2cx^2}}{b^2cf}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

input
```
Int[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)),x]
```

output
```
(Sqrt[a^2*c - b^2*c*x^2]*(-((C*Sqrt[a^2*c - b^2*c*x^2])/(b^2*c*f)) + (-(((
C*e - B*f)*ArcTan[(b*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(b*Sqrt[c]*f)) +
 ((C*e^2 - B*e*f + A*f^2)*ArcTan[(a^2*c*f + b^2*c*e*x)/(Sqrt[c]*Sqrt[b^2*e
^2 - a^2*f^2]*Sqrt[a^2*c - b^2*c*x^2])])/(Sqrt[c]*f*Sqrt[b^2*e^2 - a^2*f^2
]))/f))/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
```

3.31.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2113
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m])   Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a
*d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.31.4 Maple [A] (verified)

Time = 1.70 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.08

method	result
risch	$-\frac{C\sqrt{bx+a}\,(-bx+a)}{f\,b^2\sqrt{-c(bx-a)}}+\frac{\left(\frac{(Bf-Ce)\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{-b^2c\,x^2+a^2c}}\right)}{f\sqrt{b^2c}}-\frac{(Af^2-Bef+Ce^2)\ln\left(\frac{\frac{2c\left(a^2f^2-b^2e^2\right)}{f^2}+\frac{2b^2ce\left(x+\frac{e}{f}\right)}{f}+2\sqrt{\frac{c\left(a^2f^2-b^2e^2\right)}{f^2}}}{x+}\right.}{f^2\sqrt{\frac{c\left(a^2f^2-b^2e^2\right)}{f^2}}}\right.}{f\sqrt{bx+a}\,\sqrt{-c(bx-a)}}$
default	$\frac{\left(-A\ln\left(\frac{2b^2cex+2a^2cf+2\sqrt{\frac{c\left(a^2f^2-b^2e^2\right)}{f^2}}\sqrt{c\left(-b^2x^2+a^2\right)}\,f}{fx+e}\right)b^2c\,f^2\sqrt{b^2c}+B\ln\left(\frac{2b^2cex+2a^2cf+2\sqrt{\frac{c\left(a^2f^2-b^2e^2\right)}{f^2}}\sqrt{c\left(-b^2x^2+a^2\right)}\,f}{fx+e}\right)\right.}{}$

input
```
int((C*x^2+B*x+A)/(f*x+e)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x,method=_RETUR
NVERBOSE)
```

output
```
-C*(b*x+a)^(1/2)*(-b*x+a)/f/b^2/(-c*(b*x-a))^(1/2)+1/f*((B*f-C*e)/f/(b^2*c
)^(1/2)*arctan((b^2*c)^(1/2)*x/(-b^2*c*x^2+a^2*c)^(1/2))-(A*f^2-B*e*f+C*e^
2)/f^2/(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*ln((2*c*(a^2*f^2-b^2*e^2)/f^2+2*b^2
*c*e/f*(x+e/f)+2*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-b^2*c*(x+e/f)^2+2*b^2*c
*e/f*(x+e/f)+c*(a^2*f^2-b^2*e^2)/f^2)^(1/2))/(x+e/f)))*(-(b*x+a)*c*(b*x-a)
)^(1/2)/(b*x+a)^(1/2)/(-c*(b*x-a))^(1/2)
```

3.31.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}\,dx=\text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/(f*x+e)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algori
thm="fricas")
```

output
```
Timed out
```

3.31.6 Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}\,dx=\int \frac{A+Bx+Cx^2}{\sqrt{-c\,(-a+bx)}\sqrt{a+bx}\,(e+fx)}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(f*x+e)/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(sqrt(-c*(-a + b*x))*sqrt(a + b*x)*(e + f*x)),
 x)
```

3.31.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+B*x+A)/(f*x+e)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algori
thm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((4*b^2*c>0)', see `assume?` for
more detai
```

3.31.8 Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((C*x^2+B*x+A)/(f*x+e)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.31.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 9298, normalized size of antiderivative = 33.45

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/((e + f*x)*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),x)
```

output
```
(B*a*e*atan(((B*a*e*((4096*(32*B^3*a^(17/2)*c^3*e*f^2*(a*c)^(5/2) + 24*B^3
*a^(15/2)*b^2*c^4*e^3*(a*c)^(3/2)))/(a^6*b^8*e^6) - (4096*(32*B^3*a^(17/2)
*c^2*e*f^2*(a*c)^(5/2) - 96*B^3*a^(15/2)*b^2*c^3*e^3*(a*c)^(3/2))*((a*c -
b*c*x)^(1/2) - (a*c)^(1/2))^2)/(a^6*b^8*e^6*((a + b*x)^(1/2) - a^(1/2))^2)
 - (B*a*e*((4096*(16*B^2*a^12*c^6*f^4 + 9*B^2*a^8*b^4*c^6*e^4))/(a^6*b^8*e
^6) + (B*a*e*((4096*(24*B*a^(17/2)*b^2*c^4*e*f^4*(a*c)^(5/2) - 30*B*a^(15/
2)*b^4*c^5*e^3*f^2*(a*c)^(3/2)))/(a^6*b^8*e^6) + (16384*(20*B*a^12*c^6*f^5
 - 22*B*a^10*b^2*c^6*e^2*f^3)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^
7*e^6*((a + b*x)^(1/2) - a^(1/2))) + (B*a*e*((4096*(9*a^8*b^6*c^7*e^4*f^2
- 7*a^10*b^4*c^7*e^2*f^4))/(a^6*b^8*e^6) + (4096*(9*a^8*b^6*c^6*e^4*f^2 -
11*a^10*b^4*c^6*e^2*f^4)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(a^6*b^8*e
^6*((a + b*x)^(1/2) - a^(1/2))^2) - (16384*(5*a^(17/2)*b^2*c^4*e*f^5*(a*c)
^(5/2) - 6*a^(15/2)*b^4*c^5*e^3*f^3*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a
*c)^(1/2)))/(a^6*b^7*e^6*((a + b*x)^(1/2) - a^(1/2)))))/(f*(a^4*c*f^2 - a^
2*b^2*c*e^2)^(1/2)) + (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(96*B*a^
(17/2)*b^2*c^3*e*f^4*(a*c)^(5/2) - 90*B*a^(15/2)*b^4*c^4*e^3*f^2*(a*c)^(3/
2)))/(a^6*b^8*e^6*((a + b*x)^(1/2) - a^(1/2))^2)))/(f*(a^4*c*f^2 - a^2*b^2
*c*e^2)^(1/2)) + (16384*(8*B^2*a^(17/2)*c^3*e*f^3*(a*c)^(5/2) + 3*B^2*a^(1
5/2)*b^2*c^4*e^3*f*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*
b^7*e^6*((a + b*x)^(1/2) - a^(1/2))) + (4096*((a*c - b*c*x)^(1/2) - (a*...
```

3.32 $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2}\,dx$

3.32.1 Optimal result

Integrand size = 40, antiderivative size = 322

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2}\,dx = \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)(a^2-b^2x^2)}{(b^2e^2-a^2f^2)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}+\frac{C\sqrt{a^2c-b^2cx^2}\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}f^2\sqrt{a+bx}\sqrt{ac-bcx}} + \frac{(a^2f^2(2Ce-Bf)-b^2(Ce^3-Aef^2))\sqrt{a^2c-b^2cx^2}\arctan\left(\frac{\sqrt{c}(a^2f+b^2ex)}{\sqrt{b^2e^2-a^2f^2}\sqrt{a^2c-b^2cx^2}}\right)}{\sqrt{c}f^2(b^2e^2-a^2f^2)^{3/2}\sqrt{a+bx}\sqrt{ac-bcx}}$$

output
```
f*(A+e*(-B*f+C*e)/f^2)*(-b^2*x^2+a^2)/(-a^2*f^2+b^2*e^2)/(f*x+e)/(b*x+a)^(
1/2)/(-b*c*x+a*c)^(1/2)+C*arctan(b*x*c^(1/2)/(-b^2*c*x^2+a^2*c)^(1/2))*(-b
^2*c*x^2+a^2*c)^(1/2)/b/f^2/c^(1/2)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)+(a^2*
f^2*(-B*f+2*C*e)-b^2*(-A*e*f^2+C*e^3))*arctan((b^2*e*x+a^2*f)*c^(1/2)/(-a^
2*f^2+b^2*e^2)^(1/2)/(-b^2*c*x^2+a^2*c)^(1/2))*(-b^2*c*x^2+a^2*c)^(1/2)/f^
2/(-a^2*f^2+b^2*e^2)^(3/2)/c^(1/2)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)
```

3.32.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.71

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2}\,dx = \frac{2\left(\frac{f(Ce^2+f(-Be+Af))(-a+bx)\sqrt{a+bx}}{2(-be+af)(be+af)(e+fx)}+\frac{C\sqrt{a-bx}\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{a-bx}}\right)}{b}-\frac{(a^2f^2(-2Ce+Bf)+b^2(Ce^3-Aef^2))\sqrt{a-bx}\arctan\left(\frac{\sqrt{be+af}\sqrt{a+b}}{\sqrt{be-af}\sqrt{a-b}}\right.}{(be-af)^{3/2}(be+af)^{3/2}}}{f^2\sqrt{c(a-bx)}}$$

input
```
Integrate[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^2),
x]
```

output
```
(2*((f*(C*e^2 + f*(-(B*e) + A*f))*(-a + b*x)*Sqrt[a + b*x])/(2*(-(b*e) + a
*f)*(b*e + a*f)*(e + f*x)) + (C*Sqrt[a - b*x]*ArcTan[Sqrt[a + b*x]/Sqrt[a
- b*x]])/b - ((a^2*f^2*(-2*C*e + B*f) + b^2*(C*e^3 - A*e*f^2))*Sqrt[a - b*
x]*ArcTan[(Sqrt[b*e + a*f]*Sqrt[a + b*x])/(Sqrt[b*e - a*f]*Sqrt[a - b*x])]
)/((b*e - a*f)^(3/2)*(b*e + a*f)^(3/2))))/(f^2*Sqrt[c*(a - b*x)])
```

3.32.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2113, 2182, 27, 719, 224, 216, 488, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}(e+fx)^2\sqrt{ac-bcx}}\,dx$$

$$\Big\downarrow \ 2113$$

$$\frac{\sqrt{a^2c-b^2cx^2}\int \frac{Cx^2+Bx+A}{(e+fx)^2\sqrt{a^2c-b^2cx^2}}dx}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \ 2182$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\int \frac{c\left((Ce-Bf)a^2+Ab^2e+C\left(\frac{b^2e^2}{f}-a^2f\right)x\right)}{(e+fx)\sqrt{a^2c-b^2cx^2}}dx}{c(b^2e^2-a^2f^2)}+\frac{f\sqrt{a^2c-b^2cx^2}\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{c(e+fx)(b^2e^2-a^2f^2)}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\int \frac{(Ce-Bf)a^2+Ab^2e+C\left(\frac{b^2e^2}{f}-a^2f\right)x}{(e+fx)\sqrt{a^2c-b^2cx^2}}dx}{b^2e^2-a^2f^2}+\frac{f\sqrt{a^2c-b^2cx^2}\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{c(e+fx)(b^2e^2-a^2f^2)}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$\Big\downarrow$ 719

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{\left(a^2f^2(2Ce-Bf)-b^2\left(Ce^3-Aef^2\right)\right)\int \frac{1}{(e+fx)\sqrt{a^2c-b^2cx^2}}dx}{f^2}+\frac{C\left(b^2e^2-a^2f^2\right)\int \frac{1}{\sqrt{a^2c-b^2cx^2}}dx}{f^2}}{b^2e^2-a^2f^2}+\frac{f\sqrt{a^2c-b^2cx^2}\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{c(e+fx)(b^2e^2-a^2f^2)}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$\Big\downarrow$ 224

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{\left(a^2f^2(2Ce-Bf)-b^2\left(Ce^3-Aef^2\right)\right)\int \frac{1}{(e+fx)\sqrt{a^2c-b^2cx^2}}dx}{f^2}+\frac{C\left(b^2e^2-a^2f^2\right)\int \frac{1}{\frac{b^2cx^2}{a^2c-b^2cx^2}+1}d\frac{x}{\sqrt{a^2c-b^2cx^2}}}{f^2}}{b^2e^2-a^2f^2}+\frac{f\sqrt{a^2c-b^2cx^2}\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{c(e+fx)(b^2e^2-a^2f^2)}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$\Big\downarrow$ 216

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{\left(a^2f^2(2Ce-Bf)-b^2\left(Ce^3-Aef^2\right)\right)\int \frac{1}{(e+fx)\sqrt{a^2c-b^2cx^2}}dx}{f^2}+\frac{C\left(b^2e^2-a^2f^2\right)\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}f^2}}{b^2e^2-a^2f^2}+\frac{f\sqrt{a^2c-b^2cx^2}\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{c(e+fx)(b^2e^2-a^2f^2)}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$\Big\downarrow$ 488

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{C\left(b^2e^2-a^2f^2\right)\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}f^2}-\frac{\left(a^2f^2(2Ce-Bf)-b^2\left(Ce^3-Aef^2\right)\right)\int \frac{1}{-b^2ce^2+a^2cf^2-\frac{\left(cfa^2+b^2cex\right)^2}{a^2c-b^2cx^2}}d\frac{cfa^2+b^2cex}{\sqrt{a^2c-b^2cx^2}}}{f^2}}{b^2e^2-a^2f^2}+\frac{f\sqrt{\cdots}}{\cdots}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$\Big\downarrow$ 217

3.32. $\displaystyle\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2}\,dx$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{\left(a^2f^2(2Ce-Bf)-b^2\left(Ce^3-Aef^2\right)\right)\arctan\left(\frac{a^2cf+b^2cex}{\sqrt{c}\sqrt{a^2c-b^2cx^2}\sqrt{b^2e^2-a^2f^2}}\right)}{\sqrt{c}f^2\sqrt{b^2e^2-a^2f^2}}+\frac{C\left(b^2e^2-a^2f^2\right)\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}f^2}}{b^2e^2-a^2f^2}+\frac{f\sqrt{a^2c-b^2cx}}{c(e+fx)}\right.}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

input
```
Int[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^2),x]
```

output
```
(Sqrt[a^2*c - b^2*c*x^2]*((f*(A + (e*(C*e - B*f))/f^2)*Sqrt[a^2*c - b^2*c*
x^2])/(c*(b^2*e^2 - a^2*f^2)*(e + f*x)) + ((C*(b^2*e^2 - a^2*f^2)*ArcTan[(
b*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(b*Sqrt[c]*f^2) + ((a^2*f^2*(2*C*e
- B*f) - b^2*(C*e^3 - A*e*f^2))*ArcTan[(a^2*c*f + b^2*c*e*x)/(Sqrt[c]*Sqrt
[b^2*e^2 - a^2*f^2]*Sqrt[a^2*c - b^2*c*x^2])])/(Sqrt[c]*f^2*Sqrt[b^2*e^2 -
 a^2*f^2]))/(b^2*e^2 - a^2*f^2)))/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
```

3.32.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2113
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m])   Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a
*d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

3.32.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1165 vs. $2(290) = 580$.

Time = 1.68 (sec) , antiderivative size = 1166, normalized size of antiderivative = 3.62

method	result	size
default	Expression too large to display	1166

input
```
int((C*x^2+B*x+A)/(f*x+e)^2/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
(A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^
2))^(1/2)*f)/(f*x+e))*b^2*c*e*f^3*x*(b^2*c)^(1/2)-B*ln(2*(b^2*c*e*x+a^2*c*
f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^2
*c*f^4*x*(b^2*c)^(1/2)+C*arctan((b^2*c)^(1/2)*x/(c*(-b^2*x^2+a^2))^(1/2))*
a^2*c*f^4*x*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)-C*arctan((b^2*c)^(1/2)*x/(c*(-
b^2*x^2+a^2))^(1/2))*b^2*c*e^2*f^2*x*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)+2*C*l
n(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^
(1/2)*f)/(f*x+e))*a^2*c*e*f^3*x*(b^2*c)^(1/2)-C*ln(2*(b^2*c*e*x+a^2*c*f+(c
*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*b^2*c*e
^3*f*x*(b^2*c)^(1/2)+A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(
1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*b^2*c*e^2*f^2*(b^2*c)^(1/2)-B*ln
(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(
1/2)*f)/(f*x+e))*a^2*c*e*f^3*(b^2*c)^(1/2)+C*arctan((b^2*c)^(1/2)*x/(c*(-b
^2*x^2+a^2))^(1/2))*a^2*c*e*f^3*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)-C*arctan((
b^2*c)^(1/2)*x/(c*(-b^2*x^2+a^2))^(1/2))*b^2*c*e^3*f*(c*(a^2*f^2-b^2*e^2)/
f^2)^(1/2)+2*C*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*
(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^2*c*e^2*f^2*(b^2*c)^(1/2)-C*ln(2*(b^2*
c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/
(f*x+e))*b^2*c*e^4*(b^2*c)^(1/2)-A*f^4*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*
(-b^2*x^2+a^2))^(1/2)*(b^2*c)^(1/2)+B*e*f^3*(c*(a^2*f^2-b^2*e^2)/f^2)^(...
```

3.32.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/(f*x+e)^2/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algo
rithm="fricas")
```

output
```
Timed out
```

3.32.6 Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2}\,dx = \int \frac{A+Bx+Cx^2}{\sqrt{-c\left(-a+bx\right)}\sqrt{a+bx}\left(e+fx\right)^2}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(f*x+e)**2/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(sqrt(-c*(-a + b*x))*sqrt(a + b*x)*(e + f*x)**
2), x)
```

3.32.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+B*x+A)/(f*x+e)^2/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((4*b^2*c>0)', see `assume?` for
more detai
```

3.32.8 Giac [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 526, normalized size of antiderivative = 1.63

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2}\,dx$$
$$= \frac{\frac{2\left(Cb^3\sqrt{-c}e^3-2\,Ca^2b\sqrt{-c}ef^2-Ab^3\sqrt{-c}ef^2+Ba^2b\sqrt{-c}f^3\right)\arctan\left(-\frac{2\,bce-\left(\sqrt{bx+a}\sqrt{-c}-\sqrt{-(bx+a)c+2\,ac}\right)^2f}{2\sqrt{-b^2e^2+a^2f^2}c}\right)}{(b^2e^2f^2-a^2f^4)\sqrt{-b^2e^2+a^2f^2}c} - \frac{C\log\left(\left(\sqrt{bx+a}\sqrt{-c}-\sqrt{-(\cdots}\right.\right.}{\sqrt{-c}f^2}}{}$$

input
```
integrate((C*x^2+B*x+A)/(f*x+e)^2/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algo
rithm="giac")
```

output
```
(2*(C*b^3*sqrt(-c)*e^3 - 2*C*a^2*b*sqrt(-c)*e*f^2 - A*b^3*sqrt(-c)*e*f^2 +
 B*a^2*b*sqrt(-c)*f^3)*arctan(-1/2*(2*b*c*e - (sqrt(b*x + a)*sqrt(-c) - sq
rt(-(b*x + a)*c + 2*a*c))^2*f)/(sqrt(-b^2*e^2 + a^2*f^2)*c))/((b^2*e^2*f^2
 - a^2*f^4)*sqrt(-b^2*e^2 + a^2*f^2)*c) - C*log((sqrt(b*x + a)*sqrt(-c) -
sqrt(-(b*x + a)*c + 2*a*c))^2)/(sqrt(-c)*f^2) + 4*(C*b^3*(sqrt(b*x + a)*sq
rt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^2*sqrt(-c)*e^3 - B*b^3*(sqrt(b*x + a)
*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^2*sqrt(-c)*e^2*f - 2*C*a^2*b^2*sqr
t(-c)*c*e^2*f + A*b^3*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c)
)^2*sqrt(-c)*e*f^2 + 2*B*a^2*b^2*sqrt(-c)*c*e*f^2 - 2*A*a^2*b^2*sqrt(-c)*c
*f^3)/((b^2*e^2*f^2 - a^2*f^4)*(4*b*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x +
 a)*c + 2*a*c))^2*c*e - (sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*
c))^4*f - 4*a^2*c^2*f)))/b
```

3.32.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2}\,dx=\text{Hanged}$$

input
```
int((A + B*x + C*x^2)/((e + f*x)^2*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),x)
```

output
```
\text{Hanged}
```

3.33 $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3}\,dx$

3.33.1 Optimal result

Integrand size = 40, antiderivative size = 363

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3}\,dx = \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)(a^2-b^2x^2)}{2\left(b^2e^2-a^2f^2\right)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2} + \frac{\left(2a^2f^2(2Ce-Bf)-b^2e(Ce^2+f(Be-3Af))\right)(a^2-b^2x^2)}{2f\left(b^2e^2-a^2f^2\right)^2\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)} + \frac{\left(A(2b^4e^2+a^2b^2f^2)+a^2(2a^2Cf^2+b^2e(Ce-3Bf))\right)\sqrt{a^2c-b^2cx^2}\arctan\left(\frac{\sqrt{c}(a^2f+b^2ex)}{\sqrt{b^2e^2-a^2f^2}\sqrt{a^2c-b^2cx^2}}\right)}{2\sqrt{c}\left(b^2e^2-a^2f^2\right)^{5/2}\sqrt{a+bx}\sqrt{ac-bcx}}$$

output
```
1/2*f*(A+e*(-B*f+C*e)/f^2)*(-b^2*x^2+a^2)/(-a^2*f^2+b^2*e^2)/(f*x+e)^2/(b*
x+a)^(1/2)/(-b*c*x+a*c)^(1/2)+1/2*(2*a^2*f^2*(-B*f+2*C*e)-b^2*e*(C*e^2+f*(
-3*A*f+B*e)))*(-b^2*x^2+a^2)/f/(-a^2*f^2+b^2*e^2)^2/(f*x+e)/(b*x+a)^(1/2)/
(-b*c*x+a*c)^(1/2)+1/2*(A*(a^2*b^2*f^2+2*b^4*e^2)+a^2*(2*a^2*C*f^2+b^2*e*(
-3*B*f+C*e)))*arctan((b^2*e*x+a^2*f)*c^(1/2)/(-a^2*f^2+b^2*e^2)^(1/2)/(-b^
2*c*x^2+a^2*c)^(1/2))*(-b^2*c*x^2+a^2*c)^(1/2)/(-a^2*f^2+b^2*e^2)^(5/2)/c^
(1/2)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)
```

3.33.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.69

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3}\,dx$$
$$= \frac{\frac{(-a+bx)\sqrt{a+bx}(b^2e(Ce^2x+Be(2e+fx)-Af(4e+3fx))+a^2f(-Ce(3e+4fx)+f(Af+B(e+2fx))))}{2(be-af)^2(be+af)^2(e+fx)^2}+\frac{(2a^4Cf^2+a^2b^2e(Ce-3Bf)+A(2b^4e^2+\cdots}{(be-af)^{5/2}\cdots}}{\sqrt{c(a-bx)}}$$

input
```
Integrate[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^3),
x]
```

output
```
(((-a + b*x)*Sqrt[a + b*x]*(b^2*e*(C*e^2*x + B*e*(2*e + f*x) - A*f*(4*e +
3*f*x)) + a^2*f*(-(C*e*(3*e + 4*f*x)) + f*(A*f + B*(e + 2*f*x)))))/(2*(b*e
 - a*f)^2*(b*e + a*f)^2*(e + f*x)^2) + ((2*a^4*C*f^2 + a^2*b^2*e*(C*e - 3*
B*f) + A*(2*b^4*e^2 + a^2*b^2*f^2))*Sqrt[a - b*x]*ArcTan[(Sqrt[b*e + a*f]*
Sqrt[a + b*x])/(Sqrt[b*e - a*f]*Sqrt[a - b*x])])/((b*e - a*f)^(5/2)*(b*e +
 a*f)^(5/2)))/Sqrt[c*(a - b*x)]
```

3.33.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 357, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2113, 2182, 27, 679, 488, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}(e+fx)^3\sqrt{ac-bcx}}\,dx$$
$$\Big\downarrow \text{2113}$$
$$\frac{\sqrt{a^2c-b^2cx^2}\int \frac{Cx^2+Bx+A}{(e+fx)^3\sqrt{a^2c-b^2cx^2}}dx}{\sqrt{a+bx}\sqrt{ac-bcx}}$$
$$\Big\downarrow \text{2182}$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\int \frac{c\left(2\left((Ce-Bf)a^2+Ab^2e\right)-\left(2a^2Cf-b^2\left(\frac{Ce^2}{f}+Be-Af\right)\right)x\right)}{(e+fx)^2\sqrt{a^2c-b^2cx^2}}dx}{2c(b^2e^2-a^2f^2)}+\frac{f\sqrt{a^2c-b^2cx^2}\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{2c(e+fx)^2(b^2e^2-a^2f^2)}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\int \frac{2\left((Ce-Bf)a^2+Ab^2e\right)-\left(2a^2Cf-b^2\left(\frac{Ce^2}{f}+Be-Af\right)\right)x}{(e+fx)^2\sqrt{a^2c-b^2cx^2}}dx}{2(b^2e^2-a^2f^2)}+\frac{f\sqrt{a^2c-b^2cx^2}\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{2c(e+fx)^2(b^2e^2-a^2f^2)}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$\Big\downarrow$ 679

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{\left(2a^4Cf^2+A\left(a^2b^2f^2+2b^4e^2\right)+a^2b^2e(Ce-3Bf)\right)\int\frac{1}{(e+fx)\sqrt{a^2c-b^2cx^2}}dx}{b^2e^2-a^2f^2}+\frac{\sqrt{a^2c-b^2cx^2}\left(2a^2f^2(2Ce-Bf)-b^2\left(ef(Be-3Af)+Ce^3\right)\right)}{cf(e+fx)\left(b^2e^2-a^2f^2\right)}}{2(b^2e^2-a^2f^2)} - \cdots\right.}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$\Big\downarrow$ 488

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{\sqrt{a^2c-b^2cx^2}\left(2a^2f^2(2Ce-Bf)-b^2\left(ef(Be-3Af)+Ce^3\right)\right)}{cf(e+fx)\left(b^2e^2-a^2f^2\right)}-\frac{\left(2a^4Cf^2+A\left(a^2b^2f^2+2b^4e^2\right)+a^2b^2e(Ce-3Bf)\right)\int\frac{1}{-b^2ce^2+a^2cf^2-\frac{\left(cfa^2+\cdots\right.}{a^2c-\cdots}}\cdots}{b^2e^2-a^2f^2}}{2(b^2e^2-a^2f^2)}\cdots\right.}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$\Big\downarrow$ 217

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{f\sqrt{a^2c-b^2cx^2}\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{2c(e+fx)^2(b^2e^2-a^2f^2)}+\frac{\frac{\sqrt{a^2c-b^2cx^2}\left(2a^2f^2(2Ce-Bf)-b^2\left(ef(Be-3Af)+Ce^3\right)\right)}{cf(e+fx)\left(b^2e^2-a^2f^2\right)}+\frac{\left(2a^4Cf^2+A\left(a^2b^2f^2+2b^4e^2\right)+a^2b^2e(\cdots\right.}{\sqrt{c}(\cdots}}{2(b^2e^2-a^2f^2)}\cdots\right.}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

input
```
Int[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^3),x]
```

3.33. $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3}\,dx$

output
```
(Sqrt[a^2*c - b^2*c*x^2]*((f*(A + (e*(C*e - B*f))/f^2)*Sqrt[a^2*c - b^2*c*
x^2])/(2*c*(b^2*e^2 - a^2*f^2)*(e + f*x)^2) + (((2*a^2*f^2*(2*C*e - B*f) -
 b^2*(C*e^3 + e*f*(B*e - 3*A*f)))*Sqrt[a^2*c - b^2*c*x^2])/(c*f*(b^2*e^2 -
 a^2*f^2)*(e + f*x)) + ((2*a^4*C*f^2 + a^2*b^2*e*(C*e - 3*B*f) + A*(2*b^4*
e^2 + a^2*b^2*f^2))*ArcTan[(a^2*c*f + b^2*c*e*x)/(Sqrt[c]*Sqrt[b^2*e^2 - a
^2*f^2]*Sqrt[a^2*c - b^2*c*x^2])])/(Sqrt[c]*(b^2*e^2 - a^2*f^2)^(3/2)))/(2
*(b^2*e^2 - a^2*f^2))))/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
```

3.33.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2113
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m])   Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a
*d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

3.33.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1793 vs. $2(333) = 666$.

Time = 5.68 (sec) , antiderivative size = 1794, normalized size of antiderivative = 4.94

method	result	size
default	Expression too large to display	1794

input
```
int((C*x^2+B*x+A)/(f*x+e)^3/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/2*(A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x
^2+a^2))^(1/2)*f)/(f*x+e))*a^2*b^2*c*f^4*x^2+2*A*ln(2*(b^2*c*e*x+a^2*c*f+(
c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*b^4*c*
e^2*f^2*x^2+4*A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c
*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*b^4*c*e^3*f*x+4*C*ln(2*(b^2*c*e*x+a^2*c
*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^
4*c*e*f^3*x+A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(
-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^2*b^2*c*e^2*f^2-3*B*ln(2*(b^2*c*e*x+a^2
*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*
a^2*b^2*c*e^3*f+2*C*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2
)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^4*c*f^4*x^2+2*B*a^2*f^4*x*(c*(-b^
2*x^2+a^2))^(1/2)*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)-4*A*b^2*e^2*f^2*(c*(-b^2
*x^2+a^2))^(1/2)*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)+A*a^2*f^4*(c*(-b^2*x^2+a^
2))^(1/2)*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)+2*A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(
a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*b^4*c*e^4
+2*C*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+
a^2))^(1/2)*f)/(f*x+e))*a^4*c*e^2*f^2+C*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^
2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^2*b^2*c*e^4+B
*a^2*e*f^3*(c*(-b^2*x^2+a^2))^(1/2)*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)+2*B*b^
2*e^3*f*(c*(-b^2*x^2+a^2))^(1/2)*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)-3*C*a^...
```

3.33.5 Fricas [A] (verification not implemented)

Time = 40.27 (sec) , antiderivative size = 1355, normalized size of antiderivative = 3.73

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(f*x+e)^3/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algo
rithm="fricas")
```

output
```
[1/4*((3*B*a^2*b^2*e^3*f - (C*a^2*b^2 + 2*A*b^4)*e^4 - (2*C*a^4 + A*a^2*b^
2)*e^2*f^2 + (3*B*a^2*b^2*e*f^3 - (C*a^2*b^2 + 2*A*b^4)*e^2*f^2 - (2*C*a^4
 + A*a^2*b^2)*f^4)*x^2 + 2*(3*B*a^2*b^2*e^2*f^2 - (C*a^2*b^2 + 2*A*b^4)*e^
3*f - (2*C*a^4 + A*a^2*b^2)*e*f^3)*x)*sqrt(-b^2*c*e^2 + a^2*c*f^2)*log((2*
a^2*b^2*c*e*f*x - a^2*b^2*c*e^2 + 2*a^4*c*f^2 + (2*b^4*c*e^2 - a^2*b^2*c*f
^2)*x^2 - 2*sqrt(-b^2*c*e^2 + a^2*c*f^2)*(b^2*e*x + a^2*f)*sqrt(-b*c*x + a
*c)*sqrt(b*x + a))/(f^2*x^2 + 2*e*f*x + e^2)) - 2*(2*B*b^4*e^5 - B*a^2*b^2
*e^3*f^2 - B*a^4*e*f^4 - A*a^4*f^5 - (3*C*a^2*b^2 + 4*A*b^4)*e^4*f + (3*C*
a^4 + 5*A*a^2*b^2)*e^2*f^3 + (C*b^4*e^5 + B*b^4*e^4*f + B*a^2*b^2*e^2*f^3
- 2*B*a^4*f^5 - (5*C*a^2*b^2 + 3*A*b^4)*e^3*f^2 + (4*C*a^4 + 3*A*a^2*b^2)*
e*f^4)*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/(b^6*c*e^8 - 3*a^2*b^4*c*e^6*f
^2 + 3*a^4*b^2*c*e^4*f^4 - a^6*c*e^2*f^6 + (b^6*c*e^6*f^2 - 3*a^2*b^4*c*e^
4*f^4 + 3*a^4*b^2*c*e^2*f^6 - a^6*c*f^8)*x^2 + 2*(b^6*c*e^7*f - 3*a^2*b^4*
c*e^5*f^3 + 3*a^4*b^2*c*e^3*f^5 - a^6*c*e*f^7)*x), -1/2*((3*B*a^2*b^2*e^3*
f - (C*a^2*b^2 + 2*A*b^4)*e^4 - (2*C*a^4 + A*a^2*b^2)*e^2*f^2 + (3*B*a^2*b
^2*e*f^3 - (C*a^2*b^2 + 2*A*b^4)*e^2*f^2 - (2*C*a^4 + A*a^2*b^2)*f^4)*x^2
+ 2*(3*B*a^2*b^2*e^2*f^2 - (C*a^2*b^2 + 2*A*b^4)*e^3*f - (2*C*a^4 + A*a^2*
b^2)*e*f^3)*x)*sqrt(b^2*c*e^2 - a^2*c*f^2)*arctan(sqrt(b^2*c*e^2 - a^2*c*f
^2)*(b^2*e*x + a^2*f)*sqrt(-b*c*x + a*c)*sqrt(b*x + a)/(a^2*b^2*c*e^2 - a^
4*c*f^2 - (b^4*c*e^2 - a^2*b^2*c*f^2)*x^2)) + (2*B*b^4*e^5 - B*a^2*b^2*...
```

3.33.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/(f*x+e)**3/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

output
```
Timed out
```

3.33.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+B*x+A)/(f*x+e)^3/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((a*f-b*e)>0)', see `assume?` for
 more deta
```

3.33.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1425 vs. $2(335) = 670$.

Time = 0.72 (sec) , antiderivative size = 1425, normalized size of antiderivative = 3.93

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(f*x+e)^3/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algo
rithm="giac")
```

output
```
-((C*a^2*b^3*sqrt(-c)*e^2 + 2*A*b^5*sqrt(-c)*e^2 - 3*B*a^2*b^3*sqrt(-c)*e*
f + 2*C*a^4*b*sqrt(-c)*f^2 + A*a^2*b^3*sqrt(-c)*f^2)*arctan(-1/2*(2*b*c*e
- (sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^2*f)/(sqrt(-b^2*e^
2 + a^2*f^2)*c))/((b^4*e^4 - 2*a^2*b^2*e^2*f^2 + a^4*f^4)*sqrt(-b^2*e^2 +
a^2*f^2)*c) + 2*(4*C*b^6*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a
*c))^4*sqrt(-c)*c*e^5 - 2*C*b^5*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*
c + 2*a*c))^6*sqrt(-c)*e^4*f + 4*B*b^6*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*
x + a)*c + 2*a*c))^4*sqrt(-c)*c*e^4*f - 8*C*a^2*b^5*(sqrt(b*x + a)*sqrt(-c
) - sqrt(-(b*x + a)*c + 2*a*c))^2*sqrt(-c)*c^2*e^4*f - 14*C*a^2*b^4*(sqrt(
b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^4*sqrt(-c)*c*e^3*f^2 - 12*
A*b^6*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^4*sqrt(-c)*c*e
^3*f^2 - 16*B*a^2*b^5*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c)
)^2*sqrt(-c)*c^2*e^3*f^2 + 8*C*a^4*b^4*sqrt(-c)*c^3*e^3*f^2 + 5*C*a^2*b^3*
(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^6*sqrt(-c)*e^2*f^3 +
 2*A*b^5*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^6*sqrt(-c)*
e^2*f^3 + 10*B*a^2*b^4*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c
))^4*sqrt(-c)*c*e^2*f^3 + 44*C*a^4*b^3*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*
x + a)*c + 2*a*c))^2*sqrt(-c)*c^2*e^2*f^3 + 40*A*a^2*b^5*(sqrt(b*x + a)*sq
rt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^2*sqrt(-c)*c^2*e^2*f^3 + 8*B*a^4*b^4*
sqrt(-c)*c^3*e^2*f^3 - 3*B*a^2*b^3*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x...
```

3.33.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 9344, normalized size of antiderivative = 25.74

$$\int \frac{A + Bx + Cx^2}{\sqrt{a + bx}\sqrt{ac - bcx}(e + fx)^3} \, dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/((e + f*x)^3*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),x)
```

output
```
((((a*c - b*c*x)^(1/2) - (a*c)^(1/2))*(4*C*a^4*c^3*f^2 + 2*C*a^2*b^2*c^3*e
^2))/(((a + b*x)^(1/2) - a^(1/2))*(b^5*e^5 - 2*a^2*b^3*e^3*f^2 + a^4*b*e*f
^4)) + (((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3*(68*C*a^4*c^2*f^2 - 14*C*a^2
*b^2*c^2*e^2))/(((a + b*x)^(1/2) - a^(1/2))^3*(b^5*e^5 - 2*a^2*b^3*e^3*f^2
 + a^4*b*e*f^4)) - ((68*C*a^4*c*f^2 - 14*C*a^2*b^2*c*e^2)*((a*c - b*c*x)^(
1/2) - (a*c)^(1/2))^5)/(((a + b*x)^(1/2) - a^(1/2))^5*(b^5*e^5 - 2*a^2*b^3
*e^3*f^2 + a^4*b*e*f^4)) - ((4*C*a^4*f^2 + 2*C*a^2*b^2*e^2)*((a*c - b*c*x)
^(1/2) - (a*c)^(1/2))^7)/(((a + b*x)^(1/2) - a^(1/2))^7*(b^5*e^5 - 2*a^2*b
^3*e^3*f^2 + a^4*b*e*f^4)) - (a^(1/2)*(a*c)^(1/2)*(48*C*a^4*c*f^3 - 24*C*a
^2*b^2*c*e^2*f)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4)/(((a + b*x)^(1/2) -
 a^(1/2))^4*(b^6*e^6 - 2*a^2*b^4*e^4*f^2 + a^4*b^2*e^2*f^4)) + (a^(1/2)*(a
*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6*(24*C*a^4*f^3 + 12*C*a^2*b
^2*e^2*f))/(((a + b*x)^(1/2) - a^(1/2))^6*(b^6*e^6 - 2*a^2*b^4*e^4*f^2 + a
^4*b^2*e^2*f^4)) + (a^(1/2)*(a*c)^(1/2)*(24*C*a^4*c^2*f^3 + 12*C*a^2*b^2*c
^2*e^2*f)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(((a + b*x)^(1/2) - a^(1/
2))^2*(b^6*e^6 - 2*a^2*b^4*e^4*f^2 + a^4*b^2*e^2*f^4)))/(((a*c - b*c*x)^(1
/2) - (a*c)^(1/2))^8/((a + b*x)^(1/2) - a^(1/2))^8 + c^4 + (((a*c - b*c*x)
^(1/2) - (a*c)^(1/2))^6*(16*a^2*c*f^2 + 4*b^2*c*e^2))/(b^2*e^2*((a + b*x)^
(1/2) - a^(1/2))^6) + ((16*a^2*c^3*f^2 + 4*b^2*c^3*e^2)*((a*c - b*c*x)^(1/
2) - (a*c)^(1/2))^2)/(b^2*e^2*((a + b*x)^(1/2) - a^(1/2))^2) - ((32*a^2...
```

3.33. $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3}\,dx$

3.34 $\int \frac{x(a+bx+cx^2)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx$

3.34.1 Optimal result

Integrand size = 30, antiderivative size = 87

$$\int \frac{x(a+bx+cx^2)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{cx^2\sqrt{-1+dx}\sqrt{1+dx}}{3d^2} + \frac{\sqrt{-1+dx}\sqrt{1+dx}(2(2c+3ad^2)+3bd^2x)}{6d^4} + \frac{b\operatorname{arccosh}(dx)}{2d^3}$$

output
```
1/2*b*arccosh(d*x)/d^3+1/3*c*x^2*(d*x-1)^(1/2)*(d*x+1)^(1/2)/d^2+1/6*(3*b*
d^2*x+6*a*d^2+4*c)*(d*x-1)^(1/2)*(d*x+1)^(1/2)/d^4
```

3.34.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.85

$$\int \frac{x(a+bx+cx^2)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{-1+dx}\sqrt{1+dx}(3d^2(2a+bx)+2c(2+d^2x^2))+6bd\operatorname{arctanh}\left(\sqrt{\frac{-1+dx}{1+dx}}\right)}{6d^4}$$

input
```
Integrate[(x*(a + b*x + c*x^2))/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

output
```
(Sqrt[-1 + d*x]*Sqrt[1 + d*x]*(3*d^2*(2*a + b*x) + 2*c*(2 + d^2*x^2)) + 6*
b*d*ArcTanh[Sqrt[(-1 + d*x)/(1 + d*x)]])/(6*d^4)
```

3.34.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.57, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2113, 2340, 533, 25, 27, 455, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+bx+cx^2)}{\sqrt{dx-1}\sqrt{dx+1}}dx$$

$$\Big\downarrow \text{ 2113}$$

$$\frac{\sqrt{d^2x^2-1}\int \frac{x(cx^2+bx+a)}{\sqrt{d^2x^2-1}}dx}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{ 2340}$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{\int \frac{x(3ad^2+3bxd^2+2c)}{\sqrt{d^2x^2-1}}dx}{3d^2}+\frac{cx^2\sqrt{d^2x^2-1}}{3d^2}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{ 533}$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{\frac{3}{2}bx\sqrt{d^2x^2-1}-\frac{\int -\frac{d^2(3b+2(3ad^2+2c)x)}{\sqrt{d^2x^2-1}}dx}{2d^2}}{3d^2}+\frac{cx^2\sqrt{d^2x^2-1}}{3d^2}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{\frac{\int \frac{d^2(3b+2(3ad^2+2c)x)}{\sqrt{d^2x^2-1}}dx}{2d^2}+\frac{3}{2}bx\sqrt{d^2x^2-1}}{3d^2}+\frac{cx^2\sqrt{d^2x^2-1}}{3d^2}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{\frac{1}{2}\int \frac{3b+2(3ad^2+2c)x}{\sqrt{d^2x^2-1}}dx+\frac{3}{2}bx\sqrt{d^2x^2-1}}{3d^2}+\frac{cx^2\sqrt{d^2x^2-1}}{3d^2}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{ 455}$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{\frac{1}{2}\left(3b\int\frac{1}{\sqrt{d^2x^2-1}}dx+2\sqrt{d^2x^2-1}\left(3a+\frac{2c}{d^2}\right)\right)+\frac{3}{2}bx\sqrt{d^2x^2-1}}{3d^2}+\frac{cx^2\sqrt{d^2x^2-1}}{3d^2}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow 224$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{\frac{1}{2}\left(3b\int\frac{1}{1-\frac{d^2x^2}{d^2x^2-1}}d\frac{x}{\sqrt{d^2x^2-1}}+2\sqrt{d^2x^2-1}\left(3a+\frac{2c}{d^2}\right)\right)+\frac{3}{2}bx\sqrt{d^2x^2-1}}{3d^2}+\frac{cx^2\sqrt{d^2x^2-1}}{3d^2}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow 219$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{\frac{1}{2}\left(2\sqrt{d^2x^2-1}\left(3a+\frac{2c}{d^2}\right)+\frac{3b\text{arctanh}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{d}\right)+\frac{3}{2}bx\sqrt{d^2x^2-1}}{3d^2}+\frac{cx^2\sqrt{d^2x^2-1}}{3d^2}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

input
```
Int[(x*(a + b*x + c*x^2))/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

output
```
(Sqrt[-1 + d^2*x^2]*((c*x^2*Sqrt[-1 + d^2*x^2])/(3*d^2) + ((3*b*x*Sqrt[-1
+ d^2*x^2])/2 + (2*(3*a + (2*c)/d^2)*Sqrt[-1 + d^2*x^2] + (3*b*ArcTanh[(d*
x)/Sqrt[-1 + d^2*x^2]])/d)/2)/(3*d^2)))/(Sqrt[-1 + d*x]*Sqrt[1 + d*x])
```

3.34.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] && !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 2113
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m])   Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a
*d, 0] && EqQ[m, n] && !IntegerQ[m]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

3.34.4 Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.24

method	result
risch	$\frac{\left(2 c\,d^{2} x^{2}+3 b\,d^{2} x+6 a\,d^{2}+4 c\right) \sqrt{d x+1}\,\sqrt{d x-1}}{6 d^{4}}+\frac{b \ln\left(\frac{x\,d^{2}}{\sqrt{d^{2}}}+\sqrt{d^{2} x^{2}-1}\right) \sqrt{\left(d x+1\right) \left(d x-1\right)}}{2 d^{2} \sqrt{d^{2}}\,\sqrt{d x-1}\,\sqrt{d x+1}}$
default	$\frac{\sqrt{d x-1}\,\sqrt{d x+1}\,\left(2\,\mathrm{csgn}\left(d\right) c\,d^{2} x^{2} \sqrt{d^{2} x^{2}-1}+3 \sqrt{d^{2} x^{2}-1}\,\mathrm{csgn}\left(d\right) b\,d^{2} x+6 \sqrt{d^{2} x^{2}-1}\,\mathrm{csgn}\left(d\right) a\,d^{2}+4 \sqrt{d^{2} x^{2}-1}\,\mathrm{csgn}\left(d\right) c+3 \ln\left(\left(\sqrt{d^{2} x^{2}-\cdots\right.\right.\right.}{6 d^{4} \sqrt{d^{2} x^{2}-1}}$

input
```
int(x*(c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*c*d^2*x^2+3*b*d^2*x+6*a*d^2+4*c)*(d*x+1)^(1/2)*(d*x-1)^(1/2)/d^4+1/
2*b/d^2*ln(x*d^2/(d^2)^(1/2)+(d^2*x^2-1)^(1/2))/(d^2)^(1/2)*((d*x+1)*(d*x-
1))^(1/2)/(d*x-1)^(1/2)/(d*x+1)^(1/2)
```

3.34.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.84

$$\int \frac{x(a+bx+cx^2)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = -\frac{3\,bd\log\left(-dx+\sqrt{dx+1}\sqrt{dx-1}\right)-(2\,cd^2x^2+3\,bd^2x+6\,ad^2+4\,c)\sqrt{dx+1}\sqrt{dx-1}}{6\,d^4}$$

input
```
integrate(x*(c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas
")
```

output
```
-1/6*(3*b*d*log(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)) - (2*c*d^2*x^2 + 3*b*d
^2*x + 6*a*d^2 + 4*c)*sqrt(d*x + 1)*sqrt(d*x - 1))/d^4
```

3.34.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x(a+bx+cx^2)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \text{Timed out}$$

input
```
integrate(x*(c*x**2+b*x+a)/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

output
```
Timed out
```

3.34.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.15

$$\int \frac{x(a+bx+cx^2)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{d^2x^2-1}cx^2}{3\,d^2} + \frac{\sqrt{d^2x^2-1}bx}{2\,d^2} + \frac{\sqrt{d^2x^2-1}a}{d^2} + \frac{b\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}d\right)}{2\,d^3} + \frac{2\,\sqrt{d^2x^2-1}c}{3\,d^4}$$

input
```
integrate(x*(c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima
")
```

output
```
1/3*sqrt(d^2*x^2 - 1)*c*x^2/d^2 + 1/2*sqrt(d^2*x^2 - 1)*b*x/d^2 + sqrt(d^2
*x^2 - 1)*a/d^2 + 1/2*b*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*d)/d^3 + 2/3*sqr
t(d^2*x^2 - 1)*c/d^4
```

3.34.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.21

$$\int \frac{x(a+bx+cx^2)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{dx+1}\sqrt{dx-1}\left((dx+1)\left(\frac{2\,(dx+1)c}{d^3}+\frac{3\,bd^{10}-4\,cd^9}{d^{12}}\right)+\frac{3\,(2\,ad^{11}-bd^{10}+2\,cd^9)}{d^{12}}\right)-\frac{6\,b\log\left(\sqrt{dx+1}-\sqrt{dx-1}\right)}{d^2}}{6\,d}$$

input
```
integrate(x*(c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

output
```
1/6*(sqrt(d*x + 1)*sqrt(d*x - 1)*((d*x + 1)*(2*(d*x + 1)*c/d^3 + (3*b*d^10
 - 4*c*d^9)/d^12) + 3*(2*a*d^11 - b*d^10 + 2*c*d^9)/d^12) - 6*b*log(sqrt(d
*x + 1) - sqrt(d*x - 1))/d^2)/d
```

3.34.9 Mupad [B] (verification not implemented)

Time = 16.03 (sec) , antiderivative size = 318, normalized size of antiderivative = 3.66

$$\int \frac{x(a+bx+cx^2)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{d\,x-1}\,\left(\frac{2\,c}{3\,d^4}+\frac{c\,x^3}{3\,d}+\frac{c\,x^2}{3\,d^2}+\frac{2\,c\,x}{3\,d^3}\right)}{\sqrt{d\,x+1}}+\frac{2\,b\,\mathrm{atanh}\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)}{d^3} - \frac{\frac{14\,b\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)^3}{\left(\sqrt{d\,x+1}-1\right)^3}+\frac{14\,b\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)^5}{\left(\sqrt{d\,x+1}-1\right)^5}+\frac{2\,b\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)^7}{\left(\sqrt{d\,x+1}-1\right)^7}+\frac{2\,b\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)}{\sqrt{d\,x+1}-1}}{d^3-\frac{4\,d^3\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+\frac{6\,d^3\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)^4}{\left(\sqrt{d\,x+1}-1\right)^4}-\frac{4\,d^3\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)^6}{\left(\sqrt{d\,x+1}-1\right)^6}+\frac{d^3\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)^8}{\left(\sqrt{d\,x+1}-1\right)^8}} + \frac{a\,\sqrt{d\,x-1}\,\sqrt{d\,x+1}}{d^2}$$

input
```
int((x*(a + b*x + c*x^2))/((d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

output
```
(2*b*atanh(((d*x - 1)^(1/2) - 1i)/((d*x + 1)^(1/2) - 1)))/d^3 - ((14*b*((d
*x - 1)^(1/2) - 1i)^3)/((d*x + 1)^(1/2) - 1)^3 + (14*b*((d*x - 1)^(1/2) -
1i)^5)/((d*x + 1)^(1/2) - 1)^5 + (2*b*((d*x - 1)^(1/2) - 1i)^7)/((d*x + 1)
^(1/2) - 1)^7 + (2*b*((d*x - 1)^(1/2) - 1i))/((d*x + 1)^(1/2) - 1))/(d^3 -
 (4*d^3*((d*x - 1)^(1/2) - 1i)^2)/((d*x + 1)^(1/2) - 1)^2 + (6*d^3*((d*x -
 1)^(1/2) - 1i)^4)/((d*x + 1)^(1/2) - 1)^4 - (4*d^3*((d*x - 1)^(1/2) - 1i)
^6)/((d*x + 1)^(1/2) - 1)^6 + (d^3*((d*x - 1)^(1/2) - 1i)^8)/((d*x + 1)^(1
/2) - 1)^8) + ((d*x - 1)^(1/2)*((2*c)/(3*d^4) + (c*x^3)/(3*d) + (c*x^2)/(3
*d^2) + (2*c*x)/(3*d^3)))/(d*x + 1)^(1/2) + (a*(d*x - 1)^(1/2)*(d*x + 1)^(
1/2))/d^2
```

3.35 $\int \frac{a+bx+cx^2}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx$

3.35.1 Optimal result

Integrand size = 29, antiderivative size = 52

$$\int \frac{a+bx+cx^2}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx=\frac{(2b+cx)\sqrt{-1+dx}\sqrt{1+dx}}{2d^2}+\frac{(c+2ad^2)\operatorname{arccosh}(dx)}{2d^3}$$

output
```
1/2*(2*a*d^2+c)*arccosh(d*x)/d^3+1/2*(c*x+2*b)*(d*x-1)^(1/2)*(d*x+1)^(1/2)
/d^2
```

3.35.2 Mathematica [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.21

$$\int \frac{a+bx+cx^2}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx=\frac{d(2b+cx)\sqrt{-1+dx}\sqrt{1+dx}+2(c+2ad^2)\operatorname{arctanh}\left(\sqrt{\frac{-1+dx}{1+dx}}\right)}{2d^3}$$

input
```
Integrate[(a + b*x + c*x^2)/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

output
```
(d*(2*b + c*x)*Sqrt[-1 + d*x]*Sqrt[1 + d*x] + 2*(c + 2*a*d^2)*ArcTanh[Sqrt
[(-1 + d*x)/(1 + d*x)]])/(2*d^3)
```

3.35.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.35, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1189, 83, 646, 43}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{\sqrt{dx-1}\sqrt{dx+1}}dx$$

$$\Big\downarrow \text{1189}$$

$$\int \frac{cx^2+a}{\sqrt{dx-1}\sqrt{dx+1}}dx+b\int \frac{x}{\sqrt{dx-1}\sqrt{dx+1}}dx$$

$$\Big\downarrow \text{83}$$

$$\int \frac{cx^2+a}{\sqrt{dx-1}\sqrt{dx+1}}dx+\frac{b\sqrt{dx-1}\sqrt{dx+1}}{d^2}$$

$$\Big\downarrow \text{646}$$

$$\frac{(2ad^2+c)\int \frac{1}{\sqrt{dx-1}\sqrt{dx+1}}dx}{2d^2}+\frac{b\sqrt{dx-1}\sqrt{dx+1}}{d^2}+\frac{cx\sqrt{dx-1}\sqrt{dx+1}}{2d^2}$$

$$\Big\downarrow \text{43}$$

$$\frac{(2ad^2+c)\operatorname{arccosh}(dx)}{2d^3}+\frac{b\sqrt{dx-1}\sqrt{dx+1}}{d^2}+\frac{cx\sqrt{dx-1}\sqrt{dx+1}}{2d^2}$$

input
```
Int[(a + b*x + c*x^2)/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

output
```
(b*Sqrt[-1 + d*x]*Sqrt[1 + d*x])/d^2 + (c*x*Sqrt[-1 + d*x]*Sqrt[1 + d*x])/
(2*d^2) + ((c + 2*a*d^2)*ArcCosh[d*x])/(2*d^3)
```

3.35.3.1 Defintions of rubi rules used

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 646
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2), x_Symbol] :> Simp[b*x*(c + d*x)^(m + 1)*((e + f*x)^(n + 1)/(d*f*(2*m +
 3))), x] - Simp[(b*c*e - a*d*f*(2*m + 3))/(d*f*(2*m + 3))   Int[(c + d*x)^
m*(e + f*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m, n] &&
 EqQ[d*e + c*f, 0] &&  !LtQ[m, -1]
```

rule 1189
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2), x_Symbol] :> Simp[b   Int[x*(d + e*x)^m*(f + g*x)^n, x],
x] + Int[(d + e*x)^m*(f + g*x)^n*(a + c*x^2), x] /; FreeQ[{a, b, c, d, e, f
, g, m, n}, x] && EqQ[m, n] && EqQ[e*f + d*g, 0]
```

3.35.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(44) = 88$.

Time = 5.70 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.85

method	result
risch	$\frac{(cx+2b)\sqrt{dx-1}\,\sqrt{dx+1}}{2d^2}+\frac{(2a\,d^2+c)\ln\left(\frac{x\,d^2}{\sqrt{d^2}}+\sqrt{d^2x^2-1}\right)\sqrt{(dx+1)(dx-1)}}{2d^2\sqrt{d^2}\,\sqrt{dx-1}\,\sqrt{dx+1}}$
default	$\frac{\sqrt{dx-1}\,\sqrt{dx+1}\,\left(\sqrt{d^2x^2-1}\,\mathrm{csgn}(d)dcx+2\ln\left(\left(\sqrt{d^2x^2-1}\,\mathrm{csgn}(d)+dx\right)\mathrm{csgn}(d)\right)a\,d^2+2\,\mathrm{csgn}(d)d\sqrt{d^2x^2-1}\,b+\ln\left(\left(\sqrt{d^2x^2-1}\,\mathrm{csgn}(d\right.\right.\right.}{2d^3\sqrt{d^2x^2-1}}$

input
```
int((c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(c*x+2*b)*(d*x-1)^(1/2)*(d*x+1)^(1/2)/d^2+1/2*(2*a*d^2+c)/d^2*ln(x*d^2
/(d^2)^(1/2)+(d^2*x^2-1)^(1/2))/(d^2)^(1/2)*((d*x+1)*(d*x-1))^(1/2)/(d*x-1
)^(1/2)/(d*x+1)^(1/2)
```

3.35.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.17

$$\int \frac{a+bx+cx^2}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{(cdx+2\,bd)\sqrt{dx+1}\sqrt{dx-1}-(2\,ad^2+c)\log\left(-dx+\sqrt{dx+1}\sqrt{dx-1}\right)}{2\,d^3}$$

input
```
integrate((c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*((c*d*x + 2*b*d)*sqrt(d*x + 1)*sqrt(d*x - 1) - (2*a*d^2 + c)*log(-d*x
+ sqrt(d*x + 1)*sqrt(d*x - 1)))/d^3
```

3.35.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx+cx^2}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

output
```
Timed out
```

3.35.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(44) = 88$.

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.73

$$\int \frac{a+bx+cx^2}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{a\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}d\right)}{d} + \frac{\sqrt{d^2x^2-1}cx}{2\,d^2} + \frac{\sqrt{d^2x^2-1}b}{d^2} + \frac{c\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}d\right)}{2\,d^3}$$

input
```
integrate((c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima")
```

output
```
a*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*d)/d + 1/2*sqrt(d^2*x^2 - 1)*c*x/d^2 +
 sqrt(d^2*x^2 - 1)*b/d^2 + 1/2*c*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*d)/d^3
```

3.35.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.54

$$\int \frac{a+bx+cx^2}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{dx+1}\sqrt{dx-1}\left(\frac{(dx+1)c}{d^2}+\frac{2\,bd^5-cd^4}{d^6}\right)-\frac{2\,(2\,ad^2+c)\log\left(\sqrt{dx+1}-\sqrt{dx-1}\right)}{d^2}}{2\,d}$$

input
```
integrate((c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

output
```
1/2*(sqrt(d*x + 1)*sqrt(d*x - 1)*((d*x + 1)*c/d^2 + (2*b*d^5 - c*d^4)/d^6)
 - 2*(2*a*d^2 + c)*log(sqrt(d*x + 1) - sqrt(d*x - 1))/d^2)/d
```

3.35.9 Mupad [B] (verification not implemented)

Time = 20.38 (sec) , antiderivative size = 312, normalized size of antiderivative = 6.00

$$\int \frac{a+bx+cx^2}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{b\sqrt{d\,x-1}\,\sqrt{d\,x+1}}{d^2}+\frac{2\,c\,\mathrm{atanh}\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)}{d^3}-\frac{4\,a\,\mathrm{atan}\left(\frac{d\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)}{\left(\sqrt{d\,x+1}-1\right)\,\sqrt{-d^2}}\right)}{\sqrt{-d^2}} - \frac{\frac{14\,c\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^3}{{\left(\sqrt{d\,x+1}-1\right)}^3}+\frac{14\,c\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^5}{{\left(\sqrt{d\,x+1}-1\right)}^5}+\frac{2\,c\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^7}{{\left(\sqrt{d\,x+1}-1\right)}^7}+\frac{2\,c\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)}{\sqrt{d\,x+1}-1}}{d^3-\frac{4\,d^3\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+\frac{6\,d^3\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^4}{{\left(\sqrt{d\,x+1}-1\right)}^4}-\frac{4\,d^3\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^6}{{\left(\sqrt{d\,x+1}-1\right)}^6}+\frac{d^3\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^8}{{\left(\sqrt{d\,x+1}-1\right)}^8}}$$

input
```
int((a + b*x + c*x^2)/((d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

output
```
(2*c*atanh(((d*x - 1)^(1/2) - 1i)/((d*x + 1)^(1/2) - 1)))/d^3 - ((14*c*((d
*x - 1)^(1/2) - 1i)^3)/((d*x + 1)^(1/2) - 1)^3 + (14*c*((d*x - 1)^(1/2) -
1i)^5)/((d*x + 1)^(1/2) - 1)^5 + (2*c*((d*x - 1)^(1/2) - 1i)^7)/((d*x + 1)
^(1/2) - 1)^7 + (2*c*((d*x - 1)^(1/2) - 1i))/((d*x + 1)^(1/2) - 1))/(d^3 -
 (4*d^3*((d*x - 1)^(1/2) - 1i)^2)/((d*x + 1)^(1/2) - 1)^2 + (6*d^3*((d*x -
 1)^(1/2) - 1i)^4)/((d*x + 1)^(1/2) - 1)^4 - (4*d^3*((d*x - 1)^(1/2) - 1i)
^6)/((d*x + 1)^(1/2) - 1)^6 + (d^3*((d*x - 1)^(1/2) - 1i)^8)/((d*x + 1)^(1
/2) - 1)^8) - (4*a*atan((d*((d*x - 1)^(1/2) - 1i))/(((d*x + 1)^(1/2) - 1)*
(-d^2)^(1/2))))/(-d^2)^(1/2) + (b*(d*x - 1)^(1/2)*(d*x + 1)^(1/2))/d^2
```

3.36 $\int \frac{a+bx+cx^2}{x\sqrt{-1+dx}\sqrt{1+dx}}\,dx$

3.36.1 Optimal result

Integrand size = 32, antiderivative size = 55

$$\int \frac{a+bx+cx^2}{x\sqrt{-1+dx}\sqrt{1+dx}}\,dx=\frac{c\sqrt{-1+dx}\sqrt{1+dx}}{d^2}+\frac{b\operatorname{arccosh}(dx)}{d}+a\arctan\left(\sqrt{-1+dx}\sqrt{1+dx}\right)$$

output
```
b*arccosh(d*x)/d+a*arctan((d*x-1)^(1/2)*(d*x+1)^(1/2))+c*(d*x-1)^(1/2)*(d*
x+1)^(1/2)/d^2
```

3.36.2 Mathematica [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.25

$$\int \frac{a+bx+cx^2}{x\sqrt{-1+dx}\sqrt{1+dx}}\,dx=\frac{c\sqrt{-1+dx}\sqrt{1+dx}}{d^2}+2a\arctan\left(\sqrt{\frac{-1+dx}{1+dx}}\right)+\frac{2b\operatorname{arctanh}\left(\sqrt{\frac{-1+dx}{1+dx}}\right)}{d}$$

input
```
Integrate[(a + b*x + c*x^2)/(x*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

output
```
(c*Sqrt[-1 + d*x]*Sqrt[1 + d*x])/d^2 + 2*a*ArcTan[Sqrt[(-1 + d*x)/(1 + d*x
)]] + (2*b*ArcTanh[Sqrt[(-1 + d*x)/(1 + d*x)]])/d
```

3.36.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.62, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {2113, 2340, 27, 538, 224, 219, 243, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{x\sqrt{dx-1}\sqrt{dx+1}}dx$$

$$\Big\downarrow \text{2113}$$

$$\frac{\sqrt{d^2x^2-1}\int \frac{cx^2+bx+a}{x\sqrt{d^2x^2-1}}dx}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{2340}$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{\int \frac{d^2(a+bx)}{x\sqrt{d^2x^2-1}}dx}{d^2}+\frac{c\sqrt{d^2x^2-1}}{d^2}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d^2x^2-1}\left(\int \frac{a+bx}{x\sqrt{d^2x^2-1}}dx+\frac{c\sqrt{d^2x^2-1}}{d^2}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{538}$$

$$\frac{\sqrt{d^2x^2-1}\left(a\int \frac{1}{x\sqrt{d^2x^2-1}}dx+b\int \frac{1}{\sqrt{d^2x^2-1}}dx+\frac{c\sqrt{d^2x^2-1}}{d^2}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\sqrt{d^2x^2-1}\left(a\int \frac{1}{x\sqrt{d^2x^2-1}}dx+b\int \frac{1}{1-\frac{d^2x^2}{d^2x^2-1}}d\frac{x}{\sqrt{d^2x^2-1}}+\frac{c\sqrt{d^2x^2-1}}{d^2}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{d^2x^2-1}\left(a\int \frac{1}{x\sqrt{d^2x^2-1}}dx+\frac{b\,\text{arctanh}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{d}+\frac{c\sqrt{d^2x^2-1}}{d^2}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{243}$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{1}{2}a\int\frac{1}{x^2\sqrt{d^2x^2-1}}dx^2+\frac{b\text{arctanh}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{d}+\frac{c\sqrt{d^2x^2-1}}{d^2}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \ 73$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{a\int\frac{1}{\frac{x^4}{d^2}+\frac{1}{d^2}}d\sqrt{d^2x^2-1}}{d^2}+\frac{b\text{arctanh}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{d}+\frac{c\sqrt{d^2x^2-1}}{d^2}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \ 218$$

$$\frac{\sqrt{d^2x^2-1}\left(a\arctan\left(\sqrt{d^2x^2-1}\right)+\frac{b\text{arctanh}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{d}+\frac{c\sqrt{d^2x^2-1}}{d^2}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

input
```
Int[(a + b*x + c*x^2)/(x*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

output
```
(Sqrt[-1 + d^2*x^2]*((c*Sqrt[-1 + d^2*x^2])/d^2 + a*ArcTan[Sqrt[-1 + d^2*x
^2]] + (b*ArcTanh[(d*x)/Sqrt[-1 + d^2*x^2]])/d))/(Sqrt[-1 + d*x]*Sqrt[1 +
d*x])
```

3.36.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2113
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m])   Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a
*d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

3.36. $\int \frac{a+bx+cx^2}{x\sqrt{-1+dx}\sqrt{1+dx}}\,dx$

3.36.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.65 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.73

method	result
default	$\frac{\left(-\operatorname{csgn}(d)\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)a\,d^2+\sqrt{d^2x^2-1}\,\operatorname{csgn}(d)c+\ln\left(\left(\sqrt{(dx+1)(dx-1)}\,\operatorname{csgn}(d)+dx\right)\operatorname{csgn}(d)\right)bd\right)\sqrt{dx-1}\,\sqrt{dx+1}\,\operatorname{csgn}(d)}{d^2\sqrt{d^2x^2-1}}$

input
```
int((c*x^2+b*x+a)/x/(d*x-1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-csgn(d)*arctan(1/(d^2*x^2-1)^(1/2))*a*d^2+(d^2*x^2-1)^(1/2)*csgn(d)*c+ln
((((d*x+1)*(d*x-1))^(1/2)*csgn(d)+d*x)*csgn(d))*b*d)*(d*x-1)^(1/2)*(d*x+1)
^(1/2)/d^2*csgn(d)/(d^2*x^2-1)^(1/2)
```

3.36.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.33

$$\int \frac{a+bx+cx^2}{x\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{2\,ad^2\arctan\left(-dx+\sqrt{dx+1}\sqrt{dx-1}\right)-bd\log\left(-dx+\sqrt{dx+1}\sqrt{dx-1}\right)+\sqrt{dx+1}\sqrt{dx-1}c}{d^2}$$

input
```
integrate((c*x^2+b*x+a)/x/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas
")
```

output
```
(2*a*d^2*arctan(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)) - b*d*log(-d*x + sqrt(
d*x + 1)*sqrt(d*x - 1)) + sqrt(d*x + 1)*sqrt(d*x - 1)*c)/d^2
```

3.36.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 29.13 (sec) , antiderivative size = 240, normalized size of antiderivative = 4.36

$$\int \frac{a+bx+cx^2}{x\sqrt{-1+dx}\sqrt{1+dx}}\,dx = -\frac{aG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2} \\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0 \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{iaG_{6,6}^{2,6}\left(\begin{matrix} 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0 \end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{bG_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1 \\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d} - \frac{ibG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0 \end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d} + \frac{cG_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1 \\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2} + \frac{icG_{6,6}^{2,6}\left(\begin{matrix} -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0 \end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}$$

input
```
integrate((c*x**2+b*x+a)/x/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

output
```
-a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)),
1/(d**2*x**2))/(4*pi**(3/2)) + I*a*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()),
 ((1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/
2)) + b*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0),
 ()), 1/(d**2*x**2))/(4*pi**(3/2)*d) - I*b*meijerg(((-1/2, -1/4, 0, 1/4, 1
/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(2*I*pi)/(d**2*x**2)
)/(4*pi**(3/2)*d) + c*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4,
 0, 1/4, 1/2, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**2) + I*c*meijerg(((-
1, -3/4, -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_
polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**2)
```

3.36.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \frac{a+bx+cx^2}{x\sqrt{-1+dx}\sqrt{1+dx}}\,dx = -a\arcsin\left(\frac{1}{d|x|}\right)+\frac{b\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}d\right)}{d}+\frac{\sqrt{d^2x^2-1}c}{d^2}$$

input
```
integrate((c*x^2+b*x+a)/x/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima
")
```

output
```
-a*arcsin(1/(d*abs(x))) + b*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*d)/d + sqrt(
d^2*x^2 - 1)*c/d^2
```

3.36.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.29

$$\int \frac{a+bx+cx^2}{x\sqrt{-1+dx}\sqrt{1+dx}}\,dx = -2\,a\arctan\left(\frac{1}{2}\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right) - \frac{b\log\left(\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right)}{d}+\frac{\sqrt{dx+1}\sqrt{dx-1}c}{d^2}$$

input
```
integrate((c*x^2+b*x+a)/x/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

output
```
-2*a*arctan(1/2*(sqrt(d*x + 1) - sqrt(d*x - 1))^2) - b*log((sqrt(d*x + 1)
- sqrt(d*x - 1))^2)/d + sqrt(d*x + 1)*sqrt(d*x - 1)*c/d^2
```

3.36.9 Mupad [B] (verification not implemented)

Time = 5.82 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.15

$$\int \frac{a+bx+cx^2}{x\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{c\sqrt{d\,x-1}\,\sqrt{d\,x+1}}{d^2} - \frac{4\,b\,\mathrm{atan}\left(\frac{d\,(\sqrt{d\,x-1}-\mathrm{i})}{(\sqrt{d\,x+1}-1)\,\sqrt{-d^2}}\right)}{\sqrt{-d^2}} - a\left(\ln\left(\frac{\left(\sqrt{d\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+1\right) - \ln\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)\right)\,1\mathrm{i}$$

input
```
int((a + b*x + c*x^2)/(x*(d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

output
```
(c*(d*x - 1)^(1/2)*(d*x + 1)^(1/2))/d^2 - (4*b*atan((d*((d*x - 1)^(1/2) -
1i))/(((d*x + 1)^(1/2) - 1)*(-d^2)^(1/2))))/(-d^2)^(1/2) - a*(log(((d*x -
1)^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + 1) - log(((d*x - 1)^(1/2) - 1i)
/((d*x + 1)^(1/2) - 1)))*1i
```

3.37 $\int \frac{a+bx+cx^2}{x^2\sqrt{-1+dx}\sqrt{1+dx}}\,dx$

3.37.1 Optimal result

Integrand size = 32, antiderivative size = 55

$$\int \frac{a+bx+cx^2}{x^2\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{a\sqrt{-1+dx}\sqrt{1+dx}}{x}+\frac{c\,\text{arccosh}(dx)}{d} + b\arctan\left(\sqrt{-1+dx}\sqrt{1+dx}\right)$$

output
```
c*arccosh(d*x)/d+b*arctan((d*x-1)^(1/2)*(d*x+1)^(1/2))+a*(d*x-1)^(1/2)*(d*
x+1)^(1/2)/x
```

3.37.2 Mathematica [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.25

$$\int \frac{a+bx+cx^2}{x^2\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{a\sqrt{-1+dx}\sqrt{1+dx}}{x}+2b\arctan\left(\sqrt{\frac{-1+dx}{1+dx}}\right) + \frac{2c\,\text{arctanh}\left(\sqrt{\frac{-1+dx}{1+dx}}\right)}{d}$$

input
```
Integrate[(a + b*x + c*x^2)/(x^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

output
```
(a*Sqrt[-1 + d*x]*Sqrt[1 + d*x])/x + 2*b*ArcTan[Sqrt[(-1 + d*x)/(1 + d*x)]
] + (2*c*ArcTanh[Sqrt[(-1 + d*x)/(1 + d*x)]])/d
```

3.37.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.62, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2113, 2338, 538, 224, 219, 243, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{x^2\sqrt{dx-1}\sqrt{dx+1}}dx$$

$$\Big\downarrow \text{2113}$$

$$\frac{\sqrt{d^2x^2-1}\int \frac{cx^2+bx+a}{x^2\sqrt{d^2x^2-1}}dx}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{2338}$$

$$\frac{\sqrt{d^2x^2-1}\left(\int \frac{b+cx}{x\sqrt{d^2x^2-1}}dx+\frac{a\sqrt{d^2x^2-1}}{x}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{538}$$

$$\frac{\sqrt{d^2x^2-1}\left(b\int \frac{1}{x\sqrt{d^2x^2-1}}dx+c\int \frac{1}{\sqrt{d^2x^2-1}}dx+\frac{a\sqrt{d^2x^2-1}}{x}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\sqrt{d^2x^2-1}\left(b\int \frac{1}{x\sqrt{d^2x^2-1}}dx+c\int \frac{1}{1-\frac{d^2x^2}{d^2x^2-1}}d\frac{x}{\sqrt{d^2x^2-1}}+\frac{a\sqrt{d^2x^2-1}}{x}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{d^2x^2-1}\left(b\int \frac{1}{x\sqrt{d^2x^2-1}}dx+\frac{a\sqrt{d^2x^2-1}}{x}+\frac{c\,\text{arctanh}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{d}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{243}$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{1}{2}b\int \frac{1}{x^2\sqrt{d^2x^2-1}}dx^2+\frac{a\sqrt{d^2x^2-1}}{x}+\frac{c\,\text{arctanh}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{d}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{b\int \frac{1}{\frac{x^4}{d^2}+\frac{1}{d^2}}d\sqrt{d^2x^2-1}}{d^2}+\frac{a\sqrt{d^2x^2-1}}{x}+\frac{c\text{arctanh}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{d}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{a\sqrt{d^2x^2-1}}{x}+b\arctan\left(\sqrt{d^2x^2-1}\right)+\frac{c\text{arctanh}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{d}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

input
```
Int[(a + b*x + c*x^2)/(x^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

output
```
(Sqrt[-1 + d^2*x^2]*((a*Sqrt[-1 + d^2*x^2])/x + b*ArcTan[Sqrt[-1 + d^2*x^2
]] + (c*ArcTanh[(d*x)/Sqrt[-1 + d^2*x^2]])/d))/(Sqrt[-1 + d*x]*Sqrt[1 + d*
x])
```

3.37.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2113
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m])   Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a
*d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

3.37.4 Maple [A] (verified)

Time = 1.65 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.73

method	result	siz
risch	$\frac{a\sqrt{dx-1}\,\sqrt{dx+1}}{x}+\frac{\left(\frac{c\ln\left(\frac{x d^{2}}{\sqrt{d^{2}}}+\sqrt{d^{2}x^{2}-1}\right)}{\sqrt{d^{2}}}-b\arctan\left(\frac{1}{\sqrt{d^{2}x^{2}-1}}\right)\right)\sqrt{(dx+1)(dx-1)}}{\sqrt{dx-1}\,\sqrt{dx+1}}$	95
default	$\frac{\left(-\arctan\left(\frac{1}{\sqrt{d^{2}x^{2}-1}}\right)\operatorname{csgn}(d)dbx+\sqrt{d^{2}x^{2}-1}\,\operatorname{csgn}(d)da+\ln\left(\left(\sqrt{d^{2}x^{2}-1}\,\operatorname{csgn}(d)+dx\right)\operatorname{csgn}(d)\right)cx\right)\sqrt{dx-1}\,\sqrt{dx+1}\,\operatorname{csgn}(d)}{\sqrt{d^{2}x^{2}-1}\,xd}$	96

input
```
int((c*x^2+b*x+a)/x^2/(d*x-1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
a*(d*x-1)^(1/2)*(d*x+1)^(1/2)/x+(c*ln(x*d^2/(d^2)^(1/2)+(d^2*x^2-1)^(1/2))
/(d^2)^(1/2)-b*arctan(1/(d^2*x^2-1)^(1/2)))*((d*x+1)*(d*x-1))^(1/2)/(d*x-1
)^(1/2)/(d*x+1)^(1/2)
```

3.37.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.49

$$\int \frac{a+bx+cx^2}{x^2\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{ad^2x+2\,bdx\arctan\left(-dx+\sqrt{dx+1}\sqrt{dx-1}\right)+\sqrt{dx+1}\sqrt{dx-1}ad-cx\log\left(-dx+\sqrt{dx+1}\sqrt{dx-1}\right)}{dx}$$

input
```
integrate((c*x^2+b*x+a)/x^2/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fric
as")
```

output
```
(a*d^2*x + 2*b*d*x*arctan(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)) + sqrt(d*x +
 1)*sqrt(d*x - 1)*a*d - c*x*log(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)))/(d*x)
```

3.37.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 28.63 (sec) , antiderivative size = 216, normalized size of antiderivative = 3.93

$$\int \frac{a+bx+cx^2}{x^2\sqrt{-1+dx}\sqrt{1+dx}}\,dx = -\frac{adG_{6,6}^{5,3}\left(\begin{matrix} \frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2 \\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0 \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{iadG_{6,6}^{2,6}\left(\begin{matrix} \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0 \end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{bG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2} \\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0 \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{ibG_{6,6}^{2,6}\left(\begin{matrix} 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0 \end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{cG_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1 \\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d} - \frac{icG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0 \end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}$$

input
```
integrate((c*x**2+b*x+a)/x**2/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

output
```
-a*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,))
, 1/(d**2*x**2))/(4*pi**(3/2)) - I*a*d*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1)
, ()), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*pi*
*(3/2)) - b*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2)
, (0,)), 1/(d**2*x**2))/(4*pi**(3/2)) + I*b*meijerg(((0, 1/4, 1/2, 3/4, 1,
 1), ()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(
4*pi**(3/2)) + c*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/
4, 1, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d) - I*c*meijerg(((-1/2, -1/4,
0, 1/4, 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(2*I*pi)/(d
**2*x**2))/(4*pi**(3/2)*d)
```

3.37. $\int \frac{a+bx+cx^2}{x^2\sqrt{-1+dx}\sqrt{1+dx}}\,dx$

3.37.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \frac{a+bx+cx^2}{x^2\sqrt{-1+dx}\sqrt{1+dx}}\,dx = -b\arcsin\left(\frac{1}{d|x|}\right)+\frac{c\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}d\right)}{d}+\frac{\sqrt{d^2x^2-1}a}{x}$$

input
```
integrate((c*x^2+b*x+a)/x^2/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxi
ma")
```

output
```
-b*arcsin(1/(d*abs(x))) + c*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*d)/d + sqrt(
d^2*x^2 - 1)*a/x
```

3.37.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.51

$$\int \frac{a+bx+cx^2}{x^2\sqrt{-1+dx}\sqrt{1+dx}}\,dx = -\frac{2\,bd\arctan\left(\frac{1}{2}\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right)-\frac{8\,ad^2}{\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^4+4}+c\log\left(\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right)}{d}$$

input
```
integrate((c*x^2+b*x+a)/x^2/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac
")
```

output
```
-(2*b*d*arctan(1/2*(sqrt(d*x + 1) - sqrt(d*x - 1))^2) - 8*a*d^2/((sqrt(d*x
 + 1) - sqrt(d*x - 1))^4 + 4) + c*log((sqrt(d*x + 1) - sqrt(d*x - 1))^2))/
d
```

3.37.9 Mupad [B] (verification not implemented)

Time = 5.50 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.15

$$\int \frac{a+bx+cx^2}{x^2\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{a\sqrt{d\,x-1}\,\sqrt{d\,x+1}}{x}-\frac{4\,c\,\mathrm{atan}\left(\frac{d\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)}{\left(\sqrt{d\,x+1}-1\right)\,\sqrt{-d^2}}\right)}{\sqrt{-d^2}} - b\left(\ln\left(\frac{\left(\sqrt{d\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+1\right)-\ln\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)\right)\,1\mathrm{i}$$

input
```
int((a + b*x + c*x^2)/(x^2*(d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

output
```
(a*(d*x - 1)^(1/2)*(d*x + 1)^(1/2))/x - (4*c*atan((d*((d*x - 1)^(1/2) - 1i
))/(((d*x + 1)^(1/2) - 1)*(-d^2)^(1/2))))/(-d^2)^(1/2) - b*(log(((d*x - 1)
^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + 1) - log(((d*x - 1)^(1/2) - 1i)/(
(d*x + 1)^(1/2) - 1)))*1i
```

3.37. $\int \frac{a+bx+cx^2}{x^2\sqrt{-1+dx}\sqrt{1+dx}}\,dx$

3.38 $\int \frac{a+bx+cx^2}{x^3\sqrt{-1+dx}\sqrt{1+dx}}\,dx$

3.38.1 Optimal result

Integrand size = 32, antiderivative size = 83

$$\int \frac{a+bx+cx^2}{x^3\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{a\sqrt{-1+dx}\sqrt{1+dx}}{2x^2}+\frac{b\sqrt{-1+dx}\sqrt{1+dx}}{x} + \frac{1}{2}(2c+ad^2)\arctan\left(\sqrt{-1+dx}\sqrt{1+dx}\right)$$

output
```
1/2*(a*d^2+2*c)*arctan((d*x-1)^(1/2)*(d*x+1)^(1/2))+1/2*a*(d*x-1)^(1/2)*(d
*x+1)^(1/2)/x^2+b*(d*x-1)^(1/2)*(d*x+1)^(1/2)/x
```

3.38.2 Mathematica [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.72

$$\int \frac{a+bx+cx^2}{x^3\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{(a+2bx)\sqrt{-1+dx}\sqrt{1+dx}}{2x^2}+(2c+ad^2)\arctan\left(\sqrt{\frac{-1+dx}{1+dx}}\right)$$

input
```
Integrate[(a + b*x + c*x^2)/(x^3*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

output
```
((a + 2*b*x)*Sqrt[-1 + d*x]*Sqrt[1 + d*x])/(2*x^2) + (2*c + a*d^2)*ArcTan[
Sqrt[(-1 + d*x)/(1 + d*x)]]
```

3.38.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2113, 2338, 534, 243, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{x^3\sqrt{dx-1}\sqrt{dx+1}}dx$$

$$\Big\downarrow \text{2113}$$

$$\frac{\sqrt{d^2x^2-1}\int \frac{cx^2+bx+a}{x^3\sqrt{d^2x^2-1}}dx}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{2338}$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{1}{2}\int \frac{2b+(ad^2+2c)x}{x^2\sqrt{d^2x^2-1}}dx+\frac{a\sqrt{d^2x^2-1}}{2x^2}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{534}$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{1}{2}\left(\left(ad^2+2c\right)\int \frac{1}{x\sqrt{d^2x^2-1}}dx+\frac{2b\sqrt{d^2x^2-1}}{x}\right)+\frac{a\sqrt{d^2x^2-1}}{2x^2}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{243}$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{1}{2}\left(\frac{1}{2}\left(ad^2+2c\right)\int \frac{1}{x^2\sqrt{d^2x^2-1}}dx^2+\frac{2b\sqrt{d^2x^2-1}}{x}\right)+\frac{a\sqrt{d^2x^2-1}}{2x^2}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{1}{2}\left(\frac{\left(ad^2+2c\right)\int \frac{1}{\frac{x^4}{d^2}+\frac{1}{d^2}}d\sqrt{d^2x^2-1}}{d^2}+\frac{2b\sqrt{d^2x^2-1}}{x}\right)+\frac{a\sqrt{d^2x^2-1}}{2x^2}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{1}{2}\left(\left(ad^2+2c\right)\arctan\left(\sqrt{d^2x^2-1}\right)+\frac{2b\sqrt{d^2x^2-1}}{x}\right)+\frac{a\sqrt{d^2x^2-1}}{2x^2}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

input
```
Int[(a + b*x + c*x^2)/(x^3*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

output
```
(Sqrt[-1 + d^2*x^2]*((a*Sqrt[-1 + d^2*x^2])/(2*x^2) + ((2*b*Sqrt[-1 + d^2*
x^2])/x + (2*c + a*d^2)*ArcTan[Sqrt[-1 + d^2*x^2]])/2))/(Sqrt[-1 + d*x]*Sq
rt[1 + d*x])
```

3.38.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 2113
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m])   Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a
*d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

3.38.4 Maple [A] (verified)

Time = 5.71 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.92

method	result	size
risch	$\frac{\sqrt{dx+1}\,\sqrt{dx-1}\,(2bx+a)}{2x^{2}}-\frac{\left(c+\frac{a\,d^{2}}{2}\right)\arctan\left(\frac{1}{\sqrt{d^{2}x^{2}-1}}\right)\sqrt{(dx+1)(dx-1)}}{\sqrt{dx-1}\,\sqrt{dx+1}}$	76
default	$-\frac{\sqrt{dx-1}\,\sqrt{dx+1}\,\operatorname{csgn}(d)^{2}\left(\arctan\left(\frac{1}{\sqrt{d^{2}x^{2}-1}}\right)a\,d^{2}x^{2}+2\arctan\left(\frac{1}{\sqrt{d^{2}x^{2}-1}}\right)c\,x^{2}-2\sqrt{d^{2}x^{2}-1}\,bx-\sqrt{d^{2}x^{2}-1}\,a\right)}{2\sqrt{d^{2}x^{2}-1}\,x^{2}}$	103

input
```
int((c*x^2+b*x+a)/x^3/(d*x-1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(d*x+1)^(1/2)*(d*x-1)^(1/2)*(2*b*x+a)/x^2-(c+1/2*a*d^2)*arctan(1/(d^2*
x^2-1)^(1/2))*((d*x+1)*(d*x-1))^(1/2)/(d*x-1)^(1/2)/(d*x+1)^(1/2)
```

3.38.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.83

$$\int \frac{a+bx+cx^2}{x^3\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{2\,bdx^2+2\,(ad^2+2\,c)x^2\arctan\left(-dx+\sqrt{dx+1}\sqrt{dx-1}\right)+(2\,bx+a)\sqrt{dx+1}\sqrt{dx-1}}{2\,x^2}$$

input
```
integrate((c*x^2+b*x+a)/x^3/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fric
as")
```

output
```
1/2*(2*b*d*x^2 + 2*(a*d^2 + 2*c)*x^2*arctan(-d*x + sqrt(d*x + 1)*sqrt(d*x
- 1)) + (2*b*x + a)*sqrt(d*x + 1)*sqrt(d*x - 1))/x^2
```

3.38.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx+cx^2}{x^3\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)/x**3/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

output
```
Timed out
```

3.38.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int \frac{a+bx+cx^2}{x^3\sqrt{-1+dx}\sqrt{1+dx}}\,dx = -\frac{1}{2}\,ad^2\arcsin\left(\frac{1}{d|x|}\right) - c\arcsin\left(\frac{1}{d|x|}\right) + \frac{\sqrt{d^2x^2-1}b}{x} + \frac{\sqrt{d^2x^2-1}a}{2\,x^2}$$

input
```
integrate((c*x^2+b*x+a)/x^3/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxi
ma")
```

output
```
-1/2*a*d^2*arcsin(1/(d*abs(x))) - c*arcsin(1/(d*abs(x))) + sqrt(d^2*x^2 -
1)*b/x + 1/2*sqrt(d^2*x^2 - 1)*a/x^2
```

3.38.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. 2(67) = 134.

Time = 0.32 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.75

$$\int \frac{a+bx+cx^2}{x^3\sqrt{-1+dx}\sqrt{1+dx}}\,dx = -\frac{(ad^3+2\,cd)\arctan\left(\frac{1}{2}\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right) + \frac{2\left(ad^3\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^6-4\,bd^2\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^4-4\,ad^3\left(\sqrt{dx+1}-\cdots\right.\right.}{\left(\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^4+4\right)^2}}{d}$$

input
```
integrate((c*x^2+b*x+a)/x^3/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac
")
```

output
```
-((a*d^3 + 2*c*d)*arctan(1/2*(sqrt(d*x + 1) - sqrt(d*x - 1))^2) + 2*(a*d^3
*(sqrt(d*x + 1) - sqrt(d*x - 1))^6 - 4*b*d^2*(sqrt(d*x + 1) - sqrt(d*x - 1
))^4 - 4*a*d^3*(sqrt(d*x + 1) - sqrt(d*x - 1))^2 - 16*b*d^2)/((sqrt(d*x +
1) - sqrt(d*x - 1))^4 + 4)^2)/d
```

3.38.9 Mupad [B] (verification not implemented)

Time = 14.59 (sec) , antiderivative size = 316, normalized size of antiderivative = 3.81

$$\int \frac{a+bx+cx^2}{x^3\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\frac{a\,d^2\,1\mathrm{i}}{32}+\frac{a\,d^2\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2\,1\mathrm{i}}{16\,{\left(\sqrt{d\,x+1}-1\right)}^2}-\frac{a\,d^2\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^4\,15\mathrm{i}}{32\,{\left(\sqrt{d\,x+1}-1\right)}^4}}{\frac{{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+\frac{2\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^4}{{\left(\sqrt{d\,x+1}-1\right)}^4}+\frac{{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^6}{{\left(\sqrt{d\,x+1}-1\right)}^6}} - c\left(\ln\left(\frac{{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+1\right)-\ln\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)\right)\,1\mathrm{i} - \frac{a\,d^2\,\ln\left(\frac{{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+1\right)\,1\mathrm{i}}{2}+\frac{a\,d^2\,\ln\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)\,1\mathrm{i}}{2} + \frac{b\,\sqrt{d\,x-1}\,\sqrt{d\,x+1}}{x}+\frac{a\,d^2\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2\,1\mathrm{i}}{32\,{\left(\sqrt{d\,x+1}-1\right)}^2}$$

input
```
int((a + b*x + c*x^2)/(x^3*(d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

output
```
((a*d^2*1i)/32 + (a*d^2*((d*x - 1)^(1/2) - 1i)^2*1i)/(16*((d*x + 1)^(1/2)
- 1)^2) - (a*d^2*((d*x - 1)^(1/2) - 1i)^4*15i)/(32*((d*x + 1)^(1/2) - 1)^4
))/(((d*x - 1)^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + (2*((d*x - 1)^(1/2)
 - 1i)^4)/((d*x + 1)^(1/2) - 1)^4 + ((d*x - 1)^(1/2) - 1i)^6/((d*x + 1)^(1
/2) - 1)^6) - c*(log(((d*x - 1)^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + 1)
 - log(((d*x - 1)^(1/2) - 1i)/((d*x + 1)^(1/2) - 1)))*1i - (a*d^2*log(((d*
x - 1)^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + 1)*1i)/2 + (a*d^2*log(((d*x
 - 1)^(1/2) - 1i)/((d*x + 1)^(1/2) - 1))*1i)/2 + (b*(d*x - 1)^(1/2)*(d*x +
 1)^(1/2))/x + (a*d^2*((d*x - 1)^(1/2) - 1i)^2*1i)/(32*((d*x + 1)^(1/2) -
1)^2)
```

3.39 $\int \frac{a+bx+cx^2}{x^4\sqrt{-1+dx}\sqrt{1+dx}}\,dx$

3.39.1 Optimal result

Integrand size = 32, antiderivative size = 116

$$\int \frac{a+bx+cx^2}{x^4\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{a\sqrt{-1+dx}\sqrt{1+dx}}{3x^3}+\frac{b\sqrt{-1+dx}\sqrt{1+dx}}{2x^2} + \frac{(3c+2ad^2)\sqrt{-1+dx}\sqrt{1+dx}}{3x} + \frac{1}{2}bd^2\arctan\left(\sqrt{-1+dx}\sqrt{1+dx}\right)$$

output
```
1/2*b*d^2*arctan((d*x-1)^(1/2)*(d*x+1)^(1/2))+1/3*a*(d*x-1)^(1/2)*(d*x+1)^
(1/2)/x^3+1/2*b*(d*x-1)^(1/2)*(d*x+1)^(1/2)/x^2+1/3*(2*a*d^2+3*c)*(d*x-1)^
(1/2)*(d*x+1)^(1/2)/x
```

3.39.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.61

$$\int \frac{a+bx+cx^2}{x^4\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{-1+dx}\sqrt{1+dx}(3x(b+2cx)+a(2+4d^2x^2))}{6x^3} + bd^2\arctan\left(\sqrt{\frac{-1+dx}{1+dx}}\right)$$

input
```
Integrate[(a + b*x + c*x^2)/(x^4*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

output
```
(Sqrt[-1 + d*x]*Sqrt[1 + d*x]*(3*x*(b + 2*c*x) + a*(2 + 4*d^2*x^2)))/(6*x^
3) + b*d^2*ArcTan[Sqrt[(-1 + d*x)/(1 + d*x)]]
```

3.39.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {2113, 2338, 539, 534, 243, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{x^4\sqrt{dx-1}\sqrt{dx+1}}dx$$

$$\Big\downarrow \text{2113}$$

$$\frac{\sqrt{d^2x^2-1}\int \frac{cx^2+bx+a}{x^4\sqrt{d^2x^2-1}}dx}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{2338}$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{1}{3}\int \frac{3b+(2ad^2+3c)x}{x^3\sqrt{d^2x^2-1}}dx+\frac{a\sqrt{d^2x^2-1}}{3x^3}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{539}$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{3bxd^2+2(2ad^2+3c)}{x^2\sqrt{d^2x^2-1}}dx+\frac{3b\sqrt{d^2x^2-1}}{2x^2}\right)+\frac{a\sqrt{d^2x^2-1}}{3x^3}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{534}$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{1}{3}\left(\frac{1}{2}\left(3bd^2\int \frac{1}{x\sqrt{d^2x^2-1}}dx+\frac{2\sqrt{d^2x^2-1}(2ad^2+3c)}{x}\right)+\frac{3b\sqrt{d^2x^2-1}}{2x^2}\right)+\frac{a\sqrt{d^2x^2-1}}{3x^3}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{243}$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{1}{3}\left(\frac{1}{2}\left(\frac{3}{2}bd^2\int \frac{1}{x^2\sqrt{d^2x^2-1}}dx^2+\frac{2\sqrt{d^2x^2-1}(2ad^2+3c)}{x}\right)+\frac{3b\sqrt{d^2x^2-1}}{2x^2}\right)+\frac{a\sqrt{d^2x^2-1}}{3x^3}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{1}{3}\left(\frac{1}{2}\left(3b\int \frac{1}{\frac{x^4}{d^2}+\frac{1}{d^2}}d\sqrt{d^2x^2-1}+\frac{2\sqrt{d^2x^2-1}(2ad^2+3c)}{x}\right)+\frac{3b\sqrt{d^2x^2-1}}{2x^2}\right)+\frac{a\sqrt{d^2x^2-1}}{3x^3}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{1}{3}\left(\frac{1}{2}\left(\frac{2\sqrt{d^2x^2-1}(2ad^2+3c)}{x}+3bd^2\arctan\left(\sqrt{d^2x^2-1}\right)\right)+\frac{3b\sqrt{d^2x^2-1}}{2x^2}\right)+\frac{a\sqrt{d^2x^2-1}}{3x^3}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

input
```
Int[(a + b*x + c*x^2)/(x^4*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

output
```
(Sqrt[-1 + d^2*x^2]*((a*Sqrt[-1 + d^2*x^2])/(3*x^3) + ((3*b*Sqrt[-1 + d^2*
x^2])/(2*x^2) + ((2*(3*c + 2*a*d^2)*Sqrt[-1 + d^2*x^2])/x + 3*b*d^2*ArcTan
[Sqrt[-1 + d^2*x^2]])/2)/3))/(Sqrt[-1 + d*x]*Sqrt[1 + d*x])
```

3.39.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2113
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m])   Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a
*d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

3.39.4 Maple [A] (verified)

Time = 1.65 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.77

method	result	siz
risch	$\frac{\sqrt{dx+1}\,\sqrt{dx-1}\,(4a\,d^2x^2+6c\,x^2+3bx+2a)}{6x^3}-\frac{b\,d^2\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)\sqrt{(dx+1)(dx-1)}}{2\sqrt{dx-1}\,\sqrt{dx+1}}$	89
default	$-\frac{\sqrt{dx-1}\,\sqrt{dx+1}\,\operatorname{csgn}(d)^2\left(3\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)b\,d^2x^3-4\sqrt{d^2x^2-1}\,a\,d^2x^2-6\sqrt{d^2x^2-1}\,c\,x^2-3\sqrt{d^2x^2-1}\,bx-2\sqrt{d^2x^2-1}\,a\right)}{6\sqrt{d^2x^2-1}\,x^3}$	12

input
```
int((c*x^2+b*x+a)/x^4/(d*x-1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(d*x+1)^(1/2)*(d*x-1)^(1/2)*(4*a*d^2*x^2+6*c*x^2+3*b*x+2*a)/x^3-1/2*b*
d^2*arctan(1/(d^2*x^2-1)^(1/2))*((d*x+1)*(d*x-1))^(1/2)/(d*x-1)^(1/2)/(d*x
+1)^(1/2)
```

3.39.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.78

$$\int \frac{a+bx+cx^2}{x^4\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{6\,bd^2x^3\arctan\left(-dx+\sqrt{dx+1}\sqrt{dx-1}\right)+2\,(2\,ad^3+3\,cd)x^3+(2\,(2\,ad^2+3\,c)x^2+3\,bx+2\,a)\sqrt{dx+1}\sqrt{}}{6\,x^3}$$

input
```
integrate((c*x^2+b*x+a)/x^4/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fric
as")
```

output
```
1/6*(6*b*d^2*x^3*arctan(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)) + 2*(2*a*d^3 +
 3*c*d)*x^3 + (2*(2*a*d^2 + 3*c)*x^2 + 3*b*x + 2*a)*sqrt(d*x + 1)*sqrt(d*x
 - 1))/x^3
```

3.39.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx+cx^2}{x^4\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)/x**4/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

output
```
Timed out
```

3.39.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.74

$$\int \frac{a+bx+cx^2}{x^4\sqrt{-1+dx}\sqrt{1+dx}}\,dx = -\frac{1}{2}\,bd^2\arcsin\left(\frac{1}{d|x|}\right)+\frac{2\,\sqrt{d^2x^2-1}ad^2}{3\,x} + \frac{\sqrt{d^2x^2-1}c}{x}+\frac{\sqrt{d^2x^2-1}b}{2\,x^2}+\frac{\sqrt{d^2x^2-1}a}{3\,x^3}$$

input
```
integrate((c*x^2+b*x+a)/x^4/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxi
ma")
```

output
```
-1/2*b*d^2*arcsin(1/(d*abs(x))) + 2/3*sqrt(d^2*x^2 - 1)*a*d^2/x + sqrt(d^2
*x^2 - 1)*c/x + 1/2*sqrt(d^2*x^2 - 1)*b/x^2 + 1/3*sqrt(d^2*x^2 - 1)*a/x^3
```

3.39.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(92) = 184$.

Time = 0.33 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.70

$$\int \frac{a+bx+cx^2}{x^4\sqrt{-1+dx}\sqrt{1+dx}}\,dx =$$

$$-\frac{3\,bd^3\arctan\left(\frac{1}{2}\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right)+\frac{2\left(3\,bd^3\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^{10}-12\,cd^2\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^8-96\,ad^4\left(\sqrt{dx+1}-\sqrt{dx-\right.\right.}{\left(\left(\sqrt{dx+1}-\sqrt{\right.\right.}}}{3\,d}$$

input
```
integrate((c*x^2+b*x+a)/x^4/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac
")
```

output
```
-1/3*(3*b*d^3*arctan(1/2*(sqrt(d*x + 1) - sqrt(d*x - 1))^2) + 2*(3*b*d^3*(
sqrt(d*x + 1) - sqrt(d*x - 1))^10 - 12*c*d^2*(sqrt(d*x + 1) - sqrt(d*x - 1
))^8 - 96*a*d^4*(sqrt(d*x + 1) - sqrt(d*x - 1))^4 - 96*c*d^2*(sqrt(d*x + 1
) - sqrt(d*x - 1))^4 - 48*b*d^3*(sqrt(d*x + 1) - sqrt(d*x - 1))^2 - 128*a*
d^4 - 192*c*d^2)/((sqrt(d*x + 1) - sqrt(d*x - 1))^4 + 4)^3)/d
```

3.39.9 Mupad [B] (verification not implemented)

Time = 12.13 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.62

$$\int \frac{a+bx+cx^2}{x^4\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\frac{b\,d^2\,1\mathrm{i}}{32}+\frac{b\,d^2\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)^2\,1\mathrm{i}}{16\,\left(\sqrt{d\,x+1}-1\right)^2}-\frac{b\,d^2\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)^4\,15\mathrm{i}}{32\,\left(\sqrt{d\,x+1}-1\right)^4}}{\frac{\left(\sqrt{d\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+\frac{2\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)^4}{\left(\sqrt{d\,x+1}-1\right)^4}+\frac{\left(\sqrt{d\,x-1}-\mathrm{i}\right)^6}{\left(\sqrt{d\,x+1}-1\right)^6}} - \frac{b\,d^2\,\ln\left(\frac{\left(\sqrt{d\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+1\right)\,1\mathrm{i}}{2} + \frac{b\,d^2\,\ln\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)\,1\mathrm{i}}{2}+\frac{c\,\sqrt{d\,x-1}\,\sqrt{d\,x+1}}{x} + \frac{\sqrt{d\,x-1}\,\left(\frac{2\,a\,d^3\,x^3}{3}+\frac{2\,a\,d^2\,x^2}{3}+\frac{a\,d\,x}{3}+\frac{a}{3}\right)}{x^3\,\sqrt{d\,x+1}} + \frac{b\,d^2\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)^2\,1\mathrm{i}}{32\,\left(\sqrt{d\,x+1}-1\right)^2}$$

input
```
int((a + b*x + c*x^2)/(x^4*(d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

output
```
((b*d^2*1i)/32 + (b*d^2*((d*x - 1)^(1/2) - 1i)^2*1i)/(16*((d*x + 1)^(1/2)
- 1)^2) - (b*d^2*((d*x - 1)^(1/2) - 1i)^4*15i)/(32*((d*x + 1)^(1/2) - 1)^4
))/(((d*x - 1)^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + (2*((d*x - 1)^(1/2)
 - 1i)^4)/((d*x + 1)^(1/2) - 1)^4 + ((d*x - 1)^(1/2) - 1i)^6/((d*x + 1)^(1
/2) - 1)^6) - (b*d^2*log(((d*x - 1)^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2
+ 1)*1i)/2 + (b*d^2*log(((d*x - 1)^(1/2) - 1i)/((d*x + 1)^(1/2) - 1))*1i)/
2 + (c*(d*x - 1)^(1/2)*(d*x + 1)^(1/2))/x + ((d*x - 1)^(1/2)*(a/3 + (2*a*d
^2*x^2)/3 + (2*a*d^3*x^3)/3 + (a*d*x)/3))/(x^3*(d*x + 1)^(1/2)) + (b*d^2*(
(d*x - 1)^(1/2) - 1i)^2*1i)/(32*((d*x + 1)^(1/2) - 1)^2)
```

3.40 $\int \frac{a+bx+cx^2}{\sqrt{-1+x}\sqrt{1+x}(d+ex)^3}\,dx$

3.40.1 Optimal result

Integrand size = 32, antiderivative size = 199

$$\int \frac{a+bx+cx^2}{\sqrt{-1+x}\sqrt{1+x}(d+ex)^3}\,dx = -\frac{(cd^2-bde+ae^2)\sqrt{-1+x}\sqrt{1+x}}{2e(d^2-e^2)(d+ex)^2} + \frac{(cd^3+bd^2e-(3a+4c)de^2+2be^3)\sqrt{-1+x}\sqrt{1+x}}{2e(d^2-e^2)^2(d+ex)} + \frac{((2a+c)d^2-3bde+(a+2c)e^2)\operatorname{arctanh}\left(\frac{\sqrt{d+e}\sqrt{1+x}}{\sqrt{d-e}\sqrt{-1+x}}\right)}{(d-e)^{5/2}(d+e)^{5/2}}$$

output
```
((2*a+c)*d^2-3*b*d*e+(a+2*c)*e^2)*arctanh((d+e)^(1/2)*(1+x)^(1/2)/(d-e)^(1
/2)/(-1+x)^(1/2))/(d-e)^(5/2)/(d+e)^(5/2)-1/2*(a*e^2-b*d*e+c*d^2)*(-1+x)^(
1/2)*(1+x)^(1/2)/e/(d^2-e^2)/(e*x+d)^2+1/2*(c*d^3+b*d^2*e-(3*a+4*c)*d*e^2+
2*b*e^3)*(-1+x)^(1/2)*(1+x)^(1/2)/e/(d^2-e^2)^2/(e*x+d)
```

3.40.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.93

$$\int \frac{a+bx+cx^2}{\sqrt{-1+x}\sqrt{1+x}(d+ex)^3}\,dx = \frac{\sqrt{-1+x}\sqrt{1+x}(ae(-4d^2+e^2-3dex)+cd(-3de+d^2x-4e^2x)+b(2d^3+de^2+d^2ex+2e^3x))}{2(d-e)^2(d+e)^2(d+ex)^2} - \frac{(-3bde+a(2d^2+e^2)+c(d^2+2e^2))\arctan\left(\frac{\sqrt{d-e}\sqrt{\frac{-1+x}{1+x}}}{\sqrt{-d-e}}\right)}{(-d-e)^{5/2}(d-e)^{5/2}}$$

input
```
Integrate[(a + b*x + c*x^2)/(Sqrt[-1 + x]*Sqrt[1 + x]*(d + e*x)^3),x]
```

output
```
(Sqrt[-1 + x]*Sqrt[1 + x]*(a*e*(-4*d^2 + e^2 - 3*d*e*x) + c*d*(-3*d*e + d^
2*x - 4*e^2*x) + b*(2*d^3 + d*e^2 + d^2*e*x + 2*e^3*x)))/(2*(d - e)^2*(d +
 e)^2*(d + e*x)^2) - ((-3*b*d*e + a*(2*d^2 + e^2) + c*(d^2 + 2*e^2))*ArcTa
n[(Sqrt[d - e]*Sqrt[(-1 + x)/(1 + x)])/Sqrt[-d - e]])/((-d - e)^(5/2)*(d -
 e)^(5/2))
```

3.40.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2113, 2182, 25, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{\sqrt{x-1}\sqrt{x+1}(d+ex)^3}dx$$

$$\Big\downarrow \text{2113}$$

$$\frac{\sqrt{x^2-1}\int \frac{cx^2+bx+a}{(d+ex)^3\sqrt{x^2-1}}dx}{\sqrt{x-1}\sqrt{x+1}}$$

$$\Big\downarrow \text{2182}$$

$$\frac{\sqrt{x^2-1}\left(-\frac{\int -\frac{2(ad+cd-be)+\left(\frac{cd^2}{e}+bd-ae-2ce\right)x}{(d+ex)^2\sqrt{x^2-1}}dx}{2(d^2-e^2)}-\frac{\sqrt{x^2-1}(ae^2-bde+cd^2)}{2e(d^2-e^2)(d+ex)^2}\right)}{\sqrt{x-1}\sqrt{x+1}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{x^2-1}\left(\frac{\int \frac{2(ad+cd-be)+\left(\frac{cd^2}{e}+bd-ae-2ce\right)x}{(d+ex)^2\sqrt{x^2-1}}dx}{2(d^2-e^2)}-\frac{\sqrt{x^2-1}(ae^2-bde+cd^2)}{2e(d^2-e^2)(d+ex)^2}\right)}{\sqrt{x-1}\sqrt{x+1}}$$

$$\Big\downarrow \text{679}$$

$$\frac{\sqrt{x^2-1}\left(\frac{\frac{\sqrt{x^2-1}\left(-3ade^2+bd^2e+2be^3+cd^3-4cde^2\right)}{e\left(d^2-e^2\right)(d+ex)}-\frac{\left(-a\left(2d^2+e^2\right)+3bde-c\left(d^2+2e^2\right)\right)\int\frac{1}{(d+ex)\sqrt{x^2-1}}dx}{d^2-e^2}}{2(d^2-e^2)}-\frac{\sqrt{x^2-1}(ae^2-bde+cd^2)}{2e(d^2-e^2)(d+ex)^2}\right)}{\sqrt{x-1}\sqrt{x+1}}$$

$$\Big\downarrow \; 488$$

$$\frac{\sqrt{x^2-1}\left(\frac{\frac{\left(-a\left(2d^2+e^2\right)+3bde-c\left(d^2+2e^2\right)\right)\int\frac{1}{d^2-e^2-\frac{(-e-dx)^2}{x^2-1}}d\frac{-e-dx}{\sqrt{x^2-1}}}{d^2-e^2}+\frac{\sqrt{x^2-1}\left(-3ade^2+bd^2e+2be^3+cd^3-4cde^2\right)}{e\left(d^2-e^2\right)(d+ex)}}{2(d^2-e^2)}-\frac{\sqrt{x^2-1}(ae^2-bde+cd^2)}{2e(d^2-e^2)(d+ex)^2}\right)}{\sqrt{x-1}\sqrt{x+1}}$$

$$\Big\downarrow \; 219$$

$$\frac{\sqrt{x^2-1}\left(\frac{\frac{\text{arctanh}\left(\frac{-dx-e}{\sqrt{x^2-1}\sqrt{d^2-e^2}}\right)\left(-a\left(2d^2+e^2\right)+3bde-c\left(d^2+2e^2\right)\right)}{\left(d^2-e^2\right)^{3/2}}+\frac{\sqrt{x^2-1}\left(-3ade^2+bd^2e+2be^3+cd^3-4cde^2\right)}{e\left(d^2-e^2\right)(d+ex)}}{2(d^2-e^2)}-\frac{\sqrt{x^2-1}(ae^2-bde+cd^2)}{2e(d^2-e^2)(d+ex)^2}\right)}{\sqrt{x-1}\sqrt{x+1}}$$

input
```
Int[(a + b*x + c*x^2)/(Sqrt[-1 + x]*Sqrt[1 + x]*(d + e*x)^3),x]
```

output
```
(Sqrt[-1 + x^2]*(-1/2*((c*d^2 - b*d*e + a*e^2)*Sqrt[-1 + x^2])/(e*(d^2 - e
^2)*(d + e*x)^2) + (((c*d^3 + b*d^2*e - 3*a*d*e^2 - 4*c*d*e^2 + 2*b*e^3)*S
qrt[-1 + x^2])/(e*(d^2 - e^2)*(d + e*x)) + ((3*b*d*e - a*(2*d^2 + e^2) - c
*(d^2 + 2*e^2))*ArcTanh[(-e - d*x)/(Sqrt[d^2 - e^2]*Sqrt[-1 + x^2])])/(d^2
 - e^2)^(3/2))/(2*(d^2 - e^2))))/(Sqrt[-1 + x]*Sqrt[1 + x])
```

3.40.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2113
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m])   Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a
*d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

3.40.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1094 vs. $2(175) = 350$.

Time = 1.69 (sec) , antiderivative size = 1095, normalized size of antiderivative = 5.50

method	result	size
default	Expression too large to display	1095

input
```
int((c*x^2+b*x+a)/(e*x+d)^3/(-1+x)^(1/2)/(1+x)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
-1/2*(ln(-2*(-(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)*e+d*x+e)/(e*x+d))*c*d^2*
e^2*x^2+4*ln(-2*(-(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)*e+d*x+e)/(e*x+d))*a*
d^3*e*x+2*ln(-2*(-(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)*e+d*x+e)/(e*x+d))*a*
d*e^3*x-6*ln(-2*(-(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)*e+d*x+e)/(e*x+d))*b*
d^2*e^2*x+2*ln(-2*(-(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)*e+d*x+e)/(e*x+d))*
c*d^3*e*x+4*ln(-2*(-(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)*e+d*x+e)/(e*x+d))*
c*d*e^3*x+4*a*d^2*e^2*(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)-2*b*d^3*e*(x^2-1
)^(1/2)*((d^2-e^2)/e^2)^(1/2)-b*d*e^3*(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)+
3*c*d^2*e^2*(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)+2*ln(-2*(-(x^2-1)^(1/2)*((
d^2-e^2)/e^2)^(1/2)*e+d*x+e)/(e*x+d))*a*d^2*e^2*x^2-3*ln(-2*(-(x^2-1)^(1/2
)*((d^2-e^2)/e^2)^(1/2)*e+d*x+e)/(e*x+d))*b*d*e^3*x^2+3*a*d*e^3*x*(x^2-1)^
(1/2)*((d^2-e^2)/e^2)^(1/2)-b*d^2*e^2*x*(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2
)-c*d^3*e*x*(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)+4*c*d*e^3*x*(x^2-1)^(1/2)*
((d^2-e^2)/e^2)^(1/2)-2*b*e^4*x*(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)+ln(-2*
(-(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)*e+d*x+e)/(e*x+d))*a*e^4*x^2+2*ln(-2*
(-(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)*e+d*x+e)/(e*x+d))*c*e^4*x^2+ln(-2*(-
(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)*e+d*x+e)/(e*x+d))*a*d^2*e^2-3*ln(-2*(-
(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)*e+d*x+e)/(e*x+d))*b*d^3*e+2*ln(-2*(-(x
^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)*e+d*x+e)/(e*x+d))*c*d^2*e^2-a*e^4*(x^2-1
)^(1/2)*((d^2-e^2)/e^2)^(1/2)+2*ln(-2*(-(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(...
```

3.40.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 587 vs. $2(175) = 350$.

Time = 0.32 (sec) , antiderivative size = 1186, normalized size of antiderivative = 5.96

$$\int \frac{a + bx + cx^2}{\sqrt{-1 + x}\sqrt{1 + x}(d + ex)^3}\, dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)/(e*x+d)^3/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="f
ricas")
```

output
```
[1/2*(c*d^7 + b*d^6*e - (3*a + 5*c)*d^5*e^2 + b*d^4*e^3 + (3*a + 4*c)*d^3*
e^4 - 2*b*d^2*e^5 + (c*d^5*e^2 + b*d^4*e^3 - (3*a + 5*c)*d^3*e^4 + b*d^2*e
^5 + (3*a + 4*c)*d*e^6 - 2*b*e^7)*x^2 + ((2*a + c)*d^4*e^2 - 3*b*d^3*e^3 +
 (a + 2*c)*d^2*e^4 + ((2*a + c)*d^2*e^4 - 3*b*d*e^5 + (a + 2*c)*e^6)*x^2 +
 2*((2*a + c)*d^3*e^3 - 3*b*d^2*e^4 + (a + 2*c)*d*e^5)*x)*sqrt(d^2 - e^2)*
log((d^2*x + d*e + (d^2 - e^2 + sqrt(d^2 - e^2)*d)*sqrt(x + 1)*sqrt(x - 1)
 + sqrt(d^2 - e^2)*(d*x + e))/(e*x + d)) + (2*b*d^5*e^2 - (4*a + 3*c)*d^4*
e^3 - b*d^3*e^4 + (5*a + 3*c)*d^2*e^5 - b*d*e^6 - a*e^7 + (c*d^5*e^2 + b*d
^4*e^3 - (3*a + 5*c)*d^3*e^4 + b*d^2*e^5 + (3*a + 4*c)*d*e^6 - 2*b*e^7)*x)
*sqrt(x + 1)*sqrt(x - 1) + 2*(c*d^6*e + b*d^5*e^2 - (3*a + 5*c)*d^4*e^3 +
b*d^3*e^4 + (3*a + 4*c)*d^2*e^5 - 2*b*d*e^6)*x)/(d^8*e^2 - 3*d^6*e^4 + 3*d
^4*e^6 - d^2*e^8 + (d^6*e^4 - 3*d^4*e^6 + 3*d^2*e^8 - e^10)*x^2 + 2*(d^7*e
^3 - 3*d^5*e^5 + 3*d^3*e^7 - d*e^9)*x), 1/2*(c*d^7 + b*d^6*e - (3*a + 5*c)
*d^5*e^2 + b*d^4*e^3 + (3*a + 4*c)*d^3*e^4 - 2*b*d^2*e^5 + (c*d^5*e^2 + b*
d^4*e^3 - (3*a + 5*c)*d^3*e^4 + b*d^2*e^5 + (3*a + 4*c)*d*e^6 - 2*b*e^7)*x
^2 - 2*((2*a + c)*d^4*e^2 - 3*b*d^3*e^3 + (a + 2*c)*d^2*e^4 + ((2*a + c)*d
^2*e^4 - 3*b*d*e^5 + (a + 2*c)*e^6)*x^2 + 2*((2*a + c)*d^3*e^3 - 3*b*d^2*e
^4 + (a + 2*c)*d*e^5)*x)*sqrt(-d^2 + e^2)*arctan(-(sqrt(-d^2 + e^2)*e*sqrt
(x + 1)*sqrt(x - 1) - sqrt(-d^2 + e^2)*(e*x + d))/(d^2 - e^2)) + (2*b*d^5*
e^2 - (4*a + 3*c)*d^4*e^3 - b*d^3*e^4 + (5*a + 3*c)*d^2*e^5 - b*d*e^6 -...
```

3.40.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx+cx^2}{\sqrt{-1+x}\sqrt{1+x}(d+ex)^3}\,dx=\text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)/(e*x+d)**3/(-1+x)**(1/2)/(1+x)**(1/2),x)
```

output
```
Timed out
```

3.40.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx+cx^2}{\sqrt{-1+x}\sqrt{1+x}(d+ex)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)/(e*x+d)^3/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((e-d)*(e+d)>0)', see `assume?` f
or more de
```

3.40.8 Giac [F]

$$\int \frac{a+bx+cx^2}{\sqrt{-1+x}\sqrt{1+x}(d+ex)^3}\,dx = \int \frac{cx^2+bx+a}{(ex+d)^3\sqrt{x+1}\sqrt{x-1}}\,dx$$

input
```
integrate((c*x^2+b*x+a)/(e*x+d)^3/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="g
iac")
```

output
```
sage0*x
```

3.40.9 Mupad [B] (verification not implemented)

Time = 72.89 (sec) , antiderivative size = 7235, normalized size of antiderivative = 36.36

$$\int \frac{a+bx+cx^2}{\sqrt{-1+x}\sqrt{1+x}(d+ex)^3}\,dx = \text{Too large to display}$$

input
```
int((a + b*x + c*x^2)/((x - 1)^(1/2)*(x + 1)^(1/2)*(d + e*x)^3),x)
```

output
```
((((x - 1)^(1/2) - 1i)^2*(2*c*e^3 + c*d^2*e)*12i)/(d^2*((x + 1)^(1/2) - 1)
^2*(d^4 + e^4 - 2*d^2*e^2)) - (2*(7*c*d^4 + 14*c*d^2*e^2)*((x - 1)^(1/2) -
 1i))/(7*d^3*((x + 1)^(1/2) - 1)*(d^4 + e^4 - 2*d^2*e^2)) + (((x - 1)^(1/2
) - 1i)^4*(2*c*e^3 - c*d^2*e)*24i)/(d^2*((x + 1)^(1/2) - 1)^4*(d^4 + e^4 -
 2*d^2*e^2)) - (2*(21*c*d^4 - 102*c*d^2*e^2)*((x - 1)^(1/2) - 1i)^5)/(3*d^
3*((x + 1)^(1/2) - 1)^5*(d^4 + e^4 - 2*d^2*e^2)) - (2*(35*c*d^4 - 170*c*d^
2*e^2)*((x - 1)^(1/2) - 1i)^3)/(5*d^3*((x + 1)^(1/2) - 1)^3*(d^4 + e^4 - 2
*d^2*e^2)) + (c*((x - 1)^(1/2) - 1i)^7*(d^2*1i + e^2*2i)*2i)/(d*((x + 1)^(
1/2) - 1)^7*(d^4 + e^4 - 2*d^2*e^2)) + (12*c*e*((x - 1)^(1/2) - 1i)^6*(d^2
*1i + e^2*2i))/(d^2*((x + 1)^(1/2) - 1)^6*(d^4 + e^4 - 2*d^2*e^2)))/(((x -
 1)^(1/2) - 1i)^8/((x + 1)^(1/2) - 1)^8 - (e*((x - 1)^(1/2) - 1i)*8i)/(d*(
(x + 1)^(1/2) - 1)) + (e*((x - 1)^(1/2) - 1i)^3*8i)/(d*((x + 1)^(1/2) - 1)
^3) + (e*((x - 1)^(1/2) - 1i)^5*8i)/(d*((x + 1)^(1/2) - 1)^5) - (e*((x - 1
)^(1/2) - 1i)^7*8i)/(d*((x + 1)^(1/2) - 1)^7) - (((x - 1)^(1/2) - 1i)^2*(4
*d^2 + 16*e^2))/(d^2*((x + 1)^(1/2) - 1)^2) - (((x - 1)^(1/2) - 1i)^6*(4*d
^2 + 16*e^2))/(d^2*((x + 1)^(1/2) - 1)^6) + (((x - 1)^(1/2) - 1i)^4*(6*d^2
 - 32*e^2))/(d^2*((x + 1)^(1/2) - 1)^4) + 1) - ((2*((x - 1)^(1/2) - 1i)^3*
(16*b*e^3 + 11*b*d^2*e))/(d^2*((x + 1)^(1/2) - 1)^3*(d^4 + e^4 - 2*d^2*e^2
)) - (6*b*e*((x - 1)^(1/2) - 1i)^7)/(((x + 1)^(1/2) - 1)^7*(d^4 + e^4 - 2*
d^2*e^2)) - (6*b*e*((x - 1)^(1/2) - 1i))/(((x + 1)^(1/2) - 1)*(d^4 + e^...
```

3.40. $\int \frac{a+bx+cx^2}{\sqrt{-1+x}\sqrt{1+x}(d+ex)^3}\,dx$

3.41 $\int (a+bx)^2\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)\,dx$

3.41.1 Optimal result

Integrand size = 36, antiderivative size = 1348

$$\int (a+bx)^2\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)\,dx$$

$$= \frac{(de-cf)\left(8a^2d^2f^2(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf)))-8abdf(C(7d^3e^3+9cd^2e^2f+\right.}{}$$

$$+ \frac{(8a^2d^2f^2(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf)))-8abdf(C(7d^3e^3+9cd^2e^2f+9c^2def}{}$$

$$- \frac{(2aCdf-b(4Bdf-3C(de+cf)))(a+bx)^2(c+dx)^{3/2}(e+fx)^{3/2}}{20bd^2f^2}$$

$$+ \frac{C(a+bx)^3(c+dx)^{3/2}(e+fx)^{3/2}}{6bdf}$$

$$- \frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(64a^3Cd^3f^3-8a^2bd^2f^2(16Bdf-7C(de+cf))-8ab^2df(C(35d^2e^2+38cdef+\right.}{}$$

$$- \frac{(de-cf)^2\left(8a^2d^2f^2(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf)))-8abdf(C(7d^3e^3+9cd^2e^2\right.}{}$$

output
```
-1/20*(2*a*C*d*f-b*(4*B*d*f-3*C*(c*f+d*e)))*(b*x+a)^2*(d*x+c)^(3/2)*(f*x+e
)^(3/2)/b/d^2/f^2+1/6*C*(b*x+a)^3*(d*x+c)^(3/2)*(f*x+e)^(3/2)/b/d/f-1/960*
(d*x+c)^(3/2)*(f*x+e)^(3/2)*(64*a^3*C*d^3*f^3-8*a^2*b*d^2*f^2*(16*B*d*f-7*
C*(c*f+d*e))-8*a*b^2*d*f*(C*(35*c^2*f^2+38*c*d*e*f+35*d^2*e^2)+10*d*f*(8*A
*d*f-5*B*(c*f+d*e)))+b^3*(7*C*(15*c^3*f^3+17*c^2*d*e*f^2+17*c*d^2*e^2*f+15
*d^3*e^3)+4*d*f*(50*A*d*f*(c*f+d*e)-B*(35*c^2*f^2+38*c*d*e*f+35*d^2*e^2)))
+6*b*d*f*(10*b*d*f*(-4*A*b*d*f+C*a*c*f+C*a*d*e+2*C*b*c*e)+(4*a*d*f-7*b*(c*
f+d*e))*(2*a*C*d*f-b*(4*B*d*f-3*C*(c*f+d*e))))*x)/b/d^4/f^4-1/512*(-c*f+d*
e)^2*(8*a^2*d^2*f^2*(C*(5*c^2*f^2+6*c*d*e*f+5*d^2*e^2)+8*d*f*(2*A*d*f-B*(c
*f+d*e)))-8*a*b*d*f*(C*(7*c^3*f^3+9*c^2*d*e*f^2+9*c*d^2*e^2*f+7*d^3*e^3)+2
*d*f*(8*A*d*f*(c*f+d*e)-B*(5*c^2*f^2+6*c*d*e*f+5*d^2*e^2)))+b^2*(C*(21*c^4
*f^4+28*c^3*d*e*f^3+30*c^2*d^2*e^2*f^2+28*c*d^3*e^3*f+21*d^4*e^4)+4*d*f*(2
*A*d*f*(5*c^2*f^2+6*c*d*e*f+5*d^2*e^2)-B*(7*c^3*f^3+9*c^2*d*e*f^2+9*c*d^2*
e^2*f+7*d^3*e^3))))*arctanh(f^(1/2)*(d*x+c)^(1/2)/d^(1/2)/(f*x+e)^(1/2))/d
^(11/2)/f^(11/2)+1/256*(8*a^2*d^2*f^2*(C*(5*c^2*f^2+6*c*d*e*f+5*d^2*e^2)+8
*d*f*(2*A*d*f-B*(c*f+d*e)))-8*a*b*d*f*(C*(7*c^3*f^3+9*c^2*d*e*f^2+9*c*d^2*
e^2*f+7*d^3*e^3)+2*d*f*(8*A*d*f*(c*f+d*e)-B*(5*c^2*f^2+6*c*d*e*f+5*d^2*e^2
)))+b^2*(C*(21*c^4*f^4+28*c^3*d*e*f^3+30*c^2*d^2*e^2*f^2+28*c*d^3*e^3*f+21
*d^4*e^4)+4*d*f*(2*A*d*f*(5*c^2*f^2+6*c*d*e*f+5*d^2*e^2)-B*(7*c^3*f^3+9*c^
2*d*e*f^2+9*c*d^2*e^2*f+7*d^3*e^3))))*(d*x+c)^(3/2)*(f*x+e)^(1/2)/d^5/f...
```

3.41.2 Mathematica [A] (verified)

Time = 4.43 (sec) , antiderivative size = 1253, normalized size of antiderivative = 0.93

$$\int (a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx$$

$$= \frac{\sqrt{c+dx}\sqrt{e+fx}(40a^2d^2f^2(C(15c^3f^3-c^2df^2(7e+10fx)+cd^2f(-7e^2+4efx+8f^2x^2)+d^3(15e^3-10}{}$$

$$-\frac{(de-cf)^2\left(8a^2d^2f^2(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf)))-8abdf(C(7d^3e^3+9cd^2e^2\right.}{}$$

input
```
Integrate[(a + b*x)^2*Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2),x]
```

output
```
(Sqrt[c + d*x]*Sqrt[e + f*x]*(40*a^2*d^2*f^2*(C*(15*c^3*f^3 - c^2*d*f^2*(7
*e + 10*f*x) + c*d^2*f*(-7*e^2 + 4*e*f*x + 8*f^2*x^2) + d^3*(15*e^3 - 10*e
^2*f*x + 8*e*f^2*x^2 + 48*f^3*x^3)) + 8*d*f*(6*A*d*f*(c*f + d*(e + 2*f*x))
 + B*(-3*c^2*f^2 + 2*c*d*f*(e + f*x) + d^2*(-3*e^2 + 2*e*f*x + 8*f^2*x^2))
)) + 8*a*b*d*f*(C*(-105*c^4*f^4 + 10*c^3*d*f^3*(4*e + 7*f*x) - 2*c^2*d^2*f
^2*(-17*e^2 + 11*e*f*x + 28*f^2*x^2) + 2*c*d^3*f*(20*e^3 - 11*e^2*f*x + 8*
e*f^2*x^2 + 24*f^3*x^3) + d^4*(-105*e^4 + 70*e^3*f*x - 56*e^2*f^2*x^2 + 48
*e*f^3*x^3 + 384*f^4*x^4)) + 10*d*f*(8*A*d*f*(-3*c^2*f^2 + 2*c*d*f*(e + f*
x) + d^2*(-3*e^2 + 2*e*f*x + 8*f^2*x^2)) + B*(15*c^3*f^3 - c^2*d*f^2*(7*e
+ 10*f*x) + c*d^2*f*(-7*e^2 + 4*e*f*x + 8*f^2*x^2) + d^3*(15*e^3 - 10*e^2*
f*x + 8*e*f^2*x^2 + 48*f^3*x^3)))) + b^2*(C*(315*c^5*f^5 - 105*c^4*d*f^4*(
e + 2*f*x) + 2*c^3*d^2*f^3*(-41*e^2 + 28*e*f*x + 84*f^2*x^2) - 2*c^2*d^3*f
^2*(41*e^3 - 26*e^2*f*x + 20*e*f^2*x^2 + 72*f^3*x^3) + c*d^4*f*(-105*e^4 +
 56*e^3*f*x - 40*e^2*f^2*x^2 + 32*e*f^3*x^3 + 128*f^4*x^4) + d^5*(315*e^5
- 210*e^4*f*x + 168*e^3*f^2*x^2 - 144*e^2*f^3*x^3 + 128*e*f^4*x^4 + 1280*f
^5*x^5)) + 4*d*f*(10*A*d*f*(15*c^3*f^3 - c^2*d*f^2*(7*e + 10*f*x) + c*d^2*
f*(-7*e^2 + 4*e*f*x + 8*f^2*x^2) + d^3*(15*e^3 - 10*e^2*f*x + 8*e*f^2*x^2
+ 48*f^3*x^3)) + B*(-105*c^4*f^4 + 10*c^3*d*f^3*(4*e + 7*f*x) - 2*c^2*d^2*
f^2*(-17*e^2 + 11*e*f*x + 28*f^2*x^2) + 2*c*d^3*f*(20*e^3 - 11*e^2*f*x + 8
*e*f^2*x^2 + 24*f^3*x^3) + d^4*(-105*e^4 + 70*e^3*f*x - 56*e^2*f^2*x^2 ...
```

3.41.3 Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 815, normalized size of antiderivative = 0.60, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2118, 27, 170, 27, 164, 60, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{2118}$$

$$\frac{\int -\frac{3}{2}b(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}(2bcCe+aCde+acCf-4Abdf-(4bBdf-2aCdf-3bC(de+cf))x)dx}{6b^2df}+\frac{C(a+bx)^3(c+dx)^{3/2}(e+fx)^{3/2}}{6bdf}$$

$$\Big\downarrow \text{27}$$

$$\frac{C(a+bx)^3(c+dx)^{3/2}(e+fx)^{3/2}}{6bdf}-\frac{\int (a+bx)^2\sqrt{c+dx}\sqrt{e+fx}(2bcCe+aCde+acCf-4Abdf-(4bBdf-2aCdf-3bC(de+cf))x)dx}{4bdf}$$

$\downarrow$ 170

$$\frac{C(a+bx)^3(c+dx)^{3/2}(e+fx)^{3/2}}{6bdf}-\frac{\frac{\int \frac{1}{2}(a+bx)\sqrt{c+dx}\sqrt{e+fx}(10adf(2bcCe+aCde+acCf-4Abdf)+(4bce+3a(de+cf))(4bBdf-2aCdf-3bC(de+cf))+(10bdf(2bcCe+aCde+acCf-}{5df}}{4bdf}$$

$\downarrow$ 27

$$\frac{C(a+bx)^3(c+dx)^{3/2}(e+fx)^{3/2}}{6bdf}-\frac{\frac{\int (a+bx)\sqrt{c+dx}\sqrt{e+fx}(10adf(2bcCe+aCde+acCf-4Abdf)+(4bce+3a(de+cf))(4bBdf-2aCdf-3bC(de+cf))+(10bdf(2bcCe+aCde+acCf-4A}{10df}}{4bdf}$$

$\downarrow$ 164

$$\frac{C(a+bx)^3(c+dx)^{3/2}(e+fx)^{3/2}}{6bdf}-\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(64a^3Cd^3f^3-8a^2bd^2f^2(16Bdf-7C(cf+de))-8ab^2df\left(10df(8Adf-5B(cf+de))+C\left(35c^2f^2+38cdef+35d^2e^2\right)\right)+6bdfx(10bdf(acCf+aCde-4}{}$$

$\downarrow$ 60

$$\frac{C(a+bx)^3(c+dx)^{3/2}(e+fx)^{3/2}}{6bdf}-\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(64a^3Cd^3f^3-8a^2bd^2f^2(16Bdf-7C(cf+de))-8ab^2df\left(10df(8Adf-5B(cf+de))+C\left(35c^2f^2+38cdef+35d^2e^2\right)\right)+6bdfx(10bdf(acCf+aCde-4}{}$$

$\downarrow$ 60

$$\frac{C(a+bx)^3(c+dx)^{3/2}(e+fx)^{3/2}}{6bdf}-\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(\left(7C\left(15d^3e^3+17cd^2fe^2+17c^2df^2e+15c^3f^3\right)+4df\left(50Adf(de+cf)-B\left(35d^2e^2+38cdfe+35c^2f^2\right)\right)\right)b^3-8adf\left(C\left(35d^2e^2+38cdfe+35c^2f^2\right)+1\right.}{2}$$

$\downarrow$ 66

3.41. $\int (a+bx)^2\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)\ dx$

$$\frac{C(a+bx)^3(c+dx)^{3/2}(e+fx)^{3/2}}{6bdf}-$$

$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(\left(7C\left(15d^3e^3+17cd^2fe^2+17c^2df^2e+15c^3f^3\right)+4df\left(50Adf(de+cf)-B\left(35d^2e^2+38cdfe+35c^2f^2\right)\right)\right)b^3-8adf\left(C\left(35d^2e^2+38cdfe+35c^2f^2\right)+1\right.\right.}{2}$$

$$\Big\downarrow \text{221}$$

$$\frac{C(a+bx)^3(c+dx)^{3/2}(e+fx)^{3/2}}{6bdf}-$$

$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(\left(7C\left(15d^3e^3+17cd^2fe^2+17c^2df^2e+15c^3f^3\right)+4df\left(50Adf(de+cf)-B\left(35d^2e^2+38cdfe+35c^2f^2\right)\right)\right)b^3-8adf\left(C\left(35d^2e^2+38cdfe+35c^2f^2\right)+1\right.\right.}{2}$$

input
```
Int[(a + b*x)^2*Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2),x]
```

output
```
(C*(a + b*x)^3*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(6*b*d*f) - (-1/5*((4*b*B*
d*f - 2*a*C*d*f - 3*b*C*(d*e + c*f))*(a + b*x)^2*(c + d*x)^(3/2)*(e + f*x)
^(3/2))/(d*f) + (((c + d*x)^(3/2)*(e + f*x)^(3/2)*(64*a^3*C*d^3*f^3 - 8*a^
2*b*d^2*f^2*(16*B*d*f - 7*C*(d*e + c*f)) - 8*a*b^2*d*f*(C*(35*d^2*e^2 + 38
*c*d*e*f + 35*c^2*f^2) + 10*d*f*(8*A*d*f - 5*B*(d*e + c*f))) + b^3*(7*C*(1
5*d^3*e^3 + 17*c*d^2*e^2*f + 17*c^2*d*e*f^2 + 15*c^3*f^3) + 4*d*f*(50*A*d*
f*(d*e + c*f) - B*(35*d^2*e^2 + 38*c*d*e*f + 35*c^2*f^2))) + 6*b*d*f*(10*b
*d*f*(2*b*c*C*e + a*C*d*e + a*c*C*f - 4*A*b*d*f) - (4*a*d*f - 7*b*(d*e + c
*f))*(4*b*B*d*f - 2*a*C*d*f - 3*b*C*(d*e + c*f)))*x))/(24*d^2*f^2) - (5*b*
(8*a^2*d^2*f^2*(C*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2) + 8*d*f*(2*A*d*f - B
*(d*e + c*f))) - 8*a*b*d*f*(C*(7*d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 +
 7*c^3*f^3) + 2*d*f*(8*A*d*f*(d*e + c*f) - B*(5*d^2*e^2 + 6*c*d*e*f + 5*c^
2*f^2))) + b^2*(C*(21*d^4*e^4 + 28*c*d^3*e^3*f + 30*c^2*d^2*e^2*f^2 + 28*c
^3*d*e*f^3 + 21*c^4*f^4) + 4*d*f*(2*A*d*f*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f
^2) - B*(7*d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 7*c^3*f^3))))*(((c +
d*x)^(3/2)*Sqrt[e + f*x])/(2*d) + ((d*e - c*f)*((Sqrt[c + d*x]*Sqrt[e + f*
x])/f - ((d*e - c*f)*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x
])])/(Sqrt[d]*f^(3/2))))/(4*d)))/(16*d^2*f^2))/(10*d*f))/(4*b*d*f)
```

3.41. $\int (a+bx)^2\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)\,dx$

3.41.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

3.41.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5733 vs. 2(1304) = 2608.

Time = 1.68 (sec) , antiderivative size = 5734, normalized size of antiderivative = 4.25

method	result	size
default	Expression too large to display	5734

input
```
int((b*x+a)^2*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
result too large to display
```

3.41.5 Fricas [A] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 3096, normalized size of antiderivative = 2.30

$$\int (a+bx)^2\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algorithm
="fricas")
```

output
```
[1/30720*(15*(21*C*b^2*d^6*e^6 - 14*(C*b^2*c*d^5 + 2*(2*C*a*b + B*b^2)*d^6
)*e^5*f - 5*(C*b^2*c^2*d^4 - 4*(2*C*a*b + B*b^2)*c*d^5 - 8*(C*a^2 + 2*B*a*
b + A*b^2)*d^6)*e^4*f^2 - 4*(C*b^2*c^3*d^3 - 2*(2*C*a*b + B*b^2)*c^2*d^4 +
 8*(C*a^2 + 2*B*a*b + A*b^2)*c*d^5 + 16*(B*a^2 + 2*A*a*b)*d^6)*e^3*f^3 - (
5*C*b^2*c^4*d^2 - 128*A*a^2*d^6 - 8*(2*C*a*b + B*b^2)*c^3*d^3 + 16*(C*a^2
+ 2*B*a*b + A*b^2)*c^2*d^4 - 64*(B*a^2 + 2*A*a*b)*c*d^5)*e^2*f^4 - 2*(7*C*
b^2*c^5*d + 128*A*a^2*c*d^5 - 10*(2*C*a*b + B*b^2)*c^4*d^2 + 16*(C*a^2 + 2
*B*a*b + A*b^2)*c^3*d^3 - 32*(B*a^2 + 2*A*a*b)*c^2*d^4)*e*f^5 + (21*C*b^2*
c^6 + 128*A*a^2*c^2*d^4 - 28*(2*C*a*b + B*b^2)*c^5*d + 40*(C*a^2 + 2*B*a*b
 + A*b^2)*c^4*d^2 - 64*(B*a^2 + 2*A*a*b)*c^3*d^3)*f^6)*sqrt(d*f)*log(8*d^2
*f^2*x^2 + d^2*e^2 + 6*c*d*e*f + c^2*f^2 - 4*(2*d*f*x + d*e + c*f)*sqrt(d*
f)*sqrt(d*x + c)*sqrt(f*x + e) + 8*(d^2*e*f + c*d*f^2)*x) + 4*(1280*C*b^2*
d^6*f^6*x^5 + 315*C*b^2*d^6*e^5*f - 105*(C*b^2*c*d^5 + 4*(2*C*a*b + B*b^2)
*d^6)*e^4*f^2 - 2*(41*C*b^2*c^2*d^4 - 80*(2*C*a*b + B*b^2)*c*d^5 - 300*(C*
a^2 + 2*B*a*b + A*b^2)*d^6)*e^3*f^3 - 2*(41*C*b^2*c^3*d^3 - 68*(2*C*a*b +
B*b^2)*c^2*d^4 + 140*(C*a^2 + 2*B*a*b + A*b^2)*c*d^5 + 480*(B*a^2 + 2*A*a*
b)*d^6)*e^2*f^4 - 5*(21*C*b^2*c^4*d^2 - 384*A*a^2*d^6 - 32*(2*C*a*b + B*b^
2)*c^3*d^3 + 56*(C*a^2 + 2*B*a*b + A*b^2)*c^2*d^4 - 128*(B*a^2 + 2*A*a*b)*
c*d^5)*e*f^5 + 15*(21*C*b^2*c^5*d + 128*A*a^2*c*d^5 - 28*(2*C*a*b + B*b^2)
*c^4*d^2 + 40*(C*a^2 + 2*B*a*b + A*b^2)*c^3*d^3 - 64*(B*a^2 + 2*A*a*b)*...
```

3.41.6 Sympy [F]

$$\int (a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx = \int (a+bx)^2\,\sqrt{c+dx}\,\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx$$

input
```
integrate((b*x+a)**2*(C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2),x)
```

output
```
Integral((a + b*x)**2*sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2), x)
```

3.41.7 Maxima [F(-2)]

Exception generated.

$$\int (a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^2*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f+d*e>0)', see `assume?` for m
ore detail
```

3.41.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4656 vs. $2(1304) = 2608$.

Time = 0.86 (sec) , antiderivative size = 4656, normalized size of antiderivative = 3.45

$$\int (a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algorithm
="giac")
```

output
```
-1/7680*(7680*((d^2*e - c*d*f)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt(d^2
*e + (d*x + c)*d*f - c*d*f)))/sqrt(d*f) - sqrt(d^2*e + (d*x + c)*d*f - c*d
*f)*sqrt(d*x + c))*A*a^2*c*abs(d)/d^2 - 320*(sqrt(d^2*e + (d*x + c)*d*f -
c*d*f)*sqrt(d*x + c)*(2*(d*x + c)*(4*(d*x + c)/d^2 + (d^6*e*f^3 - 13*c*d^5
*f^4)/(d^7*f^4)) - 3*(d^7*e^2*f^2 + 2*c*d^6*e*f^3 - 11*c^2*d^5*f^4)/(d^7*f
^4)) - 3*(d^3*e^3 + c*d^2*e^2*f + 3*c^2*d*e*f^2 - 5*c^3*f^3)*log(abs(-sqrt
(d*f)*sqrt(d*x + c) + sqrt(d^2*e + (d*x + c)*d*f - c*d*f)))/(sqrt(d*f)*d*f
^2))*C*a^2*c*abs(d)/d^2 - 640*(sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*sqrt(d*
x + c)*(2*(d*x + c)*(4*(d*x + c)/d^2 + (d^6*e*f^3 - 13*c*d^5*f^4)/(d^7*f^4
)) - 3*(d^7*e^2*f^2 + 2*c*d^6*e*f^3 - 11*c^2*d^5*f^4)/(d^7*f^4)) - 3*(d^3*
e^3 + c*d^2*e^2*f + 3*c^2*d*e*f^2 - 5*c^3*f^3)*log(abs(-sqrt(d*f)*sqrt(d*x
 + c) + sqrt(d^2*e + (d*x + c)*d*f - c*d*f)))/(sqrt(d*f)*d*f^2))*B*a*b*c*a
bs(d)/d^2 - 80*(sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*(2*(d*x + c)*(4*(d*x +
 c)*(6*(d*x + c)/d^3 + (d^12*e*f^5 - 25*c*d^11*f^6)/(d^14*f^6)) - (5*d^13*
e^2*f^4 + 14*c*d^12*e*f^5 - 163*c^2*d^11*f^6)/(d^14*f^6)) + 3*(5*d^14*e^3*
f^3 + 9*c*d^13*e^2*f^4 + 15*c^2*d^12*e*f^5 - 93*c^3*d^11*f^6)/(d^14*f^6))*
sqrt(d*x + c) + 3*(5*d^4*e^4 + 4*c*d^3*e^3*f + 6*c^2*d^2*e^2*f^2 + 20*c^3*
d*e*f^3 - 35*c^4*f^4)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt(d^2*e + (d*x
 + c)*d*f - c*d*f)))/(sqrt(d*f)*d^2*f^3))*C*a*b*c*abs(d)/d^2 - 320*(sqrt(d
^2*e + (d*x + c)*d*f - c*d*f)*sqrt(d*x + c)*(2*(d*x + c)*(4*(d*x + c)/d...
```

3.41.9 Mupad [F(-1)]

Timed out.

$$\int (a+bx)^2\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)\,dx = \text{Hanged}$$

input
```
int((e + f*x)^(1/2)*(a + b*x)^2*(c + d*x)^(1/2)*(A + B*x + C*x^2),x)
```

output
```
\text{Hanged}
```

3.42 $\int (a+bx)\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)\,dx$

3.42.1 Optimal result

Integrand size = 34, antiderivative size = 721

$$\int (a+bx)\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)\,dx$$
$$= \frac{(de-cf)\,(2adf(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf)))-b(C(7d^3e^3+9cd^2e^2f+9c^2def}{128d^4f^4}$$
$$+ \frac{(2adf(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf)))-b(C(7d^3e^3+9cd^2e^2f+9c^2def^2+7c^3}{64d^4f^3}$$
$$+ \frac{C(a+bx)^2(c+dx)^{3/2}(e+fx)^{3/2}}{5bdf}$$
$$- \frac{(c+dx)^{3/2}(e+fx)^{3/2}\,(48a^2Cd^2f^2-10abdf(8Bdf-5C(de+cf))-b^2(C(35d^2e^2+38cdef+35c^2f^2)}{240bd^3f^3}$$
$$- \frac{(de-cf)^2\,(2adf(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf)))-b(C(7d^3e^3+9cd^2e^2f+9c^2}{128d^{9/2}f^{9/2}}$$

output
```
1/5*C*(b*x+a)^2*(d*x+c)^(3/2)*(f*x+e)^(3/2)/b/d/f-1/240*(d*x+c)^(3/2)*(f*x
+e)^(3/2)*(48*a^2*C*d^2*f^2-10*a*b*d*f*(8*B*d*f-5*C*(c*f+d*e))-b^2*(C*(35*
c^2*f^2+38*c*d*e*f+35*d^2*e^2)+10*d*f*(8*A*d*f-5*B*(c*f+d*e)))+6*b*d*f*(6*
a*C*d*f-b*(10*B*d*f-7*C*(c*f+d*e)))*x)/b/d^3/f^3-1/128*(-c*f+d*e)^2*(2*a*d
*f*(C*(5*c^2*f^2+6*c*d*e*f+5*d^2*e^2)+8*d*f*(2*A*d*f-B*(c*f+d*e)))-b*(C*(7
*c^3*f^3+9*c^2*d*e*f^2+9*c*d^2*e^2*f+7*d^3*e^3)+2*d*f*(8*A*d*f*(c*f+d*e)-B
*(5*c^2*f^2+6*c*d*e*f+5*d^2*e^2))))*arctanh(f^(1/2)*(d*x+c)^(1/2)/d^(1/2)/
(f*x+e)^(1/2))/d^(9/2)/f^(9/2)+1/64*(2*a*d*f*(C*(5*c^2*f^2+6*c*d*e*f+5*d^2
*e^2)+8*d*f*(2*A*d*f-B*(c*f+d*e)))-b*(C*(7*c^3*f^3+9*c^2*d*e*f^2+9*c*d^2*e
^2*f+7*d^3*e^3)+2*d*f*(8*A*d*f*(c*f+d*e)-B*(5*c^2*f^2+6*c*d*e*f+5*d^2*e^2)
)))*(d*x+c)^(3/2)*(f*x+e)^(1/2)/d^4/f^3+1/128*(-c*f+d*e)*(2*a*d*f*(C*(5*c^
2*f^2+6*c*d*e*f+5*d^2*e^2)+8*d*f*(2*A*d*f-B*(c*f+d*e)))-b*(C*(7*c^3*f^3+9*
c^2*d*e*f^2+9*c*d^2*e^2*f+7*d^3*e^3)+2*d*f*(8*A*d*f*(c*f+d*e)-B*(5*c^2*f^2
+6*c*d*e*f+5*d^2*e^2))))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/d^4/f^4
```

3.42.2 Mathematica [A] (verified)

Time = 2.73 (sec) , antiderivative size = 662, normalized size of antiderivative = 0.92

$$\int (a+bx)\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx$$
$$= \frac{\sqrt{c+dx}\sqrt{e+fx}(10adf(C(15c^3f^3-c^2df^2(7e+10fx)+cd^2f(-7e^2+4efx+8f^2x^2)+d^3(15e^3-10e^2f\cdots}{\cdots}$$
$$+\frac{(de-cf)^2\left(-2adf(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf)))+b(C(7d^3e^3+9cd^2e^2f+9c\cdots\right.}{128d^{9/2}f^{9/2}}$$

input
```
Integrate[(a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2),x]
```

output
```
(Sqrt[c + d*x]*Sqrt[e + f*x]*(10*a*d*f*(C*(15*c^3*f^3 - c^2*d*f^2*(7*e + 1
0*f*x) + c*d^2*f*(-7*e^2 + 4*e*f*x + 8*f^2*x^2) + d^3*(15*e^3 - 10*e^2*f*x
 + 8*e*f^2*x^2 + 48*f^3*x^3)) + 8*d*f*(6*A*d*f*(c*f + d*(e + 2*f*x)) + B*(
-3*c^2*f^2 + 2*c*d*f*(e + f*x) + d^2*(-3*e^2 + 2*e*f*x + 8*f^2*x^2)))) + b
*(C*(-105*c^4*f^4 + 10*c^3*d*f^3*(4*e + 7*f*x) - 2*c^2*d^2*f^2*(-17*e^2 +
11*e*f*x + 28*f^2*x^2) + 2*c*d^3*f*(20*e^3 - 11*e^2*f*x + 8*e*f^2*x^2 + 24
*f^3*x^3) + d^4*(-105*e^4 + 70*e^3*f*x - 56*e^2*f^2*x^2 + 48*e*f^3*x^3 + 3
84*f^4*x^4)) + 10*d*f*(8*A*d*f*(-3*c^2*f^2 + 2*c*d*f*(e + f*x) + d^2*(-3*e
^2 + 2*e*f*x + 8*f^2*x^2)) + B*(15*c^3*f^3 - c^2*d*f^2*(7*e + 10*f*x) + c*
d^2*f*(-7*e^2 + 4*e*f*x + 8*f^2*x^2) + d^3*(15*e^3 - 10*e^2*f*x + 8*e*f^2*
x^2 + 48*f^3*x^3))))))/(1920*d^4*f^4) + ((d*e - c*f)^2*(-2*a*d*f*(C*(5*d^2
*e^2 + 6*c*d*e*f + 5*c^2*f^2) + 8*d*f*(2*A*d*f - B*(d*e + c*f))) + b*(C*(7
*d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 7*c^3*f^3) + 2*d*f*(8*A*d*f*(d*
e + c*f) - B*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2))))*ArcTanh[(Sqrt[d]*Sqrt[
e + f*x])/(Sqrt[f]*Sqrt[c + d*x])])/(128*d^(9/2)*f^(9/2))
```

3.42.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 461, normalized size of antiderivative = 0.64, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.206, Rules used = {2118, 27, 164, 60, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{2118}$$

$$\frac{\int -\frac{1}{2}b(a+bx)\sqrt{c+dx}\sqrt{e+fx}(4bcCe+3aCde+3acCf-10Abdf-(10bBdf-6aCdf-7bC(de+cf))x)dx}{5b^2df}+\frac{C(a+bx)^2(c+dx)^{3/2}(e+fx)^{3/2}}{5bdf}$$

$$\Big\downarrow \text{27}$$

$$\frac{C(a+bx)^2(c+dx)^{3/2}(e+fx)^{3/2}}{5bdf}-\frac{\int (a+bx)\sqrt{c+dx}\sqrt{e+fx}(4bcCe+3aCde+3acCf-10Abdf-(10bBdf-6aCdf-7bC(de+cf))x)dx}{10bdf}$$

$$\Big\downarrow \text{164}$$

$$\frac{C(a+bx)^2(c+dx)^{3/2}(e+fx)^{3/2}}{5bdf}-$$

$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}(48a^2Cd^2f^2-6bdfx(-6aCdf+10bBdf-7bC(cf+de))-10abdf(8Bdf-5C(cf+de))-(b^2(10df(8Adf-5B(cf+de))+C(35c^2f^2}{24d^2f^2}$$

$$\Big\downarrow \; 60$$

$$\frac{C(a+bx)^2(c+dx)^{3/2}(e+fx)^{3/2}}{5bdf}-$$

$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}(48a^2Cd^2f^2-6bdfx(-6aCdf+10bBdf-7bC(cf+de))-10abdf(8Bdf-5C(cf+de))-(b^2(10df(8Adf-5B(cf+de))+C(35c^2f^2}{24d^2f^2}$$

$$\Big\downarrow \; 60$$

$$\frac{C(a+bx)^2(c+dx)^{3/2}(e+fx)^{3/2}}{5bdf}-$$

$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}(48a^2Cd^2f^2-6bdfx(-6aCdf+10bBdf-7bC(cf+de))-10abdf(8Bdf-5C(cf+de))-(b^2(10df(8Adf-5B(cf+de))+C(35c^2f^2}{24d^2f^2}$$

$$\Big\downarrow \; 66$$

$$\frac{C(a+bx)^2(c+dx)^{3/2}(e+fx)^{3/2}}{5bdf}-$$

$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}(48a^2Cd^2f^2-6bdfx(-6aCdf+10bBdf-7bC(cf+de))-10abdf(8Bdf-5C(cf+de))-(b^2(10df(8Adf-5B(cf+de))+C(35c^2f^2}{24d^2f^2}$$

$$\Big\downarrow \; 221$$

$$\frac{C(a+bx)^2(c+dx)^{3/2}(e+fx)^{3/2}}{5bdf}-$$

$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}(48a^2Cd^2f^2-6bdfx(-6aCdf+10bBdf-7bC(cf+de))-10abdf(8Bdf-5C(cf+de))-(b^2(10df(8Adf-5B(cf+de))+C(35c^2f^2}{24d^2f^2}$$

3.42. $\int (a+bx)\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)\,dx$

input
```
Int[(a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2),x]
```

output
```
(C*(a + b*x)^2*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(5*b*d*f) - (((c + d*x)^(3
/2)*(e + f*x)^(3/2)*(48*a^2*C*d^2*f^2 - 10*a*b*d*f*(8*B*d*f - 5*C*(d*e + c
*f)) - b^2*(C*(35*d^2*e^2 + 38*c*d*e*f + 35*c^2*f^2) + 10*d*f*(8*A*d*f - 5
*B*(d*e + c*f))) - 6*b*d*f*(10*b*B*d*f - 6*a*C*d*f - 7*b*C*(d*e + c*f))*x)
)/(24*d^2*f^2) - (5*b*(2*a*d*f*(C*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2) + 8*
d*f*(2*A*d*f - B*(d*e + c*f))) - b*(C*(7*d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d
*e*f^2 + 7*c^3*f^3) + 2*d*f*(8*A*d*f*(d*e + c*f) - B*(5*d^2*e^2 + 6*c*d*e*
f + 5*c^2*f^2))))*(((c + d*x)^(3/2)*Sqrt[e + f*x])/(2*d) + ((d*e - c*f)*((
Sqrt[c + d*x]*Sqrt[e + f*x])/f - ((d*e - c*f)*ArcTanh[(Sqrt[f]*Sqrt[c + d*
x])/(Sqrt[d]*Sqrt[e + f*x])])/(Sqrt[d]*f^(3/2))))/(4*d)))/(16*d^2*f^2))/(1
0*b*d*f)
```

3.42.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

3.42.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3024 vs. $2(683) = 1366$.

Time = 1.67 (sec) , antiderivative size = 3025, normalized size of antiderivative = 4.20

method	result	size
default	Expression too large to display	3025

input
```
int((b*x+a)*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
-1/3840*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(-32*C*b*c*d^3*e*f^3*x^2*((d*x+c)*(f*x
+e))^(1/2)*(d*f)^(1/2)+200*B*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b*d^4*e^2
*f^2*x+480*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e
)/(d*f)^(1/2))*a*d^5*e^2*f^3-240*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/
2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c^3*d^2*f^5-960*A*ln(1/2*(2*d*f*x+2
*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c*d^4*e*f^4+2
40*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^
(1/2))*b*c^2*d^3*e*f^4+200*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*a*d^4*e^2
*f^2*x-140*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b*c^3*d*f^4*x+240*B*ln(1/
2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c
*d^4*e^2*f^3-120*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c
*f+d*e)/(d*f)^(1/2))*b*c^3*d^2*e*f^4+75*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+
e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c*d^4*e^4*f-960*A*((d*x+c)*(
f*x+e))^(1/2)*(d*f)^(1/2)*a*c*d^3*f^4-960*A*((d*x+c)*(f*x+e))^(1/2)*(d*f)^
(1/2)*a*d^4*e*f^3+480*A*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b*c^2*d^2*f^4+
480*A*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b*d^4*e^2*f^2+480*B*((d*x+c)*(f*
x+e))^(1/2)*(d*f)^(1/2)*a*c^2*d^2*f^4-300*B*((d*x+c)*(f*x+e))^(1/2)*(d*f)^
(1/2)*b*d^4*e^3*f-1920*A*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*a*d^4*f^4*x-7
68*C*b*d^4*f^4*x^4*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)-960*B*b*d^4*f^4*x^3
*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)-960*C*a*d^4*f^4*x^3*((d*x+c)*(f*x+...
```

3.42.5 Fricas [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 1620, normalized size of antiderivative = 2.25

$$\int (a+bx)\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algorithm="
fricas")
```

output
```
[-1/7680*(15*(7*C*b*d^5*e^5 - 5*(C*b*c*d^4 + 2*(C*a + B*b)*d^5)*e^4*f - 2*
(C*b*c^2*d^3 - 4*(C*a + B*b)*c*d^4 - 8*(B*a + A*b)*d^5)*e^3*f^2 - 2*(C*b*c
^3*d^2 + 16*A*a*d^5 - 2*(C*a + B*b)*c^2*d^3 + 8*(B*a + A*b)*c*d^4)*e^2*f^3
 - (5*C*b*c^4*d - 64*A*a*c*d^4 - 8*(C*a + B*b)*c^3*d^2 + 16*(B*a + A*b)*c^
2*d^3)*e*f^4 + (7*C*b*c^5 - 32*A*a*c^2*d^3 - 10*(C*a + B*b)*c^4*d + 16*(B*
a + A*b)*c^3*d^2)*f^5)*sqrt(d*f)*log(8*d^2*f^2*x^2 + d^2*e^2 + 6*c*d*e*f +
 c^2*f^2 - 4*(2*d*f*x + d*e + c*f)*sqrt(d*f)*sqrt(d*x + c)*sqrt(f*x + e) +
 8*(d^2*e*f + c*d*f^2)*x) - 4*(384*C*b*d^5*f^5*x^4 - 105*C*b*d^5*e^4*f + 1
0*(4*C*b*c*d^4 + 15*(C*a + B*b)*d^5)*e^3*f^2 + 2*(17*C*b*c^2*d^3 - 35*(C*a
 + B*b)*c*d^4 - 120*(B*a + A*b)*d^5)*e^2*f^3 + 10*(4*C*b*c^3*d^2 + 48*A*a*
d^5 - 7*(C*a + B*b)*c^2*d^3 + 16*(B*a + A*b)*c*d^4)*e*f^4 - 15*(7*C*b*c^4*
d - 32*A*a*c*d^4 - 10*(C*a + B*b)*c^3*d^2 + 16*(B*a + A*b)*c^2*d^3)*f^5 +
48*(C*b*d^5*e*f^4 + (C*b*c*d^4 + 10*(C*a + B*b)*d^5)*f^5)*x^3 - 8*(7*C*b*d
^5*e^2*f^3 - 2*(C*b*c*d^4 + 5*(C*a + B*b)*d^5)*e*f^4 + (7*C*b*c^2*d^3 - 10
*(C*a + B*b)*c*d^4 - 80*(B*a + A*b)*d^5)*f^5)*x^2 + 2*(35*C*b*d^5*e^3*f^2
- (11*C*b*c*d^4 + 50*(C*a + B*b)*d^5)*e^2*f^3 - (11*C*b*c^2*d^3 - 20*(C*a
+ B*b)*c*d^4 - 80*(B*a + A*b)*d^5)*e*f^4 + 5*(7*C*b*c^3*d^2 + 96*A*a*d^5 -
 10*(C*a + B*b)*c^2*d^3 + 16*(B*a + A*b)*c*d^4)*f^5)*x)*sqrt(d*x + c)*sqrt
(f*x + e))/(d^5*f^5), -1/3840*(15*(7*C*b*d^5*e^5 - 5*(C*b*c*d^4 + 2*(C*a +
 B*b)*d^5)*e^4*f - 2*(C*b*c^2*d^3 - 4*(C*a + B*b)*c*d^4 - 8*(B*a + A*b)...
```

3.42.6 Sympy [F]

$$\int (a+bx)\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx$$
$$=\int (a+bx)\,\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx$$

input
```
integrate((b*x+a)*(C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2),x)
```

output
```
Integral((a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2), x)
```

3.42.7 Maxima [F(-2)]

Exception generated.

$$\int (a+bx)\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right) \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f+d*e>0)', see `assume?` for m
ore detail
```

3.42.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2592 vs. 2(683) = 1366.

Time = 0.62 (sec) , antiderivative size = 2592, normalized size of antiderivative = 3.60

$$\int (a+bx)\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right) \, dx = \text{Too large to display}$$

input
```
integrate((b*x+a)*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algorithm="
giac")
```

output
```
-1/1920*(1920*((d^2*e - c*d*f)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt(d^2
*e + (d*x + c)*d*f - c*d*f)))/sqrt(d*f) - sqrt(d^2*e + (d*x + c)*d*f - c*d
*f)*sqrt(d*x + c))*A*a*c*abs(d)/d^2 - 80*(sqrt(d^2*e + (d*x + c)*d*f - c*d
*f)*sqrt(d*x + c)*(2*(d*x + c)*(4*(d*x + c)/d^2 + (d^6*e*f^3 - 13*c*d^5*f^
4)/(d^7*f^4)) - 3*(d^7*e^2*f^2 + 2*c*d^6*e*f^3 - 11*c^2*d^5*f^4)/(d^7*f^4)
) - 3*(d^3*e^3 + c*d^2*e^2*f + 3*c^2*d*e*f^2 - 5*c^3*f^3)*log(abs(-sqrt(d*
f)*sqrt(d*x + c) + sqrt(d^2*e + (d*x + c)*d*f - c*d*f)))/(sqrt(d*f)*d*f^2)
)*C*a*c*abs(d)/d^2 - 80*(sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*sqrt(d*x + c)
*(2*(d*x + c)*(4*(d*x + c)/d^2 + (d^6*e*f^3 - 13*c*d^5*f^4)/(d^7*f^4)) - 3
*(d^7*e^2*f^2 + 2*c*d^6*e*f^3 - 11*c^2*d^5*f^4)/(d^7*f^4)) - 3*(d^3*e^3 +
c*d^2*e^2*f + 3*c^2*d*e*f^2 - 5*c^3*f^3)*log(abs(-sqrt(d*f)*sqrt(d*x + c)
+ sqrt(d^2*e + (d*x + c)*d*f - c*d*f)))/(sqrt(d*f)*d*f^2))*B*b*c*abs(d)/d^
2 - 10*(sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*(2*(d*x + c)*(4*(d*x + c)*(6*(
d*x + c)/d^3 + (d^12*e*f^5 - 25*c*d^11*f^6)/(d^14*f^6)) - (5*d^13*e^2*f^4
+ 14*c*d^12*e*f^5 - 163*c^2*d^11*f^6)/(d^14*f^6)) + 3*(5*d^14*e^3*f^3 + 9*
c*d^13*e^2*f^4 + 15*c^2*d^12*e*f^5 - 93*c^3*d^11*f^6)/(d^14*f^6))*sqrt(d*x
 + c) + 3*(5*d^4*e^4 + 4*c*d^3*e^3*f + 6*c^2*d^2*e^2*f^2 + 20*c^3*d*e*f^3
- 35*c^4*f^4)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt(d^2*e + (d*x + c)*d*
f - c*d*f)))/(sqrt(d*f)*d^2*f^3))*C*b*c*abs(d)/d^2 - 80*(sqrt(d^2*e + (d*x
 + c)*d*f - c*d*f)*sqrt(d*x + c)*(2*(d*x + c)*(4*(d*x + c)/d^2 + (d^6*e...
```

3.42.9 Mupad [F(-1)]

Timed out.

$$\int (a+bx)\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)\,dx = \text{Hanged}$$

input
```
int((e + f*x)^(1/2)*(a + b*x)*(c + d*x)^(1/2)*(A + B*x + C*x^2),x)
```

output
```
\text{Hanged}
```

3.43 $\int \sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)\,dx$

3.43.1 Optimal result

Integrand size = 29, antiderivative size = 330

$$\begin{aligned}
&\int \sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)\,dx \\
&= \frac{(de-cf)\left(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf))\right)\sqrt{c+dx}\sqrt{e+fx}}{64d^3f^3} \\
&+ \frac{\left(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf))\right)(c+dx)^{3/2}\sqrt{e+fx}}{32d^3f^2} \\
&- \frac{(5Cde+11cCf-8Bdf)(c+dx)^{3/2}(e+fx)^{3/2}}{24d^2f^2} + \frac{C(c+dx)^{5/2}(e+fx)^{3/2}}{4d^2f} \\
&- \frac{(de-cf)^2\left(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf))\right)\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)}{64d^{7/2}f^{7/2}}
\end{aligned}$$

output
```
-1/24*(-8*B*d*f+11*C*c*f+5*C*d*e)*(d*x+c)^(3/2)*(f*x+e)^(3/2)/d^2/f^2+1/4*
C*(d*x+c)^(5/2)*(f*x+e)^(3/2)/d^2/f-1/64*(-c*f+d*e)^2*(C*(5*c^2*f^2+6*c*d*
e*f+5*d^2*e^2)+8*d*f*(2*A*d*f-B*(c*f+d*e)))*arctanh(f^(1/2)*(d*x+c)^(1/2)/
d^(1/2)/(f*x+e)^(1/2))/d^(7/2)/f^(7/2)+1/32*(C*(5*c^2*f^2+6*c*d*e*f+5*d^2*
e^2)+8*d*f*(2*A*d*f-B*(c*f+d*e)))*(d*x+c)^(3/2)*(f*x+e)^(1/2)/d^3/f^2+1/64
*(-c*f+d*e)*(C*(5*c^2*f^2+6*c*d*e*f+5*d^2*e^2)+8*d*f*(2*A*d*f-B*(c*f+d*e))
)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/d^3/f^3
```

3.43.2 Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.86

$$\int \sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx$$
$$= \frac{\sqrt{c+dx}\sqrt{e+fx}(C(15c^3f^3-c^2df^2(7e+10fx)+cd^2f(-7e^2+4efx+8f^2x^2)+d^3(15e^3-10e^2fx+8e}{192}$$
$$-\frac{(de-cf)^2\left(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf))\right)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{f}\sqrt{c+dx}}\right)}{64d^{7/2}f^{7/2}}$$

input
```
Integrate[Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2),x]
```

output
```
(Sqrt[c + d*x]*Sqrt[e + f*x]*(C*(15*c^3*f^3 - c^2*d*f^2*(7*e + 10*f*x) + c
*d^2*f*(-7*e^2 + 4*e*f*x + 8*f^2*x^2) + d^3*(15*e^3 - 10*e^2*f*x + 8*e*f^2
*x^2 + 48*f^3*x^3)) + 8*d*f*(6*A*d*f*(c*f + d*(e + 2*f*x)) + B*(-3*c^2*f^2
 + 2*c*d*f*(e + f*x) + d^2*(-3*e^2 + 2*e*f*x + 8*f^2*x^2)))))/(192*d^3*f^3
) - ((d*e - c*f)^2*(C*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2) + 8*d*f*(2*A*d*f
 - B*(d*e + c*f)))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/(Sqrt[f]*Sqrt[c + d*x])
])/(64*d^(7/2)*f^(7/2))
```

3.43.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.75, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1194, 27, 90, 60, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx$$
$$\Big\downarrow \text{1194}$$
$$\frac{\int -\frac{1}{2}\sqrt{c+dx}\sqrt{e+fx}(3Cfc^2+5Cdec-8Ad^2f+d(5Cde+11cCf-8Bdf)x)\,dx}{4d^2f}+\frac{C(c+dx)^{5/2}(e+fx)^{3/2}}{4d^2f}$$
$$\Big\downarrow \text{27}$$

$$\frac{C(c+dx)^{5/2}(e+fx)^{3/2}}{4d^2f}-\frac{\int \sqrt{c+dx}\sqrt{e+fx}\left(3Cfc^2+5Cdec-8Ad^2f+d(5Cde+11cCf-8Bdf)x\right)dx}{8d^2f}$$

$\downarrow$ 90

$$\frac{C(c+dx)^{5/2}(e+fx)^{3/2}}{4d^2f}-\frac{\frac{(c+dx)^{3/2}(e+fx)^{3/2}(-8Bdf+11cCf+5Cde)}{3f}-\frac{\left(8df(2Adf-B(cf+de))+C\left(5c^2f^2+6cdef+5d^2e^2\right)\right)\int \sqrt{c+dx}\sqrt{e+fx}dx}{2f}}{8d^2f}$$

$\downarrow$ 60

$$\frac{C(c+dx)^{5/2}(e+fx)^{3/2}}{4d^2f}-\frac{\frac{(c+dx)^{3/2}(e+fx)^{3/2}(-8Bdf+11cCf+5Cde)}{3f}-\frac{\left(8df(2Adf-B(cf+de))+C\left(5c^2f^2+6cdef+5d^2e^2\right)\right)\left(\frac{(de-cf)\int \frac{\sqrt{c+dx}}{\sqrt{e+fx}}dx}{4d}+\frac{(c+dx)^{3/2}\sqrt{e+fx}}{2d}\right)}{2f}}{8d^2f}$$

$\downarrow$ 60

$$\frac{C(c+dx)^{5/2}(e+fx)^{3/2}}{4d^2f}-\frac{\frac{(c+dx)^{3/2}(e+fx)^{3/2}(-8Bdf+11cCf+5Cde)}{3f}-\frac{\left(8df(2Adf-B(cf+de))+C\left(5c^2f^2+6cdef+5d^2e^2\right)\right)\left(\frac{(de-cf)\left(\frac{\sqrt{c+dx}\sqrt{e+fx}}{f}-\frac{(de-cf)\int \frac{1}{\sqrt{c+dx}\sqrt{\cdots}}}{2f}\right.}{4d}\cdots\right.}{2f}}{8d^2f}$$

$\downarrow$ 66

$$\frac{C(c+dx)^{5/2}(e+fx)^{3/2}}{4d^2f}-\frac{\frac{(c+dx)^{3/2}(e+fx)^{3/2}(-8Bdf+11cCf+5Cde)}{3f}-\frac{\left(8df(2Adf-B(cf+de))+C\left(5c^2f^2+6cdef+5d^2e^2\right)\right)\left(\frac{(de-cf)\left(\frac{\sqrt{c+dx}\sqrt{e+fx}}{f}-\frac{(de-cf)\int \frac{1}{d-\frac{f(c+\cdots}{e+f\cdots}}}{f}\right.}{4d}\cdots\right.}{2f}}{8d^2f}$$

$\downarrow$ 221

3.43. $\int \sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)\,dx$

$$\frac{C(c+dx)^{5/2}(e+fx)^{3/2}}{4d^2f} - \frac{\frac{(c+dx)^{3/2}(e+fx)^{3/2}(-8Bdf+11cCf+5Cde)}{3f} - \frac{\left(\frac{(de-cf)\left(\frac{\sqrt{c+dx}\sqrt{e+fx}}{f}-\frac{(de-cf)\text{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)}{\sqrt{d}f^{3/2}}\right)}{4d}+\frac{(c+dx)^{3/2}\sqrt{e+fx}}{2d}\right)(8df(2Adf-B(c}{2f}}{8d^2f}$$

input
```
Int[Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2),x]
```

output
```
(C*(c + d*x)^(5/2)*(e + f*x)^(3/2))/(4*d^2*f) - (((5*C*d*e + 11*c*C*f - 8*
B*d*f)*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(3*f) - ((C*(5*d^2*e^2 + 6*c*d*e*f
 + 5*c^2*f^2) + 8*d*f*(2*A*d*f - B*(d*e + c*f)))*(((c + d*x)^(3/2)*Sqrt[e
+ f*x])/(2*d) + ((d*e - c*f)*((Sqrt[c + d*x]*Sqrt[e + f*x])/f - ((d*e - c*
f)*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(Sqrt[d]*f^(3
/2))))/(4*d)))/(2*f))/(8*d^2*f)
```

3.43.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1194
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Simp[1/(g*e^(2*p)*(m + n + 2
*p + 1))   Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(
2*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)
*(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ
[p, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && NeQ[m + n + 2*p + 1, 0]
```

3.43.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1206 vs. $2(292) = 584$.

Time = 1.66 (sec) , antiderivative size = 1207, normalized size of antiderivative = 3.66

method	result	size
default	Expression too large to display	1207

input
```
int((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/384*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(-6*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e
))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c^2*d^2*e^2*f^2+15*C*ln(1/2*(2*
d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c^4*f^4+
15*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^
(1/2))*d^4*e^4-12*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+
c*f+d*e)/(d*f)^(1/2))*c^3*d*e*f^3-16*C*c*d^2*f^3*x^2*((d*x+c)*(f*x+e))^(1/
2)*(d*f)^(1/2)-16*C*d^3*e*f^2*x^2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)-32*B
*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*d^3*e*f^2*x+20*C*(d*f)^(1/2)*((d*x+c)
*(f*x+e))^(1/2)*c^2*d*f^3*x-96*A*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*c*d^2
*f^3-96*A*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*d^3*e*f^2+48*B*(d*f)^(1/2)*(
(d*x+c)*(f*x+e))^(1/2)*c^2*d*f^3+48*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*
d^3*e^2*f-96*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d
*e)/(d*f)^(1/2))*c*d^3*e*f^3-192*A*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*d^3
*f^3*x-8*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*c*d^2*e*f^2*x+48*A*ln(1/2*(
2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c^2*d^
2*f^4+48*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/
(d*f)^(1/2))*d^4*e^2*f^2-24*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d
*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c^3*d*f^4-24*B*ln(1/2*(2*d*f*x+2*((d*x+c)*
(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*d^4*e^3*f-30*C*(d*f)^(1/2
)*((d*x+c)*(f*x+e))^(1/2)*c^3*f^3-30*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1...
```

3.43.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 840, normalized size of antiderivative = 2.55

$$\int \sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)\ dx$$

$$= \left[\frac{3\,(5\,Cd^4e^4-4\,(Ccd^3+2\,Bd^4)e^3f-2\,(Cc^2d^2-4\,Bcd^3-8\,Ad^4)e^2f^2-4\,(Cc^3d-2\,Bc^2d^2+8\,Acd^3)ef^3}{}\right.$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
[1/768*(3*(5*C*d^4*e^4 - 4*(C*c*d^3 + 2*B*d^4)*e^3*f - 2*(C*c^2*d^2 - 4*B*
c*d^3 - 8*A*d^4)*e^2*f^2 - 4*(C*c^3*d - 2*B*c^2*d^2 + 8*A*c*d^3)*e*f^3 + (
5*C*c^4 - 8*B*c^3*d + 16*A*c^2*d^2)*f^4)*sqrt(d*f)*log(8*d^2*f^2*x^2 + d^2
*e^2 + 6*c*d*e*f + c^2*f^2 - 4*(2*d*f*x + d*e + c*f)*sqrt(d*f)*sqrt(d*x +
c)*sqrt(f*x + e) + 8*(d^2*e*f + c*d*f^2)*x) + 4*(48*C*d^4*f^4*x^3 + 15*C*d
^4*e^3*f - (7*C*c*d^3 + 24*B*d^4)*e^2*f^2 - (7*C*c^2*d^2 - 16*B*c*d^3 - 48
*A*d^4)*e*f^3 + 3*(5*C*c^3*d - 8*B*c^2*d^2 + 16*A*c*d^3)*f^4 + 8*(C*d^4*e*
f^3 + (C*c*d^3 + 8*B*d^4)*f^4)*x^2 - 2*(5*C*d^4*e^2*f^2 - 2*(C*c*d^3 + 4*B
*d^4)*e*f^3 + (5*C*c^2*d^2 - 8*B*c*d^3 - 48*A*d^4)*f^4)*x)*sqrt(d*x + c)*s
qrt(f*x + e))/(d^4*f^4), 1/384*(3*(5*C*d^4*e^4 - 4*(C*c*d^3 + 2*B*d^4)*e^3
*f - 2*(C*c^2*d^2 - 4*B*c*d^3 - 8*A*d^4)*e^2*f^2 - 4*(C*c^3*d - 2*B*c^2*d^
2 + 8*A*c*d^3)*e*f^3 + (5*C*c^4 - 8*B*c^3*d + 16*A*c^2*d^2)*f^4)*sqrt(-d*f
)*arctan(1/2*(2*d*f*x + d*e + c*f)*sqrt(-d*f)*sqrt(d*x + c)*sqrt(f*x + e)/
(d^2*f^2*x^2 + c*d*e*f + (d^2*e*f + c*d*f^2)*x)) + 2*(48*C*d^4*f^4*x^3 + 1
5*C*d^4*e^3*f - (7*C*c*d^3 + 24*B*d^4)*e^2*f^2 - (7*C*c^2*d^2 - 16*B*c*d^3
 - 48*A*d^4)*e*f^3 + 3*(5*C*c^3*d - 8*B*c^2*d^2 + 16*A*c*d^3)*f^4 + 8*(C*d
^4*e*f^3 + (C*c*d^3 + 8*B*d^4)*f^4)*x^2 - 2*(5*C*d^4*e^2*f^2 - 2*(C*c*d^3
+ 4*B*d^4)*e*f^3 + (5*C*c^2*d^2 - 8*B*c*d^3 - 48*A*d^4)*f^4)*x)*sqrt(d*x +
 c)*sqrt(f*x + e))/(d^4*f^4)]
```

3.43.6 Sympy [F]

$$\int \sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx=\int \sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx$$

input
```
integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2),x)
```

output
```
Integral(sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2), x)
```

3.43.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f+d*e>0)', see `assume?` for m
ore detail
```

3.43.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1073 vs. $2(292) = 584$.

Time = 0.43 (sec) , antiderivative size = 1073, normalized size of antiderivative = 3.25

$$\int \sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
-1/192*(192*((d^2*e - c*d*f)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt(d^2*e
 + (d*x + c)*d*f - c*d*f)))/sqrt(d*f) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f
)*sqrt(d*x + c))*A*c*abs(d)/d^2 - 8*(sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*s
qrt(d*x + c)*(2*(d*x + c)*(4*(d*x + c)/d^2 + (d^6*e*f^3 - 13*c*d^5*f^4)/(d
^7*f^4)) - 3*(d^7*e^2*f^2 + 2*c*d^6*e*f^3 - 11*c^2*d^5*f^4)/(d^7*f^4)) - 3
*(d^3*e^3 + c*d^2*e^2*f + 3*c^2*d*e*f^2 - 5*c^3*f^3)*log(abs(-sqrt(d*f)*sq
rt(d*x + c) + sqrt(d^2*e + (d*x + c)*d*f - c*d*f)))/(sqrt(d*f)*d*f^2))*C*c
*abs(d)/d^2 - 8*(sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*sqrt(d*x + c)*(2*(d*x
 + c)*(4*(d*x + c)/d^2 + (d^6*e*f^3 - 13*c*d^5*f^4)/(d^7*f^4)) - 3*(d^7*e^
2*f^2 + 2*c*d^6*e*f^3 - 11*c^2*d^5*f^4)/(d^7*f^4)) - 3*(d^3*e^3 + c*d^2*e^
2*f + 3*c^2*d*e*f^2 - 5*c^3*f^3)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt(d
^2*e + (d*x + c)*d*f - c*d*f)))/(sqrt(d*f)*d*f^2))*B*abs(d)/d - (sqrt(d^2*
e + (d*x + c)*d*f - c*d*f)*(2*(d*x + c)*(4*(d*x + c)*(6*(d*x + c)/d^3 + (d
^12*e*f^5 - 25*c*d^11*f^6)/(d^14*f^6)) - (5*d^13*e^2*f^4 + 14*c*d^12*e*f^5
 - 163*c^2*d^11*f^6)/(d^14*f^6)) + 3*(5*d^14*e^3*f^3 + 9*c*d^13*e^2*f^4 +
15*c^2*d^12*e*f^5 - 93*c^3*d^11*f^6)/(d^14*f^6))*sqrt(d*x + c) + 3*(5*d^4*
e^4 + 4*c*d^3*e^3*f + 6*c^2*d^2*e^2*f^2 + 20*c^3*d*e*f^3 - 35*c^4*f^4)*log
(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt(d^2*e + (d*x + c)*d*f - c*d*f)))/(sqr
t(d*f)*d^2*f^3))*C*abs(d)/d - 48*(sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*(2*d
*x + 2*c + (d*e*f - 5*c*f^2)/f^2)*sqrt(d*x + c) + (d^3*e^2 + 2*c*d^2*e*...
```

3.43.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx = \text{Hanged}$$

input
```
int((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2),x)
```

output
```
\text{Hanged}
```

3.44 $\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{a+bx}\,dx$

3.44.1 Optimal result

Integrand size = 36, antiderivative size = 450

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{a+bx}\,dx$$
$$= \frac{(4bdf(2Abdf-aC(de+cf))+(bde-bcf+4adf)(2aCdf+b(Cde+cCf-2Bdf)))\sqrt{c+dx}\sqrt{e+fx}}{8b^3d^2f^2}$$
$$-\frac{(2aCdf+b(Cde+cCf-2Bdf))\sqrt{c+dx}(e+fx)^{3/2}}{4b^2df^2}+\frac{C(c+dx)^{3/2}(e+fx)^{3/2}}{3bdf}$$
$$-\frac{(16a^3Cd^3f^3-8a^2bd^2f^2(Cde+cCf+2Bdf)-2ab^2df(C(de-cf)^2-4df(Bde+Bcf+2Adf))-b^3(}{8b^4d^{5/2}f^{5/2}}$$
$$-\frac{2(Ab^2-a(bB-aC))\sqrt{bc-ad}\sqrt{be-af}\,\text{arctanh}\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{bc-ad}\sqrt{e+fx}}\right)}{b^4}$$

output
```
1/3*C*(d*x+c)^(3/2)*(f*x+e)^(3/2)/b/d/f-1/8*(16*a^3*C*d^3*f^3-8*a^2*b*d^2*
f^2*(2*B*d*f+C*c*f+C*d*e)-2*a*b^2*d*f*(C*(-c*f+d*e)^2-4*d*f*(2*A*d*f+B*c*f
+B*d*e))-b^3*(C*(-c*f+d*e)^2*(c*f+d*e)-2*d*f*(B*(-c*f+d*e)^2-4*A*d*f*(c*f+
d*e))))*arctanh(f^(1/2)*(d*x+c)^(1/2)/d^(1/2)/(f*x+e)^(1/2))/b^4/d^(5/2)/f
^(5/2)-2*(A*b^2-a*(B*b-C*a))*arctanh((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-a*d+
b*c)^(1/2)/(f*x+e)^(1/2))*(-a*d+b*c)^(1/2)*(-a*f+b*e)^(1/2)/b^4-1/4*(2*a*C
*d*f+b*(-2*B*d*f+C*c*f+C*d*e))*(f*x+e)^(3/2)*(d*x+c)^(1/2)/b^2/d/f^2+1/8*(
4*b*d*f*(2*A*b*d*f-a*C*(c*f+d*e))+(4*a*d*f-b*c*f+b*d*e)*(2*a*C*d*f+b*(-2*B
*d*f+C*c*f+C*d*e)))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^3/d^2/f^2
```

3.44.2 Mathematica [A] (verified)

Time = 1.54 (sec) , antiderivative size = 404, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{a+bx}\,dx$$
$$= \frac{\frac{b\sqrt{c+dx}\sqrt{e+fx}\left(24a^2Cd^2f^2-6abdf(cCf+4Bdf+Cd(e+2fx))+b^2\left(6df(Bcf+4Adf+Bd(e+2fx))+C\left(-3c^2f^2+2cdf(e+fx)+d^2\left(-3e^2+2efx+8\right.\right.\right.\right.}{d^2f^2}}{}$$

input
```
Integrate[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x),x]
```

output
```
((b*Sqrt[c + d*x]*Sqrt[e + f*x]*(24*a^2*C*d^2*f^2 - 6*a*b*d*f*(c*C*f + 4*B
*d*f + C*d*(e + 2*f*x)) + b^2*(6*d*f*(B*c*f + 4*A*d*f + B*d*(e + 2*f*x)) +
 C*(-3*c^2*f^2 + 2*c*d*f*(e + f*x) + d^2*(-3*e^2 + 2*e*f*x + 8*f^2*x^2))))
)/(d^2*f^2) - 48*(A*b^2 + a*(-(b*B) + a*C))*Sqrt[b*c - a*d]*Sqrt[-(b*e) +
a*f]*ArcTan[(Sqrt[b*c - a*d]*Sqrt[e + f*x])/(Sqrt[-(b*e) + a*f]*Sqrt[c + d
*x])] + (3*(-16*a^3*C*d^3*f^3 + 8*a^2*b*d^2*f^2*(C*d*e + c*C*f + 2*B*d*f)
+ 2*a*b^2*d*f*(C*(d*e - c*f)^2 - 4*d*f*(B*d*e + B*c*f + 2*A*d*f)) + b^3*(C
*(d*e - c*f)^2*(d*e + c*f) + 2*d*f*(-(B*(d*e - c*f)^2) + 4*A*d*f*(d*e + c*
f))))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/(Sqrt[f]*Sqrt[c + d*x])])/(d^(5/2)*f
^(5/2)))/(24*b^4)
```

3.44.3 Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {2118, 27, 171, 27, 171, 27, 175, 66, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{a+bx}\,dx$$
$$\Big\downarrow \ 2118$$
$$\frac{\int \frac{3b\sqrt{c+dx}\sqrt{e+fx}(2Abdf-aC(de+cf)-(2aCdf+b(Cde+cCf-2Bdf))x)}{2(a+bx)}dx}{3b^2df}+\frac{C(c+dx)^{3/2}(e+fx)^{3/2}}{3bdf}$$
$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\sqrt{c+dx}\sqrt{e+fx}(2Abdf-aC(de+cf)-(2aCdf+b(Cde+cCf-2Bdf))x)}{a+bx}dx}{2bdf}+\frac{C(c+dx)^{3/2}(e+fx)^{3/2}}{3bdf}$$

$\downarrow$ 171

$$\frac{\frac{\int \frac{\sqrt{e+fx}(4bcf(2Abdf-aC(de+cf))+a(de+3cf)(2aCdf+b(Cde+cCf-2Bdf))+(4bdf(2Abdf-aC(de+cf))+(bde-bcf+4adf)(2aCdf+b(Cde+cCf-2Bdf)))x)}{2(a+bx)\sqrt{c+dx}}dx}{2bf}-}{2bdf}$$

$$\frac{C(c+dx)^{3/2}(e+fx)^{3/2}}{3bdf}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{\sqrt{e+fx}(4bcf(2Abdf-aC(de+cf))+a(de+3cf)(2aCdf+b(Cde+cCf-2Bdf))+(4bdf(2Abdf-aC(de+cf))+(bde-bcf+4adf)(2aCdf+b(Cde+cCf-2Bdf)))x)}{(a+bx)\sqrt{c+dx}}dx}{4bf}-}{2bdf}$$

$$\frac{C(c+dx)^{3/2}(e+fx)^{3/2}}{3bdf}$$

$\downarrow$ 171

$$\int \frac{2bde(4bcf(2Abdf-aC(de+cf))+a(de+3cf)(2aCdf+b(Cde+cCf-2Bdf)))-a(de+cf)(4bdf(2Abdf-aC(de+cf))+(bde-bcf+4adf)(2aCdf+b(Cde+cCf-2Bdf)))-}{2(a+bx)\sqrt{c+}} \quad bd$$

$$\frac{C(c+dx)^{3/2}(e+fx)^{3/2}}{3bdf}$$

$\downarrow$ 27

$$\int \frac{2bde(4bcf(2Abdf-aC(de+cf))+a(de+3cf)(2aCdf+b(Cde+cCf-2Bdf)))-a(de+cf)(4bdf(2Abdf-aC(de+cf))+(bde-bcf+4adf)(2aCdf+b(Cde+cCf-2Bdf)))-}{(a+bx)\sqrt{c+}} \quad 2bd$$

$$\frac{C(c+dx)^{3/2}(e+fx)^{3/2}}{3bdf}$$

$\downarrow$ 175

$$\frac{\frac{16d^2f^2(bc-ad)(be-af)\left(Ab^2-a(bB-aC)\right)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}dx}{b}-\frac{\left(16a^3Cd^3f^3-8a^2bd^2f^2(2Bdf+cCf+Cde)-2ab^2df\left(C(de-cf)^2-4df(2Adf+Bcf+Bde)\right)-\right.}{b}}{2bd}$$

$$\frac{C(c+dx)^{3/2}(e+fx)^{3/2}}{3bdf}$$

$\downarrow$ 66

3.44. $\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{a+bx}\,dx$

$$\frac{16d^2f^2(bc-ad)(be-af)\left(Ab^2-a(bB-aC)\right)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}dx}{b} - \frac{2\left(16a^3Cd^3f^3-8a^2bd^2f^2(2Bdf+cCf+Cde)-2ab^2df\left(C(de-cf)^2-4df(2Adf+Bcf+Bde)\right)\right.}{2bd\quad b}$$

$$\frac{C(c+dx)^{3/2}(e+fx)^{3/2}}{3bdf}$$

$$\Big\downarrow \; 104$$

$$\frac{32d^2f^2(bc-ad)(be-af)\left(Ab^2-a(bB-aC)\right)\int \frac{1}{-bc+ad+\frac{(be-af)(c+dx)}{e+fx}}d\frac{\sqrt{c+dx}}{\sqrt{e+fx}}}{b} - \frac{2\left(16a^3Cd^3f^3-8a^2bd^2f^2(2Bdf+cCf+Cde)-2ab^2df\left(C(de-cf)^2-4df(2Adf+\right.\right.}{2bd}$$

$$\frac{C(c+dx)^{3/2}(e+fx)^{3/2}}{3bdf}$$

$$\Big\downarrow \; 221$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)\left(16a^3Cd^3f^3-8a^2bd^2f^2(2Bdf+cCf+Cde)-2ab^2df\left(C(de-cf)^2-4df(2Adf+Bcf+Bde)\right)-\left(b^3\left(C(de-cf)^2(cf+de)-2df\left(B(de-cf)^2\right.\right.\right.\right.}{b\sqrt{d}\sqrt{f}\quad 2bd}$$

$$\frac{C(c+dx)^{3/2}(e+fx)^{3/2}}{3bdf}$$

input
```
Int[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x),x]
```

output
```
(C*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(3*b*d*f) + (-1/2*((2*a*C*d*f + b*(C*d
*e + c*C*f - 2*B*d*f))*Sqrt[c + d*x]*(e + f*x)^(3/2))/(b*f) + (((4*b*d*f*(
2*A*b*d*f - a*C*(d*e + c*f)) + (b*d*e - b*c*f + 4*a*d*f)*(2*a*C*d*f + b*(C
*d*e + c*C*f - 2*B*d*f)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(b*d) + ((-2*(16*a^
3*C*d^3*f^3 - 8*a^2*b*d^2*f^2*(C*d*e + c*C*f + 2*B*d*f) - 2*a*b^2*d*f*(C*(
d*e - c*f)^2 - 4*d*f*(B*d*e + B*c*f + 2*A*d*f)) - b^3*(C*(d*e - c*f)^2*(d*
e + c*f) - 2*d*f*(B*(d*e - c*f)^2 - 4*A*d*f*(d*e + c*f))))*ArcTanh[(Sqrt[f
]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(b*Sqrt[d]*Sqrt[f]) - (32*(A*b^
2 - a*(b*B - a*C))*d^2*Sqrt[b*c - a*d]*f^2*Sqrt[b*e - a*f]*ArcTanh[(Sqrt[b
*e - a*f]*Sqrt[c + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x])])/b)/(2*b*d))/(4*
b*f))/(2*b*d*f)
```

3.44. $\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{a+bx}\,dx$

3.44.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.44. $\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{a+bx}\,dx$

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

3.44.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3897 vs. $2(406) = 812$.

Time = 5.72 (sec) , antiderivative size = 3898, normalized size of antiderivative = 8.66

method	result	size
default	Expression too large to display	3898

input
```
int((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a),x,method=_RETURNVERB
OSE)
```

output
```
-1/48*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(-3*C*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)
/b^2)^(1/2)*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)
/(d*f)^(1/2))*b^4*d^3*e^3+24*B*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/
2)*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1
/2))*a*b^3*d^3*e*f^2+24*B*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*ln
(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*
a*b^3*c*d^2*f^3-3*C*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*ln(1/2*(
2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^4*c^
3*f^3-6*C*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*ln(1/2*(2*d*f*x+2*
((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b^3*d^3*e^2*f+
48*C*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*ln(1/2*(2*d*f*x+2*((d*x
+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^3*b*d^3*f^3-24*C*((
a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*
x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*b^2*d^3*e*f^2-12*B*((a^2
*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/
2)*b^4*d^2*e*f+48*C*(d*f)^(1/2)*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f
-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e
+2*b*c*e)/(b*x+a))*a^4*d^3*f^3-48*A*(d*f)^(1/2)*ln((-2*a*d*f*x+b*c*f*x+b*d
*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/
2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a*b^3*c*d^2*f^3-6*C*((a^2*d*f-a*b*c*...
```

3.44.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{a+bx}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a),x, algorithm="
fricas")
```

output
```
Timed out
```

3.44.6 Sympy [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{a+bx}\,dx = \int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{a+bx}\,dx$$

input
```
integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2)/(b*x+a),x)
```

output
```
Integral(sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2)/(a + b*x), x)
```

3.44.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{a+bx}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(2*a*d*f-b*c*f>0)', see `assume?`
 for more
```

3.44.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{a+bx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a),x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.44.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{a+bx}\,dx=\text{Hanged}$$

input
```
int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x),x)
```

output
```
\text{Hanged}
```

3.45 $\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^2}\,dx$

3.45.1 Optimal result

Integrand size = 36, antiderivative size = 521

$$
\begin{aligned}
&\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^2}\,dx \\
&= \frac{(12a^2Cdf^2-abf(7Cde+cCf+8Bdf)+b^2(4df(Be+Af)-Ce(de-cf)))\sqrt{c+dx}\sqrt{e+fx}}{4b^3df(be-af)} \\
&\quad+\frac{(3a^2Cdf+b^2(cCe+2Adf)-ab(Cde+cCf+2Bdf))\sqrt{c+dx}(e+fx)^{3/2}}{2b^2(bc-ad)f(be-af)} \\
&\quad-\frac{(Ab^2-a(bB-aC))(c+dx)^{3/2}(e+fx)^{3/2}}{b(bc-ad)(be-af)(a+bx)} \\
&\quad+\frac{(24a^2Cd^2f^2-8abdf(Cde+cCf+2Bdf)-b^2(C(de-cf)^2-4df(Bde+Bcf+2Adf)))\operatorname{arctanh}\left(\frac{\sqrt{f}}{\sqrt{d}}\right.}{4b^4d^{3/2}f^{3/2}} \\
&\quad+\frac{(6a^3Cdf-b^3(2Bce+Ade+Acf)+ab^2(4cCe+3Bde+3Bcf+2Adf)-a^2b(4Bdf+5C(de+cf)))}{b^4\sqrt{bc-ad}\sqrt{be-af}}
\end{aligned}
$$

output
```
-(A*b^2-a*(B*b-C*a))*(d*x+c)^(3/2)*(f*x+e)^(3/2)/b/(-a*d+b*c)/(-a*f+b*e)/(
b*x+a)+1/4*(24*a^2*C*d^2*f^2-8*a*b*d*f*(2*B*d*f+C*c*f+C*d*e)-b^2*(C*(-c*f+
d*e)^2-4*d*f*(2*A*d*f+B*c*f+B*d*e)))*arctanh(f^(1/2)*(d*x+c)^(1/2)/d^(1/2)
/(f*x+e)^(1/2))/b^4/d^(3/2)/f^(3/2)+(6*a^3*C*d*f-b^3*(A*c*f+A*d*e+2*B*c*e)
+a*b^2*(2*A*d*f+3*B*c*f+3*B*d*e+4*C*c*e)-a^2*b*(4*B*d*f+5*C*(c*f+d*e)))*ar
ctanh((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2)/(f*x+e)^(1/2))/b^4/(
-a*d+b*c)^(1/2)/(-a*f+b*e)^(1/2)+1/2*(3*a^2*C*d*f+b^2*(2*A*d*f+C*c*e)-a*b*
(2*B*d*f+C*c*f+C*d*e))*(f*x+e)^(3/2)*(d*x+c)^(1/2)/b^2/(-a*d+b*c)/f/(-a*f+
b*e)+1/4*(12*a^2*C*d*f^2-a*b*f*(8*B*d*f+C*c*f+7*C*d*e)+b^2*(4*d*f*(A*f+B*e
)-C*e*(-c*f+d*e)))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^3/d/f/(-a*f+b*e)
```

3.45.2 Mathematica [A] (verified)

Time = 2.13 (sec) , antiderivative size = 358, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^2}\,dx$$

$$= \frac{\frac{b\sqrt{c+dx}\sqrt{e+fx}(-12a^2Cdf+ab(cCf+8Bdf+Cd(e-6fx))+b^2(-4Adf+x(cCf+4Bdf+Cd(e+2fx))))}{df(a+bx)} + \frac{4(-6a^3Cdf+b^3(2Bce+Ade+Acf)-a}{}}{}$$

input
```
Integrate[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^2,x]
```

output
```
((b*Sqrt[c + d*x]*Sqrt[e + f*x]*(-12*a^2*C*d*f + a*b*(c*C*f + 8*B*d*f + C*
d*(e - 6*f*x)) + b^2*(-4*A*d*f + x*(c*C*f + 4*B*d*f + C*d*(e + 2*f*x)))))/
(d*f*(a + b*x)) + (4*(-6*a^3*C*d*f + b^3*(2*B*c*e + A*d*e + A*c*f) - a*b^2
*(4*c*C*e + 3*B*d*e + 3*B*c*f + 2*A*d*f) + a^2*b*(4*B*d*f + 5*C*(d*e + c*f
)))*ArcTan[(Sqrt[b*c - a*d]*Sqrt[e + f*x])/(Sqrt[-(b*e) + a*f]*Sqrt[c + d*
x])])/(Sqrt[b*c - a*d]*Sqrt[-(b*e) + a*f]) - ((-24*a^2*C*d^2*f^2 + 8*a*b*d
*f*(C*d*e + c*C*f + 2*B*d*f) + b^2*(C*(d*e - c*f)^2 - 4*d*f*(B*d*e + B*c*f
 + 2*A*d*f)))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/(Sqrt[f]*Sqrt[c + d*x])])/(d
^(3/2)*f^(3/2)))/(4*b^4)
```

3.45.3 Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 546, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {2116, 27, 171, 27, 171, 27, 175, 66, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)}{(a+bx)^2}\,dx$$

$$\Big\downarrow \text{2116}$$

$$-\frac{\int -\frac{\sqrt{c+dx}\sqrt{e+fx}\left(3C(de+cf)a^2-b(2cCe+3Bde+3Bcf-2Adf)a+b^2(2Bce+Ade+Acf)+2b\left(\frac{3Cdfa^2}{b}-(Cde+cCf+2Bdf)a+b(cCe+2Adf)\right)\right.}{2b(a+bx)}}{(bc-ad)(be-af)}$$

$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{b(a+bx)(bc-ad)(be-af)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(3C(de+cf)a^2-b(2cCe+3Bde+3Bcf-2Adf)a+b^2(2Bce+Ade+Acf)+2b\left(\frac{3Cdfa^2}{b}-(Cde+cCf+2Bdf)a+b(cCe+2Adf)\right)x\right)}{a+bx}d}{2b(bc-ad)(be-af)}$$

$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{b(a+bx)(bc-ad)(be-af)}$$

$$\Big\downarrow \text{171}$$

$$\frac{\frac{\int \frac{(bc-ad)\sqrt{e+fx}\left(3Cf(de+3cf)a^2-b(2Bf(de+3cf)+Ce(de+7cf))a+2b^2f(2Bce+Ade+Acf)+\left((4df(Be+Af)-Ce(de-cf))b^2-af(7Cde+cCf+8Bdf)b+12a^2Cdf^2\right.\right.}{(a+bx)\sqrt{c+dx}}}{2bf}}{2b(bc-ad)(be-af)}$$

$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{b(a+bx)(bc-ad)(be-af)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{(bc-ad)\int \frac{\sqrt{e+fx}\left(3Cf(de+3cf)a^2-b(2Bf(de+3cf)+Ce(de+7cf))a+2b^2f(2Bce+Ade+Acf)+\left((4df(Be+Af)-Ce(de-cf))b^2-af(7Cde+cCf+8Bdf)b+12a^2Cdf\right.\right.}{(a+bx)\sqrt{c+dx}}}{2bf}}{2b(bc-ad)(be-af)}$$

$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{b(a+bx)(bc-ad)(be-af)}$$

$$\Big\downarrow \text{171}$$

$$(bc-ad)\left(\frac{\int \frac{(be-af)\left(12Cdf(de+cf)a^2-b\left(8Bdf(de+cf)+C\left(d^2e^2+14cdfe+c^2f^2\right)\right)a+4b^2df(2Bce+Ade+Acf)+\left(-\left(\left(C(de-cf)^2-4df(Bde+Bcf+2Adf)\right)b^2\right)-8a\cdots}{2(a+bx)\sqrt{c+dx}\sqrt{e+fx}}}{bd} \cdots\right.$$
$$\overline{\qquad 2bf \qquad}$$

$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{b(a+bx)(bc-ad)(be-af)}$$

$$\Big\downarrow \ 27$$

$$(bc-ad)\left(\frac{(be-af)\int \frac{12Cdf(de+cf)a^2-b\left(8Bdf(de+cf)+C\left(d^2e^2+14cdfe+c^2f^2\right)\right)a+4b^2df(2Bce+Ade+Acf)+\left(-\left(\left(C(de-cf)^2-4df(Bde+Bcf+2Adf)\right)b^2\right)-8a\cdots}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}}{2bd} \cdots\right.$$
$$\overline{\qquad 2bf \qquad}$$

$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{b(a+bx)(bc-ad)(be-af)}$$

$$\Big\downarrow \ 175$$

$$(bc-ad)\left(\frac{(be-af)\left(\frac{\left(24a^2Cd^2f^2-8abdf(2Bdf+cCf+Cde)-\left(b^2\left(C(de-cf)^2-4df(2Adf+Bcf+Bde)\right)\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}}dx}{b}-\frac{4df\left(6a^3Cdf-a^2b(4Bdf+5C(cf+c\cdots\right.}{\cdots}\right.}{2bd} \cdots\right.$$

$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{b(a+bx)(bc-ad)(be-af)}$$

$$\Big\downarrow \ 66$$

$$(bc-ad)\left(\frac{(be-af)\left(\frac{2\left(24a^2Cd^2f^2-8abdf(2Bdf+cCf+Cde)-\left(b^2\left(C(de-cf)^2-4df(2Adf+Bcf+Bde)\right)\right)\right)\int \frac{1}{d-\frac{f(c+dx)}{e+fx}}d\frac{\sqrt{c+dx}}{\sqrt{e+fx}}}{b}-\frac{4df\left(6a^3Cdf-a^2b(4Bdf+5C\cdots\right.}{\cdots}\right.}{2bd} \cdots\right.$$

$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{b(a+bx)(bc-ad)(be-af)}$$

$$\Big\downarrow \ 104$$

3.45. $\displaystyle\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)}{(a+bx)^2}\,dx$

$$\frac{(bc-ad)\left(\frac{(be-af)\left(\frac{2\left(24a^2Cd^2f^2-8abdf(2Bdf+cCf+Cde)-\left(b^2\left(C(de-cf)^2-4df(2Adf+Bcf+Bde)\right)\right)\right)\int \frac{1}{d-\frac{f(c+dx)}{e+fx}}d\frac{\sqrt{c+dx}}{\sqrt{e+fx}}}{b}-\frac{8df\left(6a^3Cdf-a^2b(4Bdf+5C\right.}{}\right.}{2bd}\right.}{}$$

$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{b(a+bx)(bc-ad)(be-af)}$$

$\downarrow$ 221

$$\frac{\frac{\sqrt{c+dx}(e+fx)^{3/2}\left(\frac{3a^2Cdf}{b}-a(2Bdf+cCf+Cde)+b(2Adf+cCe)\right)}{f}+\frac{(bc-ad)\left(\frac{\sqrt{c+dx}\sqrt{e+fx}\left(12a^2Cdf^2-abf(8Bdf+cCf+7Cde)+b^2(4df(Af+Be)-C\right.}{bd}\right.}{}}{}$$

$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{b(a+bx)(bc-ad)(be-af)}$$

input

```
Int[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^2,x]
```

output

```
-(((A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(b*(b*c - a*d)
*(b*e - a*f)*(a + b*x))) + ((((3*a^2*C*d*f)/b + b*(c*C*e + 2*A*d*f) - a*(C
*d*e + c*C*f + 2*B*d*f))*Sqrt[c + d*x]*(e + f*x)^(3/2))/f + ((b*c - a*d)*(
((12*a^2*C*d*f^2 - a*b*f*(7*C*d*e + c*C*f + 8*B*d*f) + b^2*(4*d*f*(B*e + A
*f) - C*e*(d*e - c*f)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(b*d) + ((b*e - a*f)*
((2*(24*a^2*C*d^2*f^2 - 8*a*b*d*f*(C*d*e + c*C*f + 2*B*d*f) - b^2*(C*(d*e
- c*f)^2 - 4*d*f*(B*d*e + B*c*f + 2*A*d*f)))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x
])/(Sqrt[d]*Sqrt[e + f*x])])/(b*Sqrt[d]*Sqrt[f]) + (8*d*f*(6*a^3*C*d*f - b
^3*(2*B*c*e + A*d*e + A*c*f) + a*b^2*(4*c*C*e + 3*B*d*e + 3*B*c*f + 2*A*d*
f) - a^2*b*(4*B*d*f + 5*C*(d*e + c*f)))*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c +
d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x])])/(b*Sqrt[b*c - a*d]*Sqrt[b*e - a*f]
)))/(2*b*d)))/(2*b*f))/(2*b*(b*c - a*d)*(b*e - a*f))
```

3.45. $\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^2}\,dx$

3.45.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.45. $\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^2}\,dx$

rule 2116
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Si
mp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*
(e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1
) - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x]
, x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && ILtQ[m,
-1]
```

3.45.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4679 vs. $2(479) = 958$.

Time = 1.69 (sec) , antiderivative size = 4680, normalized size of antiderivative = 8.98

method	result	size
default	Expression too large to display	4680

input
```
int((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^2,x,method=_RETURNVE
RBOSE)
```

output
```
1/8*(8*A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*
e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^2*
b^2*d^2*f^2*(d*f)^(1/2)-16*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*
f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b^3*d^2*f^2*x*((a^2*d*f-a*b*c*f-a*b*d*e+b
^2*c*e)/b^2)^(1/2)-8*A*b^4*d*f*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*((a^2*d
*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+8*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x
+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^4*d^2*f^2*x*((a^2*d*f-a*b*c
*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)
*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^4*c^2*f^2*x*((a^2*d*f-a*b*c*f-a*b*d*e
+b^2*c*e)/b^2)^(1/2)-C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/
2)+c*f+d*e)/(d*f)^(1/2))*b^4*d^2*e^2*x*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/
b^2)^(1/2)+4*C*b^4*d*f*x^2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*((a^2*d*f-a
*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+12*B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*(
(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c
*f-a*d*e+2*b*c*e)/(b*x+a))*a^2*b^2*d^2*e*f*(d*f)^(1/2)+4*B*ln(1/2*(2*d*f*x
+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b^3*c*d*f^2
*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+8*B*b^4*d*f*x*((d*x+c)*(f*x
+e))^(1/2)*(d*f)^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+2*C*b
^4*c*f*x*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2
*c*e)/b^2)^(1/2)+2*C*b^4*d*e*x*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*((a^...
```

3.45.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^2}\,dx=\text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^2,x, algorithm
="fricas")
```

output
```
Timed out
```

3.45.6 Sympy [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^2}\,dx = \int \frac{\sqrt{c+dx}\sqrt{e+fx}\,(A+Bx+Cx^2)}{(a+bx)^2}\,dx$$

input
```
integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2)/(b*x+a)**2,x)
```

output
```
Integral(sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2)/(a + b*x)**2, x)
```

3.45.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^2,x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(2*a*d*f-b*c*f>0)', see `assume?`
 for more
```

3.45.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1507 vs. $2(478) = 956$.

Time = 1.54 (sec) , antiderivative size = 1507, normalized size of antiderivative = 2.89

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^2,x, algorithm
="giac")
```

output
```
1/4*sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*sqrt(d*x + c)*(2*(d*x + c)*C*abs(d
)/(b^2*d^3) + (C*b^7*d^4*e*f*abs(d) - C*b^7*c*d^3*f^2*abs(d) - 8*C*a*b^6*d
^4*f^2*abs(d) + 4*B*b^7*d^4*f^2*abs(d))/(b^9*d^6*f^2)) + (4*sqrt(d*f)*C*a*
b^2*c*e*abs(d) - 2*sqrt(d*f)*B*b^3*c*e*abs(d) - 5*sqrt(d*f)*C*a^2*b*d*e*ab
s(d) + 3*sqrt(d*f)*B*a*b^2*d*e*abs(d) - sqrt(d*f)*A*b^3*d*e*abs(d) - 5*sqr
t(d*f)*C*a^2*b*c*f*abs(d) + 3*sqrt(d*f)*B*a*b^2*c*f*abs(d) - sqrt(d*f)*A*b
^3*c*f*abs(d) + 6*sqrt(d*f)*C*a^3*d*f*abs(d) - 4*sqrt(d*f)*B*a^2*b*d*f*abs
(d) + 2*sqrt(d*f)*A*a*b^2*d*f*abs(d))*arctan(-1/2*(b*d^2*e + b*c*d*f - 2*a
*d^2*f - (sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2
*b)/(sqrt(-b^2*c*d*e*f + a*b*d^2*e*f + a*b*c*d*f^2 - a^2*d^2*f^2)*d))/(sqr
t(-b^2*c*d*e*f + a*b*d^2*e*f + a*b*c*d*f^2 - a^2*d^2*f^2)*b^4*d) - 2*(sqrt
(d*f)*C*a^2*b*d^3*e^2*abs(d) - sqrt(d*f)*B*a*b^2*d^3*e^2*abs(d) + sqrt(d*f
)*A*b^3*d^3*e^2*abs(d) - 2*sqrt(d*f)*C*a^2*b*c*d^2*e*f*abs(d) + 2*sqrt(d*f
)*B*a*b^2*c*d^2*e*f*abs(d) - 2*sqrt(d*f)*A*b^3*c*d^2*e*f*abs(d) + sqrt(d*f
)*C*a^2*b*c^2*d*f^2*abs(d) - sqrt(d*f)*B*a*b^2*c^2*d*f^2*abs(d) + sqrt(d*f
)*A*b^3*c^2*d*f^2*abs(d) - sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e
 + (d*x + c)*d*f - c*d*f))^2*C*a^2*b*d*e*abs(d) + sqrt(d*f)*(sqrt(d*f)*sqr
t(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a*b^2*d*e*abs(d) - s
qrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2
*A*b^3*d*e*abs(d) - sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (...
```

3.45.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^2}\,dx = \text{Hanged}$$

input
```
int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x)^2,x)
```

output
```
\text{Hanged}
```

3.46 $\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^3}\,dx$

3.46.1 Optimal result

Integrand size = 36, antiderivative size = 658

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^3}\,dx =$$

$$-\frac{(12a^3Cdf^2 - a^2bf(17Cde + 11cCf + 4Bdf) + ab^2(Bf(5de + 3cf) + 4Ce(de + 4cf)) - b^3(Adef + c(4}{4b^3(bc - ad)(be - af)^2}$$

$$+\frac{(6a^3Cdf - b^3(4Bce - Ade - Acf) + ab^2(8cCe + 3Bde + 3Bcf - 2Adf) - a^2b(2Bdf + 7C(de + cf)))}{4b^2(bc - ad)(be - af)^2(a + bx)}$$

$$-\frac{(Ab^2 - a(bB - aC))\,(c + dx)^{3/2}(e + fx)^{3/2}}{2b(bc - ad)(be - af)(a + bx)^2}$$

$$-\frac{(6aCdf - b(Cde + cCf + 2Bdf))\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)}{b^4\sqrt{d}\sqrt{f}}$$

$$-\frac{(24a^4Cd^2f^2 - 3ab^3(Bd^2e^2 + c^2f(8Ce + Bf) + 2cde(4Ce + 3Bf)) - 8a^3bdf(Bdf + 5C(de + cf)) - b^4}{}$$

output
```
-1/2*(A*b^2-a*(B*b-C*a))*(d*x+c)^(3/2)*(f*x+e)^(3/2)/b/(-a*d+b*c)/(-a*f+b*
e)/(b*x+a)^2-1/4*(24*a^4*C*d^2*f^2-3*a*b^3*(B*d^2*e^2+c^2*f*(B*f+8*C*e)+2*
c*d*e*(3*B*f+4*C*e))-8*a^3*b*d*f*(B*d*f+5*C*(c*f+d*e))-b^4*(A*d^2*e^2-2*c*
d*e*(A*f+2*B*e)-c^2*(-A*f^2+4*B*e*f+8*C*e^2))+3*a^2*b^2*(4*B*d*f*(c*f+d*e)
+C*(5*c^2*f^2+22*c*d*e*f+5*d^2*e^2)))*arctanh((-a*f+b*e)^(1/2)*(d*x+c)^(1/
2)/(-a*d+b*c)^(1/2)/(f*x+e)^(1/2))/b^4/(-a*d+b*c)^(3/2)/(-a*f+b*e)^(3/2)-(
6*a*C*d*f-b*(2*B*d*f+C*c*f+C*d*e))*arctanh(f^(1/2)*(d*x+c)^(1/2)/d^(1/2)/(
f*x+e)^(1/2))/b^4/d^(1/2)/f^(1/2)+1/4*(6*a^3*C*d*f-b^3*(-A*c*f-A*d*e+4*B*c
*e)+a*b^2*(-2*A*d*f+3*B*c*f+3*B*d*e+8*C*c*e)-a^2*b*(2*B*d*f+7*C*(c*f+d*e))
)*(f*x+e)^(3/2)*(d*x+c)^(1/2)/b^2/(-a*d+b*c)/(-a*f+b*e)^2/(b*x+a)-1/4*(12*
a^3*C*d*f^2-a^2*b*f*(4*B*d*f+11*C*c*f+17*C*d*e)+a*b^2*(B*f*(3*c*f+5*d*e)+4
*C*e*(4*c*f+d*e))-b^3*(A*d*e*f+c*(-A*f^2+4*B*e*f+4*C*e^2)))*(d*x+c)^(1/2)*
(f*x+e)^(1/2)/b^3/(-a*d+b*c)/(-a*f+b*e)^2
```

3.46.2 Mathematica [A] (verified)

Time = 4.75 (sec) , antiderivative size = 536, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^3} \, dx$$

$$= \frac{\frac{b\sqrt{c+dx}\sqrt{e+fx}\left(12a^4Cdf+4b^4cex(-B+Cx)+Ab^3(acf+ad(e+2fx)-b(2ce+dex+cfx))+ab^3(-4Cx(-4ce+dex+cfx)+B(-2ce+5dex+5cfx))+\right.}{(bc-ad)(be-af)(a+bx)^2}}{}$$

input
```
Integrate[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^3,x]
```

output
```
((b*Sqrt[c + d*x]*Sqrt[e + f*x]*(12*a^4*C*d*f + 4*b^4*c*e*x*(-B + C*x) + A
*b^3*(a*c*f + a*d*(e + 2*f*x) - b*(2*c*e + d*e*x + c*f*x)) + a*b^3*(-4*C*x
*(-4*c*e + d*e*x + c*f*x) + B*(-2*c*e + 5*d*e*x + 5*c*f*x)) + a^2*b^2*(3*B
*d*(e - 2*f*x) + C*d*x*(-17*e + 4*f*x) + c*(10*C*e + 3*B*f - 17*C*f*x)) -
a^3*b*(4*B*d*f + C*(11*d*e + 11*c*f - 18*d*f*x))))/((b*c - a*d)*(b*e - a*f
)*(a + b*x)^2) - ((24*a^4*C*d^2*f^2 - 3*a*b^3*(B*d^2*e^2 + c^2*f*(8*C*e +
B*f) + 2*c*d*e*(4*C*e + 3*B*f)) - 8*a^3*b*d*f*(B*d*f + 5*C*(d*e + c*f)) +
b^4*(-(A*d^2*e^2) + 2*c*d*e*(2*B*e + A*f) + c^2*(8*C*e^2 + 4*B*e*f - A*f^2
)) + 3*a^2*b^2*(4*B*d*f*(d*e + c*f) + C*(5*d^2*e^2 + 22*c*d*e*f + 5*c^2*f^
2)))*ArcTan[(Sqrt[b*c - a*d]*Sqrt[e + f*x])/(Sqrt[-(b*e) + a*f]*Sqrt[c + d
*x])])/((b*c - a*d)^(3/2)*(-(b*e) + a*f)^(3/2)) + (4*(-6*a*C*d*f + b*(C*d*
e + c*C*f + 2*B*d*f))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/(Sqrt[f]*Sqrt[c + d*
x])])/(Sqrt[d]*Sqrt[f]))/(4*b^4)
```

3.46.3 Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 695, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {2116, 27, 166, 27, 171, 27, 175, 66, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^3}\,dx$$

$$\Big\downarrow \text{2116}$$

$$-\frac{\int -\frac{\sqrt{c+dx}\sqrt{e+fx}\left(3C(de+cf)a^2-b(4cCe+3Bde+3Bcf-4Adf)a+b^2(4Bce-Ade-Acf)-2b\left(-\frac{3Cdfa^2}{b}+Bdfa+2C(de+cf)a-b(2cCe+Adf)\right)\ldots}{2b(a+bx)^2}}{2(bc-ad)(be-af)}$$
$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{2b(a+bx)^2(bc-ad)(be-af)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(3C(de+cf)a^2-b(4cCe+3Bde+3Bcf-4Adf)a+b^2(4Bce-A(de+cf))-2b\left(-\frac{3Cdfa^2}{b}+Bdfa+2C(de+cf)a-b(2cCe+Adf)\right)x\right)\ldots}{(a+bx)^2}}{4b(bc-ad)(be-af)}$$
$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{2b(a+bx)^2(bc-ad)(be-af)}$$

$$\Big\downarrow \text{166}$$

$$\frac{\int -\frac{\sqrt{e+fx}\left(6Cdf(de+3cf)a^3-b\left(2Bdf(de+3cf)+C\left(7d^2e^2+34cdfe+15c^2f^2\right)\right)a^2+b^2\left(3f(8Ce+Bf)c^2+2d\left(8Ce^2+5Bfe+Af^2\right)c+d^2e(3Be-2Af)\right)a+b^3\left(-\left(\left(8Ce^2\ldots\right.\right.\right.}{2(a+\ldots}}{b(b\ldots}$$

$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{2b(a+bx)^2(bc-ad)(be-af)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\sqrt{c+dx}(e+fx)^{3/2}\left(6a^3Cdf-a^2b(2Bdf+7C(cf+de))+ab^2(-2Adf+3Bcf+3Bde+8cCe)-b^3(-Acf-Ade+4Bce)\right)}{b(a+bx)(be-af)}-\int\frac{\sqrt{e+fx}\left(6Cdf(de+3cf)a^3-b\ldots\right.}{\ldots}}{\ldots}$$

$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{2b(a+bx)^2(bc-ad)(be-af)}$$

$$\Big\downarrow \text{171}$$

$$\frac{\frac{\sqrt{c+dx}(e+fx)^{3/2}\left(6a^3Cdf-a^2b(2Bdf+7C(cf+de))+ab^2(-2Adf+3Bcf+3Bde+8cCe)-b^3(-Acf-Ade+4Bce)\right)}{b(a+bx)(be-af)}-\frac{\int \frac{d(be-af)\left(12Cdf(de+cf)a^3-\cdots\right.}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{2b(a+bx)^2(bc-ad)(be-af)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\sqrt{c+dx}(e+fx)^{3/2}\left(6a^3Cdf-a^2b(2Bdf+7C(cf+de))+ab^2(-2Adf+3Bcf+3Bde+8cCe)-b^3(-Acf-Ade+4Bce)\right)}{b(a+bx)(be-af)}-\frac{(be-af)\int \frac{12Cdf(de+cf)a^3-b\cdots}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{2b(a+bx)^2(bc-ad)(be-af)}$$

$$\Big\downarrow \; 175$$

$$\frac{\frac{\sqrt{c+dx}(e+fx)^{3/2}\left(6a^3Cdf-a^2b(2Bdf+7C(cf+de))+ab^2(-2Adf+3Bcf+3Bde+8cCe)-b^3(-Acf-Ade+4Bce)\right)}{b(a+bx)(be-af)}-\frac{(be-af)\left(\frac{4(bc-ad)(be-af)(6a\cdots}{\cdots}\right.}{\cdots}}{\cdots}$$

$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{2b(a+bx)^2(bc-ad)(be-af)}$$

$$\Big\downarrow \; 66$$

$$\frac{\frac{\sqrt{c+dx}(e+fx)^{3/2}\left(6a^3Cdf-a^2b(2Bdf+7C(cf+de))+ab^2(-2Adf+3Bcf+3Bde+8cCe)-b^3(-Acf-Ade+4Bce)\right)}{b(a+bx)(be-af)}-\frac{(be-af)\left(\frac{8(bc-ad)(be-af)(6a\cdots}{\cdots}\right.}{\cdots}}{\cdots}$$

$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{2b(a+bx)^2(bc-ad)(be-af)}$$

$$\Big\downarrow \; 104$$

$$\frac{\frac{\sqrt{c+dx}(e+fx)^{3/2}\left(6a^3Cdf-a^2b(2Bdf+7C(cf+de))+ab^2(-2Adf+3Bcf+3Bde+8cCe)-b^3(-Acf-Ade+4Bce)\right)}{b(a+bx)(be-af)}-\frac{(be-af)\left(\frac{8(bc-ad)(be-af)(6a\cdots}{\cdots}\right.}{\cdots}}{\cdots}$$

$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{2b(a+bx)^2(bc-ad)(be-af)}$$

$$\Big\downarrow \; 221$$

3.46. $\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^3}\,dx$

$$\frac{\frac{\sqrt{c+dx}(e+fx)^{3/2}\left(6a^3Cdf-a^2b(2Bdf+7C(cf+de))+ab^2(-2Adf+3Bcf+3Bde+8cCe)-b^3(-Acf-Ade+4Bce)\right)}{b(a+bx)(be-af)} - \frac{2\sqrt{c+dx}\sqrt{e+fx}\left(12a^3Cdf^2-a^2 \right.}{}}{}$$

$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{2b(a+bx)^2(bc-ad)(be-af)}$$

input
```
Int[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^3,x]
```

output
```
-1/2*((A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(b*(b*c - a
*d)*(b*e - a*f)*(a + b*x)^2) + (((6*a^3*C*d*f - b^3*(4*B*c*e - A*d*e - A*c
*f) + a*b^2*(8*c*C*e + 3*B*d*e + 3*B*c*f - 2*A*d*f) - a^2*b*(2*B*d*f + 7*C
*(d*e + c*f)))*Sqrt[c + d*x]*(e + f*x)^(3/2))/(b*(b*e - a*f)*(a + b*x)) -
((2*(12*a^3*C*d*f^2 - a^2*b*f*(17*C*d*e + 11*c*C*f + 4*B*d*f) - b^3*(4*c*C
*e^2 + A*d*e*f + c*f*(4*B*e - A*f)) + a*b^2*(B*f*(5*d*e + 3*c*f) + 4*C*e*(
d*e + 4*c*f)))*Sqrt[c + d*x]*Sqrt[e + f*x])/b + ((b*e - a*f)*((8*(b*c - a*
d)*(b*e - a*f)*(6*a*C*d*f - b*(C*d*e + c*C*f + 2*B*d*f))*ArcTanh[(Sqrt[f]*
Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(b*Sqrt[d]*Sqrt[f]) + (2*(24*a^4*
C*d^2*f^2 - 3*a*b^3*(B*d^2*e^2 + c^2*f*(8*C*e + B*f) + 2*c*d*e*(4*C*e + 3*
B*f)) - 8*a^3*b*d*f*(B*d*f + 5*C*(d*e + c*f)) - b^4*(A*d^2*e^2 - 2*c*d*e*(
2*B*e + A*f) - c^2*(8*C*e^2 + 4*B*e*f - A*f^2)) + 3*a^2*b^2*(4*B*d*f*(d*e
+ c*f) + C*(5*d^2*e^2 + 22*c*d*e*f + 5*c^2*f^2)))*ArcTanh[(Sqrt[b*e - a*f]
*Sqrt[c + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x])])/(b*Sqrt[b*c - a*d]*Sqrt[
b*e - a*f])))/b)/(2*b*(b*e - a*f)))/(4*b*(b*c - a*d)*(b*e - a*f))
```

3.46.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.46. $\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^3} \, dx$

rule 2116
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Si
mp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*
(e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1
) - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x]
, x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && ILtQ[m,
-1]
```

3.46.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 11203 vs. $2(614) = 1228$.

Time = 1.69 (sec) , antiderivative size = 11204, normalized size of antiderivative = 17.03

method	result	size
default	Expression too large to display	11204

input
```
int((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^3,x,method=_RETURNVE
RBOSE)
```

output
```
result too large to display
```

3.46.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^3}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^3,x, algorithm
="fricas")
```

output
```
Timed out
```

3.46.6 Sympy [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^3}\,dx = \int \frac{\sqrt{c+dx}\sqrt{e+fx}\,(A+Bx+Cx^2)}{(a+bx)^3}\,dx$$

input
```
integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2)/(b*x+a)**3,x)
```

output
```
Integral(sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2)/(a + b*x)**3, x)
```

3.46.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^3,x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((a*d-b*c)>0)', see `assume?` for
 more deta
```

3.46.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8241 vs. $2(613) = 1226$.

Time = 4.81 (sec) , antiderivative size = 8241, normalized size of antiderivative = 12.52

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^3,x, algorithm
="giac")
```

output
```
-1/4*(8*sqrt(d*f)*C*b^4*c^2*e^2*abs(d) - 24*sqrt(d*f)*C*a*b^3*c*d*e^2*abs(
d) + 4*sqrt(d*f)*B*b^4*c*d*e^2*abs(d) + 15*sqrt(d*f)*C*a^2*b^2*d^2*e^2*abs
(d) - 3*sqrt(d*f)*B*a*b^3*d^2*e^2*abs(d) - sqrt(d*f)*A*b^4*d^2*e^2*abs(d)
- 24*sqrt(d*f)*C*a*b^3*c^2*e*f*abs(d) + 4*sqrt(d*f)*B*b^4*c^2*e*f*abs(d) +
 66*sqrt(d*f)*C*a^2*b^2*c*d*e*f*abs(d) - 18*sqrt(d*f)*B*a*b^3*c*d*e*f*abs(
d) + 2*sqrt(d*f)*A*b^4*c*d*e*f*abs(d) - 40*sqrt(d*f)*C*a^3*b*d^2*e*f*abs(d
) + 12*sqrt(d*f)*B*a^2*b^2*d^2*e*f*abs(d) + 15*sqrt(d*f)*C*a^2*b^2*c^2*f^2
*abs(d) - 3*sqrt(d*f)*B*a*b^3*c^2*f^2*abs(d) - sqrt(d*f)*A*b^4*c^2*f^2*abs
(d) - 40*sqrt(d*f)*C*a^3*b*c*d*f^2*abs(d) + 12*sqrt(d*f)*B*a^2*b^2*c*d*f^2
*abs(d) + 24*sqrt(d*f)*C*a^4*d^2*f^2*abs(d) - 8*sqrt(d*f)*B*a^3*b*d^2*f^2*
abs(d))*arctan(-1/2*(b*d^2*e + b*c*d*f - 2*a*d^2*f - (sqrt(d*f)*sqrt(d*x +
 c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*b)/(sqrt(-b^2*c*d*e*f + a*b*d
^2*e*f + a*b*c*d*f^2 - a^2*d^2*f^2)*d))/((b^6*c*e - a*b^5*d*e - a*b^5*c*f
+ a^2*b^4*d*f)*sqrt(-b^2*c*d*e*f + a*b*d^2*e*f + a*b*c*d*f^2 - a^2*d^2*f^2
)*d) + 1/2*(8*sqrt(d*f)*C*a*b^4*c*d^7*e^5*abs(d) - 4*sqrt(d*f)*B*b^5*c*d^7
*e^5*abs(d) - 9*sqrt(d*f)*C*a^2*b^3*d^8*e^5*abs(d) + 5*sqrt(d*f)*B*a*b^4*d
^8*e^5*abs(d) - sqrt(d*f)*A*b^5*d^8*e^5*abs(d) - 32*sqrt(d*f)*C*a*b^4*c^2*
d^6*e^4*f*abs(d) + 16*sqrt(d*f)*B*b^5*c^2*d^6*e^4*f*abs(d) + 27*sqrt(d*f)*
C*a^2*b^3*c*d^7*e^4*f*abs(d) - 15*sqrt(d*f)*B*a*b^4*c*d^7*e^4*f*abs(d) + 3
*sqrt(d*f)*A*b^5*c*d^7*e^4*f*abs(d) + 10*sqrt(d*f)*C*a^3*b^2*d^8*e^4*f*...
```

3.46.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^3}\,dx = \text{Hanged}$$

input
```
int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x)^3,x)
```

output
```
\text{Hanged}
```

3.47 $\int \frac{(a+bx)^2\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$

3.47.1 Optimal result

Integrand size = 36, antiderivative size = 1032

$$\int \frac{(a+bx)^2\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx =$$

$$-\frac{(16a^2d^2f^2(2df(3Bde+Bcf-4Adf)-C(5d^2e^2+2cdef+c^2f^2))+4abdf(C(35d^3e^3+15cd^2e^2f+9c^2$$

$$-\frac{(4aCdf+b(9Cde+7cCf-10Bdf))(a+bx)^2(c+dx)^{3/2}\sqrt{e+fx}}{40bd^2f^2}$$

$$+\frac{C(a+bx)^3(c+dx)^{3/2}\sqrt{e+fx}}{5bdf}$$

$$-\frac{(c+dx)^{3/2}\sqrt{e+fx}(96a^3Cd^3f^3+8a^2bd^2f^2(23Cde+9cCf-30Bdf)+20ab^2df(8df(5Bde+3Bcf-$$

$$+\frac{(de-cf)\,(16a^2d^2f^2(2df(3Bde+Bcf-4Adf)-C(5d^2e^2+2cdef+c^2f^2))+4abdf(C(35d^3e^3+15cd^2$$

output
```
1/128*(-c*f+d*e)*(16*a^2*d^2*f^2*(2*d*f*(-4*A*d*f+B*c*f+3*B*d*e)-C*(c^2*f^
2+2*c*d*e*f+5*d^2*e^2))+4*a*b*d*f*(C*(5*c^3*f^3+9*c^2*d*e*f^2+15*c*d^2*e^2
*f+35*d^3*e^3)+8*d*f*(2*A*d*f*(c*f+3*d*e)-B*(c^2*f^2+2*c*d*e*f+5*d^2*e^2))
)-b^2*(C*(7*c^4*f^4+12*c^3*d*e*f^3+18*c^2*d^2*e^2*f^2+28*c*d^3*e^3*f+63*d^
4*e^4)+2*d*f*(8*A*d*f*(c^2*f^2+2*c*d*e*f+5*d^2*e^2)-B*(5*c^3*f^3+9*c^2*d*e
*f^2+15*c*d^2*e^2*f+35*d^3*e^3))))*arctanh(f^(1/2)*(d*x+c)^(1/2)/d^(1/2)/(
f*x+e)^(1/2))/d^(9/2)/f^(11/2)-1/40*(4*a*C*d*f+b*(-10*B*d*f+7*C*c*f+9*C*d*
e))*(b*x+a)^2*(d*x+c)^(3/2)*(f*x+e)^(1/2)/b/d^2/f^2+1/5*C*(b*x+a)^3*(d*x+c
)^(3/2)*(f*x+e)^(1/2)/b/d/f-1/960*(d*x+c)^(3/2)*(96*a^3*C*d^3*f^3+8*a^2*b*
d^2*f^2*(-30*B*d*f+9*C*c*f+23*C*d*e)+20*a*b^2*d*f*(8*d*f*(-6*A*d*f+3*B*c*f
+5*B*d*e)-C*(15*c^2*f^2+22*c*d*e*f+35*d^2*e^2))+b^3*(C*(105*c^3*f^3+145*c^
2*d*e*f^2+203*c*d^2*e^2*f+315*d^3*e^3)+10*d*f*(8*A*d*f*(3*c*f+5*d*e)-B*(15
*c^2*f^2+22*c*d*e*f+35*d^2*e^2)))+4*b*d*f*(8*b*d*f*(-10*A*b*d*f+C*a*c*f+3*
C*a*d*e+6*C*b*c*e)-(-4*a*d*f+5*b*c*f+7*b*d*e)*(4*a*C*d*f+b*(-10*B*d*f+7*C*
c*f+9*C*d*e)))*x)*(f*x+e)^(1/2)/b/d^4/f^4-1/128*(16*a^2*d^2*f^2*(2*d*f*(-4
*A*d*f+B*c*f+3*B*d*e)-C*(c^2*f^2+2*c*d*e*f+5*d^2*e^2))+4*a*b*d*f*(C*(5*c^3
*f^3+9*c^2*d*e*f^2+15*c*d^2*e^2*f+35*d^3*e^3)+8*d*f*(2*A*d*f*(c*f+3*d*e)-B
*(c^2*f^2+2*c*d*e*f+5*d^2*e^2)))-b^2*(C*(7*c^4*f^4+12*c^3*d*e*f^3+18*c^2*d
^2*e^2*f^2+28*c*d^3*e^3*f+63*d^4*e^4)+2*d*f*(8*A*d*f*(c^2*f^2+2*c*d*e*f+5*
d^2*e^2)-B*(5*c^3*f^3+9*c^2*d*e*f^2+15*c*d^2*e^2*f+35*d^3*e^3))))*(d*x+...
```

3.47.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 3220 vs. $2(1032) = 2064$.

Time = 16.92 (sec) , antiderivative size = 3220, normalized size of antiderivative = 3.12

$$\int \frac{(a+bx)^2\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[((a + b*x)^2*Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]
```

output
```
((-(b*e) + a*f)^2*(d*e - c*f)^2*(C*e^2 - B*e*f + A*f^2)*Sqrt[d/((d^2*e)/(d
*e - c*f) - (c*d*f)/(d*e - c*f))]*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*
f))^2*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[1 + (d*f*(c + d*x))/((d*e - c*f
)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))]*((2*d*f*(c + d*x))/((d*e -
 c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))) - (2*Sqrt[d]*Sqrt[f]*Sq
rt[c + d*x]*ArcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*f]*Sqrt[
(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)])])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e
)/(d*e - c*f) - (c*d*f)/(d*e - c*f)]*Sqrt[1 + (d*f*(c + d*x))/((d*e - c*f)
*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))])))/(2*d^3*f^6*Sqrt[c + d*x]
*Sqrt[e + f*x]) + (2*b^2*C*(d*e - c*f)^3*(c + d*x)^(3/2)*Sqrt[e + f*x]*(1
+ (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))
))^(9/2)*((3*(35/(64*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f
) - (c*d*f)/(d*e - c*f))))^4) + 35/(48*(1 + (d*f*(c + d*x))/((d*e - c*f)*(
(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^3) + 7/(8*(1 + (d*f*(c + d*x)
)/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^2) + (1 + (d*
f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(-
1)))/10 + (21*(d*e - c*f)^2*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))^2*
((2*d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)
)) - (2*Sqrt[d]*Sqrt[f]*Sqrt[c + d*x]*ArcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*
x])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)])])...
```

3.47.3 Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 778, normalized size of antiderivative = 0.75, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2118, 27, 170, 27, 164, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}dx$$

$$\Big\downarrow \text{2118}$$

$$\frac{\int -\frac{b(a+bx)^2\sqrt{c+dx}(6bcCe+3aCde+acCf-10Abdf+(4aCdf+b(9Cde+7cCf-10Bdf))x)}{2\sqrt{e+fx}}dx}{5b^2df}+\frac{C(a+bx)^3(c+dx)^{3/2}\sqrt{e+fx}}{5bdf}$$

$$\Big\downarrow \text{27}$$

$$\frac{C(a+bx)^3(c+dx)^{3/2}\sqrt{e+fx}}{5bdf}-\frac{\int \frac{(a+bx)^2\sqrt{c+dx}(6bcCe+3aCde+acCf-10Abdf+(4aCdf+b(9Cde+7cCf-10Bdf))x)}{\sqrt{e+fx}}dx}{10bdf}$$

$\downarrow$ 170

$$\frac{C(a+bx)^3(c+dx)^{3/2}\sqrt{e+fx}}{5bdf}-$$

$$\int \frac{(a+bx)\sqrt{c+dx}(8adf(6bcCe+3aCde+acCf-10Abdf)-(4bce+3ade+acf)(4aCdf+b(9Cde+7cCf-10Bdf))+(8bdf(6bcCe+3aCde+acCf-10Abdf)-(7bde+5bcf-4a}{2\sqrt{e+fx}}$$
$$4df$$
$$10bdf$$

$\downarrow$ 27

$$\frac{C(a+bx)^3(c+dx)^{3/2}\sqrt{e+fx}}{5bdf}-$$

$$\int \frac{(a+bx)\sqrt{c+dx}(8adf(6bcCe+3aCde+acCf-10Abdf)-(4bce+3ade+acf)(4aCdf+b(9Cde+7cCf-10Bdf))+(8bdf(6bcCe+3aCde+acCf-10Abdf)-(7bde+5bcf-4a}{\sqrt{e+fx}}$$
$$8df$$
$$10bdf$$

$\downarrow$ 164

$$\frac{C(a+bx)^3(c+dx)^{3/2}\sqrt{e+fx}}{5bdf}-$$

$$5b\left(16a^2d^2f^2\left(2df(-4Adf+Bcf+3Bde)-C\left(c^2f^2+2cdef+5d^2e^2\right)\right)+4abdf\left(8df\left(2Adf(cf+3de)-B\left(c^2f^2+2cdef+5d^2e^2\right)\right)+C\left(5c^3f^3+9c^2def^2+15cd^2e^2f+35d^3e^3\right.\right.\right.$$
$$8d^2f$$

$\downarrow$ 60

$$\frac{C(a+bx)^3(c+dx)^{3/2}\sqrt{e+fx}}{5bdf}-$$

$$5b\left(16a^2d^2f^2\left(2df(-4Adf+Bcf+3Bde)-C\left(c^2f^2+2cdef+5d^2e^2\right)\right)+4abdf\left(8df\left(2Adf(cf+3de)-B\left(c^2f^2+2cdef+5d^2e^2\right)\right)+C\left(5c^3f^3+9c^2def^2+15cd^2e^2f+35d^3e^3\right.\right.\right.$$

$\downarrow$ 66

$$\frac{C(a+bx)^3(c+dx)^{3/2}\sqrt{e+fx}}{5bdf}-$$

$$5b\left(16a^2d^2f^2\left(2df(-4Adf+Bcf+3Bde)-C\left(c^2f^2+2cdef+5d^2e^2\right)\right)+4abdf\left(8df\left(2Adf(cf+3de)-B\left(c^2f^2+2cdef+5d^2e^2\right)\right)+C\left(5c^3f^3+9c^2def^2+15cd^2e^2f+35d^3e^3\right.\right.\right.$$

$\downarrow$ 221

3.47. $\int \frac{(a+bx)^2\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$

$$\frac{C(a+bx)^3(c+dx)^{3/2}\sqrt{e+fx}}{5bdf} -$$

$$\frac{5b\left(\frac{\sqrt{c+dx}\sqrt{e+fx}}{f}-\frac{(de-cf)\text{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)}{\sqrt{d}f^{3/2}}\right)\left(16a^2d^2f^2\left(2df(-4Adf+Bcf+3Bde)-C\left(c^2f^2+2cdef+5d^2e^2\right)\right)+4abdf\left(8df\left(2Adf(cf+3de)-B\left(c^2f^2+2\right.\right.\right.\right.}{}$$

input
```
Int[((a + b*x)^2*Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]
```

output
```
(C*(a + b*x)^3*(c + d*x)^(3/2)*Sqrt[e + f*x])/(5*b*d*f) - (((4*a*C*d*f + b
*(9*C*d*e + 7*c*C*f - 10*B*d*f))*(a + b*x)^2*(c + d*x)^(3/2)*Sqrt[e + f*x]
)/(4*d*f) + (((c + d*x)^(3/2)*Sqrt[e + f*x]*(96*a^3*C*d^3*f^3 + 8*a^2*b*d^
2*f^2*(23*C*d*e + 9*c*C*f - 30*B*d*f) + 20*a*b^2*d*f*(8*d*f*(5*B*d*e + 3*B
*c*f - 6*A*d*f) - C*(35*d^2*e^2 + 22*c*d*e*f + 15*c^2*f^2)) + b^3*(C*(315*
d^3*e^3 + 203*c*d^2*e^2*f + 145*c^2*d*e*f^2 + 105*c^3*f^3) + 10*d*f*(8*A*d
*f*(5*d*e + 3*c*f) - B*(35*d^2*e^2 + 22*c*d*e*f + 15*c^2*f^2))) + 4*b*d*f*
(8*b*d*f*(6*b*c*C*e + 3*a*C*d*e + a*c*C*f - 10*A*b*d*f) - (7*b*d*e + 5*b*c
*f - 4*a*d*f)*(4*a*C*d*f + b*(9*C*d*e + 7*c*C*f - 10*B*d*f)))*x))/(12*d^2*
f^2) + (5*b*(16*a^2*d^2*f^2*(2*d*f*(3*B*d*e + B*c*f - 4*A*d*f) - C*(5*d^2*
e^2 + 2*c*d*e*f + c^2*f^2)) + 4*a*b*d*f*(C*(35*d^3*e^3 + 15*c*d^2*e^2*f +
9*c^2*d*e*f^2 + 5*c^3*f^3) + 8*d*f*(2*A*d*f*(3*d*e + c*f) - B*(5*d^2*e^2 +
 2*c*d*e*f + c^2*f^2))) - b^2*(C*(63*d^4*e^4 + 28*c*d^3*e^3*f + 18*c^2*d^2
*e^2*f^2 + 12*c^3*d*e*f^3 + 7*c^4*f^4) + 2*d*f*(8*A*d*f*(5*d^2*e^2 + 2*c*d
*e*f + c^2*f^2) - B*(35*d^3*e^3 + 15*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 5*c^3*f
^3))))*((Sqrt[c + d*x]*Sqrt[e + f*x])/f - ((d*e - c*f)*ArcTanh[(Sqrt[f]*Sq
rt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(Sqrt[d]*f^(3/2))))/(8*d^2*f^2))/(8
*d*f))/(10*b*d*f)
```

3.47.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

3.47. $\int \frac{(a+bx)^2\sqrt{c+dx}\,(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$

3.47.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3957 vs. 2(994) = 1988.

Time = 1.68 (sec) , antiderivative size = 3958, normalized size of antiderivative = 3.84

method	result	size
default	Expression too large to display	3958

input
```
int((b*x+a)^2*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
1/3840*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(-4200*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^
(1/2)*a*b*d^4*e^3*f+90*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(
1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*c^3*d^2*e^2*f^3+150*C*ln(1/2*(2*d*f*x+2*((d
*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*c^2*d^3*e^3*f^2
+640*B*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*a*b*c*d^3*f^4*x-3200*B*((d*x+c)
*(f*x+e))^(1/2)*(d*f)^(1/2)*a*b*d^4*e*f^3*x+1440*B*ln(1/2*(2*d*f*x+2*((d*x
+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*d^5*e^2*f^3+1050*
B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/
2))*b^2*d^5*e^4*f-1200*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(
1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*d^5*e^3*f^2-300*C*ln(1/2*(2*d*f*x+2*((d*x+c
)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b*c^4*d*f^5+75*C*ln(1
/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^
2*c^4*d*e*f^4+3840*A*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*a*b*d^4*f^4*x-192
0*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(
1/2))*a*b*c*d^4*e*f^4-120*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f
)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*c^3*d^2*e*f^4+300*B*((d*x+c)*(f*x+e))^(1
/2)*(d*f)^(1/2)*b^2*c^3*d*f^4+1890*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b
^2*d^4*e^4+105*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f
+d*e)/(d*f)^(1/2))*b^2*c^5*f^5+720*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(
1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*c*d^4*e^2*f^3+960*B*((d*x+c)...
```

3.47.5 Fricas [A] (verification not implemented)

Time = 3.08 (sec) , antiderivative size = 2176, normalized size of antiderivative = 2.11

$$\int \frac{(a+bx)^2\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm
="fricas")
```

output
```
[-1/7680*(15*(63*C*b^2*d^5*e^5 - 35*(C*b^2*c*d^4 + 2*(2*C*a*b + B*b^2)*d^5
)*e^4*f - 10*(C*b^2*c^2*d^3 - 4*(2*C*a*b + B*b^2)*c*d^4 - 8*(C*a^2 + 2*B*a
*b + A*b^2)*d^5)*e^3*f^2 - 6*(C*b^2*c^3*d^2 - 2*(2*C*a*b + B*b^2)*c^2*d^3
+ 8*(C*a^2 + 2*B*a*b + A*b^2)*c*d^4 + 16*(B*a^2 + 2*A*a*b)*d^5)*e^2*f^3 -
(5*C*b^2*c^4*d - 128*A*a^2*d^5 - 8*(2*C*a*b + B*b^2)*c^3*d^2 + 16*(C*a^2 +
 2*B*a*b + A*b^2)*c^2*d^3 - 64*(B*a^2 + 2*A*a*b)*c*d^4)*e*f^4 - (7*C*b^2*c
^5 + 128*A*a^2*c*d^4 - 10*(2*C*a*b + B*b^2)*c^4*d + 16*(C*a^2 + 2*B*a*b +
A*b^2)*c^3*d^2 - 32*(B*a^2 + 2*A*a*b)*c^2*d^3)*f^5)*sqrt(d*f)*log(8*d^2*f^
2*x^2 + d^2*e^2 + 6*c*d*e*f + c^2*f^2 + 4*(2*d*f*x + d*e + c*f)*sqrt(d*f)*
sqrt(d*x + c)*sqrt(f*x + e) + 8*(d^2*e*f + c*d*f^2)*x) - 4*(384*C*b^2*d^5*
f^5*x^4 + 945*C*b^2*d^5*e^4*f - 210*(C*b^2*c*d^4 + 5*(2*C*a*b + B*b^2)*d^5
)*e^3*f^2 - 2*(68*C*b^2*c^2*d^3 - 125*(2*C*a*b + B*b^2)*c*d^4 - 600*(C*a^2
 + 2*B*a*b + A*b^2)*d^5)*e^2*f^3 - 10*(11*C*b^2*c^3*d^2 - 17*(2*C*a*b + B*
b^2)*c^2*d^3 + 32*(C*a^2 + 2*B*a*b + A*b^2)*c*d^4 + 144*(B*a^2 + 2*A*a*b)*
d^5)*e*f^4 - 15*(7*C*b^2*c^4*d - 128*A*a^2*d^5 - 10*(2*C*a*b + B*b^2)*c^3*
d^2 + 16*(C*a^2 + 2*B*a*b + A*b^2)*c^2*d^3 - 32*(B*a^2 + 2*A*a*b)*c*d^4)*f
^5 - 48*(9*C*b^2*d^5*e*f^4 - (C*b^2*c*d^4 + 10*(2*C*a*b + B*b^2)*d^5)*f^5)
*x^3 + 8*(63*C*b^2*d^5*e^2*f^3 - 2*(4*C*b^2*c*d^4 + 35*(2*C*a*b + B*b^2)*d
^5)*e*f^4 - (7*C*b^2*c^2*d^3 - 10*(2*C*a*b + B*b^2)*c*d^4 - 80*(C*a^2 + 2*
B*a*b + A*b^2)*d^5)*f^5)*x^2 - 2*(315*C*b^2*d^5*e^3*f^2 - 7*(7*C*b^2*c*...
```

3.47.6 Sympy [F]

$$\int \frac{(a+bx)^2\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \int \frac{(a+bx)^2\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$$

input
```
integrate((b*x+a)**2*(C*x**2+B*x+A)*(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

output
```
Integral((a + b*x)**2*sqrt(c + d*x)*(A + B*x + C*x**2)/sqrt(e + f*x), x)
```

3.47.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^2\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^2*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

3.47.8 Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 1509, normalized size of antiderivative = 1.46

$$\int \frac{(a+bx)^2\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm
="giac")
```

output
```
1/1920*(sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*(2*(4*(d*x + c)*(6*(d*x + c)*(
8*(d*x + c)*C*b^2/(d^5*f) - (9*C*b^2*d^21*e*f^7 + 31*C*b^2*c*d^20*f^8 - 20
*C*a*b*d^21*f^8 - 10*B*b^2*d^21*f^8)/(d^25*f^9)) + (63*C*b^2*d^22*e^2*f^6
+ 154*C*b^2*c*d^21*e*f^7 - 140*C*a*b*d^22*e*f^7 - 70*B*b^2*d^22*e*f^7 + 26
3*C*b^2*c^2*d^20*f^8 - 340*C*a*b*c*d^21*f^8 - 170*B*b^2*c*d^21*f^8 + 80*C*
a^2*d^22*f^8 + 160*B*a*b*d^22*f^8 + 80*A*b^2*d^22*f^8)/(d^25*f^9)) - 5*(63
*C*b^2*d^23*e^3*f^5 + 91*C*b^2*c*d^22*e^2*f^6 - 140*C*a*b*d^23*e^2*f^6 - 7
0*B*b^2*d^23*e^2*f^6 + 109*C*b^2*c^2*d^21*e*f^7 - 200*C*a*b*c*d^22*e*f^7 -
 100*B*b^2*c*d^22*e*f^7 + 80*C*a^2*d^23*e*f^7 + 160*B*a*b*d^23*e*f^7 + 80*
A*b^2*d^23*e*f^7 + 121*C*b^2*c^3*d^20*f^8 - 236*C*a*b*c^2*d^21*f^8 - 118*B
*b^2*c^2*d^21*f^8 + 112*C*a^2*c*d^22*f^8 + 224*B*a*b*c*d^22*f^8 + 112*A*b^
2*c*d^22*f^8 - 96*B*a^2*d^23*f^8 - 192*A*a*b*d^23*f^8)/(d^25*f^9))*(d*x +
c) + 15*(63*C*b^2*d^24*e^4*f^4 + 28*C*b^2*c*d^23*e^3*f^5 - 140*C*a*b*d^24*
e^3*f^5 - 70*B*b^2*d^24*e^3*f^5 + 18*C*b^2*c^2*d^22*e^2*f^6 - 60*C*a*b*c*d
^23*e^2*f^6 - 30*B*b^2*c*d^23*e^2*f^6 + 80*C*a^2*d^24*e^2*f^6 + 160*B*a*b*
d^24*e^2*f^6 + 80*A*b^2*d^24*e^2*f^6 + 12*C*b^2*c^3*d^21*e*f^7 - 36*C*a*b*
c^2*d^22*e*f^7 - 18*B*b^2*c^2*d^22*e*f^7 + 32*C*a^2*c*d^23*e*f^7 + 64*B*a*
b*c*d^23*e*f^7 + 32*A*b^2*c*d^23*e*f^7 - 96*B*a^2*d^24*e*f^7 - 192*A*a*b*d
^24*e*f^7 + 7*C*b^2*c^4*d^20*f^8 - 20*C*a*b*c^3*d^21*f^8 - 10*B*b^2*c^3*d^
21*f^8 + 16*C*a^2*c^2*d^22*f^8 + 32*B*a*b*c^2*d^22*f^8 + 16*A*b^2*c^2*d...
```

3.47.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \text{Hanged}$$

input
```
int(((a + b*x)^2*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(e + f*x)^(1/2),x)
```

output
```
\text{Hanged}
```

3.48 $\int \frac{(a+bx)\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$

3.48.1 Optimal result

Integrand size = 34, antiderivative size = 540

$$\int \frac{(a+bx)\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx =$$

$$-\frac{(8adf(2df(3Bde+Bcf-4Adf)-C(5d^2e^2+2cdef+c^2f^2))+b(C(35d^3e^3+15cd^2e^2f+9c^2def^2+5}{64d^3f^4}$$

$$+\frac{C(a+bx)^2(c+dx)^{3/2}\sqrt{e+fx}}{4bdf}$$

$$-\frac{(c+dx)^{3/2}\sqrt{e+fx}(24a^2Cd^2f^2+8abdf(5Cde+3cCf-6Bdf)+b^2(8df(5Bde+3Bcf-6Adf)-C(}{96bd^3f^3}$$

$$+\frac{(de-cf)\,(8adf(2df(3Bde+Bcf-4Adf)-C(5d^2e^2+2cdef+c^2f^2))+b(C(35d^3e^3+15cd^2e^2f+9c}{64d^{7/2}f^{9/2}}$$

output
```
1/64*(-c*f+d*e)*(8*a*d*f*(2*d*f*(-4*A*d*f+B*c*f+3*B*d*e)-C*(c^2*f^2+2*c*d*
e*f+5*d^2*e^2))+b*(C*(5*c^3*f^3+9*c^2*d*e*f^2+15*c*d^2*e^2*f+35*d^3*e^3)+8
*d*f*(2*A*d*f*(c*f+3*d*e)-B*(c^2*f^2+2*c*d*e*f+5*d^2*e^2))))*arctanh(f^(1/
2)*(d*x+c)^(1/2)/d^(1/2)/(f*x+e)^(1/2))/d^(7/2)/f^(9/2)+1/4*C*(b*x+a)^2*(d
*x+c)^(3/2)*(f*x+e)^(1/2)/b/d/f-1/96*(d*x+c)^(3/2)*(24*a^2*C*d^2*f^2+8*a*b
*d*f*(-6*B*d*f+3*C*c*f+5*C*d*e)+b^2*(8*d*f*(-6*A*d*f+3*B*c*f+5*B*d*e)-C*(1
5*c^2*f^2+22*c*d*e*f+35*d^2*e^2))+4*b*d*f*(4*a*C*d*f+b*(-8*B*d*f+5*C*c*f+7
*C*d*e))*x)*(f*x+e)^(1/2)/b/d^3/f^3-1/64*(8*a*d*f*(2*d*f*(-4*A*d*f+B*c*f+3
*B*d*e)-C*(c^2*f^2+2*c*d*e*f+5*d^2*e^2))+b*(C*(5*c^3*f^3+9*c^2*d*e*f^2+15*
c*d^2*e^2*f+35*d^3*e^3)+8*d*f*(2*A*d*f*(c*f+3*d*e)-B*(c^2*f^2+2*c*d*e*f+5*
d^2*e^2))))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/d^3/f^4
```

3.48.2 Mathematica [A] (verified)

Time = 8.63 (sec) , antiderivative size = 474, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx)\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$$
$$= \frac{\sqrt{d}\sqrt{f}\sqrt{c+dx}\sqrt{e+fx}(8adf(6df(4Adf+B(-3de+cf+2dfx))+C(-3c^2f^2+2cdf(-2e+fx)+d^2(1}{}$$

input
```
Integrate[((a + b*x)*Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]
```

output
```
(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x]*Sqrt[e + f*x]*(8*a*d*f*(6*d*f*(4*A*d*f + B*
(-3*d*e + c*f + 2*d*f*x)) + C*(-3*c^2*f^2 + 2*c*d*f*(-2*e + f*x) + d^2*(15
*e^2 - 10*e*f*x + 8*f^2*x^2))) + b*(C*(15*c^3*f^3 + c^2*d*f^2*(17*e - 10*f
*x) + c*d^2*f*(25*e^2 - 12*e*f*x + 8*f^2*x^2) + d^3*(-105*e^3 + 70*e^2*f*x
 - 56*e*f^2*x^2 + 48*f^3*x^3)) + 8*d*f*(6*A*d*f*(-3*d*e + c*f + 2*d*f*x) +
 B*(-3*c^2*f^2 + 2*c*d*f*(-2*e + f*x) + d^2*(15*e^2 - 10*e*f*x + 8*f^2*x^2
))))) - 6*(d*e - c*f)*(-8*a*d*f*(2*d*f*(-3*B*d*e - B*c*f + 4*A*d*f) + C*(5
*d^2*e^2 + 2*c*d*e*f + c^2*f^2)) + b*(C*(35*d^3*e^3 + 15*c*d^2*e^2*f + 9*c
^2*d*e*f^2 + 5*c^3*f^3) + 8*d*f*(2*A*d*f*(3*d*e + c*f) - B*(5*d^2*e^2 + 2*
c*d*e*f + c^2*f^2))))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/(Sqrt[f]*(Sqrt[c - (
d*e)/f] - Sqrt[c + d*x]))])/(192*d^(7/2)*f^(9/2))
```

3.48.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 419, normalized size of antiderivative = 0.78, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2118, 27, 164, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$$
$$\Big\downarrow \; 2118$$

$$\frac{\int -\frac{b(a+bx)\sqrt{c+dx}(4bcCe+3aCde+acCf-8Abdf+(4aCdf+b(7Cde+5cCf-8Bdf))x)}{2\sqrt{e+fx}}dx}{4b^2df}+\frac{C(a+bx)^2(c+dx)^{3/2}\sqrt{e+fx}}{4bdf}$$

$\Big\downarrow$ 27

$$\frac{C(a+bx)^2(c+dx)^{3/2}\sqrt{e+fx}}{4bdf}-\frac{\int \frac{(a+bx)\sqrt{c+dx}(4bcCe+3aCde+acCf-8Abdf+(4aCdf+b(7Cde+5cCf-8Bdf))x)}{\sqrt{e+fx}}dx}{8bdf}$$

$\Big\downarrow$ 164

$$\frac{C(a+bx)^2(c+dx)^{3/2}\sqrt{e+fx}}{4bdf}-$$

$$b(8adf(2df(-4Adf+Bcf+3Bde)-C(c^2f^2+2cdef+5d^2e^2))+b(8df(2Adf(cf+3de)-B(c^2f^2+2cdef+5d^2e^2))+C(5c^3f^3+9c^2def^2+15cd^2e^2f+35$$

$$8d^2f^2$$

$\Big\downarrow$ 60

$$\frac{C(a+bx)^2(c+dx)^{3/2}\sqrt{e+fx}}{4bdf}-$$

$$b(8adf(2df(-4Adf+Bcf+3Bde)-C(c^2f^2+2cdef+5d^2e^2))+b(8df(2Adf(cf+3de)-B(c^2f^2+2cdef+5d^2e^2))+C(5c^3f^3+9c^2def^2+15cd^2e^2f+35$$

$$8d^2f^2$$

$\Big\downarrow$ 66

$$\frac{C(a+bx)^2(c+dx)^{3/2}\sqrt{e+fx}}{4bdf}-$$

$$b(8adf(2df(-4Adf+Bcf+3Bde)-C(c^2f^2+2cdef+5d^2e^2))+b(8df(2Adf(cf+3de)-B(c^2f^2+2cdef+5d^2e^2))+C(5c^3f^3+9c^2def^2+15cd^2e^2f+35$$

$$8d^2f^2$$

$\Big\downarrow$ 221

$$\frac{C(a+bx)^2(c+dx)^{3/2}\sqrt{e+fx}}{4bdf}-$$

$$(c+dx)^{3/2}\sqrt{e+fx}(24a^2Cd^2f^2+4bdfx(4aCdf+b(-8Bdf+5cCf+7Cde))+8abdf(-6Bdf+3cCf+5Cde)+b^2(8df(-6Adf+3Bcf+5Bde)-C(15c^2f$$

$$12d^2f^2$$

input

```
Int[((a + b*x)*Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]
```

3.48. $\int \frac{(a+bx)\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$

output
```
(C*(a + b*x)^2*(c + d*x)^(3/2)*Sqrt[e + f*x])/(4*b*d*f) - (((c + d*x)^(3/2
)*Sqrt[e + f*x]*(24*a^2*C*d^2*f^2 + 8*a*b*d*f*(5*C*d*e + 3*c*C*f - 6*B*d*f
) + b^2*(8*d*f*(5*B*d*e + 3*B*c*f - 6*A*d*f) - C*(35*d^2*e^2 + 22*c*d*e*f
+ 15*c^2*f^2)) + 4*b*d*f*(4*a*C*d*f + b*(7*C*d*e + 5*c*C*f - 8*B*d*f))*x))
/(12*d^2*f^2) + (b*(8*a*d*f*(2*d*f*(3*B*d*e + B*c*f - 4*A*d*f) - C*(5*d^2*
e^2 + 2*c*d*e*f + c^2*f^2)) + b*(C*(35*d^3*e^3 + 15*c*d^2*e^2*f + 9*c^2*d*
e*f^2 + 5*c^3*f^3) + 8*d*f*(2*A*d*f*(3*d*e + c*f) - B*(5*d^2*e^2 + 2*c*d*e
*f + c^2*f^2))))*((Sqrt[c + d*x]*Sqrt[e + f*x])/f - ((d*e - c*f)*ArcTanh[(
Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(Sqrt[d]*f^(3/2))))/(8*d^
2*f^2))/(8*b*d*f)
```

3.48.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

3.48.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2001 vs. $2(508) = 1016$.

Time = 1.67 (sec) , antiderivative size = 2002, normalized size of antiderivative = 3.71

method	result	size
default	Expression too large to display	2002

input
```
int((b*x+a)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
1/384*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(-48*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/
2)*a*c^2*d*f^3-24*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b*c*d^2*e*f^2*x-12
*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1
/2))*b*c^3*d*e*f^3-18*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1
/2)+c*f+d*e)/(d*f)^(1/2))*b*c^2*d^2*e^2*f^2+24*C*ln(1/2*(2*d*f*x+2*((d*x+c
)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c^2*d^2*e*f^3+144*A*l
n(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))
*b*d^4*e^2*f^2-48*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+
c*f+d*e)/(d*f)^(1/2))*a*c^2*d^2*f^4+192*B*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1
/2)*a*d^3*f^3*x+144*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2
)+c*f+d*e)/(d*f)^(1/2))*a*d^4*e^2*f^2-120*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*
x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*d^4*e^3*f+96*A*((d*x+c)*(f
*x+e))^(1/2)*(d*f)^(1/2)*b*c*d^2*f^3+192*A*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(
1/2)*b*d^3*f^3*x+384*A*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*a*d^3*f^3-120*C
*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2
))*a*d^4*e^3*f-64*B*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b*c*d^2*e*f^2-64*C
*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*a*c*d^2*e*f^2+50*C*((d*x+c)*(f*x+e))^
(1/2)*(d*f)^(1/2)*b*c*d^2*e^2*f-48*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(
1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c^2*d^2*f^4+32*C*((d*x+c)*(f*x+e)
)^(1/2)*(d*f)^(1/2)*a*c*d^2*f^3*x+140*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(...
```

3.48.5 Fricas [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 1114, normalized size of antiderivative = 2.06

$$\int \frac{(a+bx)\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="
fricas")
```

output
```
[1/768*(3*(35*C*b*d^4*e^4 - 20*(C*b*c*d^3 + 2*(C*a + B*b)*d^4)*e^3*f - 6*(
C*b*c^2*d^2 - 4*(C*a + B*b)*c*d^3 - 8*(B*a + A*b)*d^4)*e^2*f^2 - 4*(C*b*c^
3*d + 16*A*a*d^4 - 2*(C*a + B*b)*c^2*d^2 + 8*(B*a + A*b)*c*d^3)*e*f^3 - (5
*C*b*c^4 - 64*A*a*c*d^3 - 8*(C*a + B*b)*c^3*d + 16*(B*a + A*b)*c^2*d^2)*f^
4)*sqrt(d*f)*log(8*d^2*f^2*x^2 + d^2*e^2 + 6*c*d*e*f + c^2*f^2 + 4*(2*d*f*
x + d*e + c*f)*sqrt(d*f)*sqrt(d*x + c)*sqrt(f*x + e) + 8*(d^2*e*f + c*d*f^
2)*x) + 4*(48*C*b*d^4*f^4*x^3 - 105*C*b*d^4*e^3*f + 5*(5*C*b*c*d^3 + 24*(C
*a + B*b)*d^4)*e^2*f^2 + (17*C*b*c^2*d^2 - 32*(C*a + B*b)*c*d^3 - 144*(B*a
 + A*b)*d^4)*e*f^3 + 3*(5*C*b*c^3*d + 64*A*a*d^4 - 8*(C*a + B*b)*c^2*d^2 +
 16*(B*a + A*b)*c*d^3)*f^4 - 8*(7*C*b*d^4*e*f^3 - (C*b*c*d^3 + 8*(C*a + B*
b)*d^4)*f^4)*x^2 + 2*(35*C*b*d^4*e^2*f^2 - 2*(3*C*b*c*d^3 + 20*(C*a + B*b)
*d^4)*e*f^3 - (5*C*b*c^2*d^2 - 8*(C*a + B*b)*c*d^3 - 48*(B*a + A*b)*d^4)*f
^4)*x)*sqrt(d*x + c)*sqrt(f*x + e))/(d^4*f^5), -1/384*(3*(35*C*b*d^4*e^4 -
 20*(C*b*c*d^3 + 2*(C*a + B*b)*d^4)*e^3*f - 6*(C*b*c^2*d^2 - 4*(C*a + B*b)
*c*d^3 - 8*(B*a + A*b)*d^4)*e^2*f^2 - 4*(C*b*c^3*d + 16*A*a*d^4 - 2*(C*a +
 B*b)*c^2*d^2 + 8*(B*a + A*b)*c*d^3)*e*f^3 - (5*C*b*c^4 - 64*A*a*c*d^3 - 8
*(C*a + B*b)*c^3*d + 16*(B*a + A*b)*c^2*d^2)*f^4)*sqrt(-d*f)*arctan(1/2*(2
*d*f*x + d*e + c*f)*sqrt(-d*f)*sqrt(d*x + c)*sqrt(f*x + e)/(d^2*f^2*x^2 +
c*d*e*f + (d^2*e*f + c*d*f^2)*x)) - 2*(48*C*b*d^4*f^4*x^3 - 105*C*b*d^4*e^
3*f + 5*(5*C*b*c*d^3 + 24*(C*a + B*b)*d^4)*e^2*f^2 + (17*C*b*c^2*d^2 - ...
```

3.48.6 Sympy [F]

$$\int \frac{(a+bx)\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \int \frac{(a+bx)\,\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$$

input
```
integrate((b*x+a)*(C*x**2+B*x+A)*(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

output
```
Integral((a + b*x)*sqrt(c + d*x)*(A + B*x + C*x**2)/sqrt(e + f*x), x)
```

3.48.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

3.48.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 733, normalized size of antiderivative = 1.36

$$\int \frac{(a+bx)\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$$
$$= \frac{\left(\sqrt{d^2e+(dx+c)df-cdf}\left(2\,(dx+c)\left(4\,(dx+c)\left(\frac{6\,(dx+c)Cb}{d^4f}-\frac{7\,Cbd^{13}ef^5+17\,Cbcd^{12}f^6-8\,Cad^{13}f^6-8\,Bbd^{13}f^6}{d^{16}f^7}\right)+\right.\right.\right.}{}$$

input
```
integrate((b*x+a)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="
giac")
```

output
```
1/192*(sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*(2*(d*x + c)*(4*(d*x + c)*(6*(d
*x + c)*C*b/(d^4*f) - (7*C*b*d^13*e*f^5 + 17*C*b*c*d^12*f^6 - 8*C*a*d^13*f
^6 - 8*B*b*d^13*f^6)/(d^16*f^7)) + (35*C*b*d^14*e^2*f^4 + 50*C*b*c*d^13*e*
f^5 - 40*C*a*d^14*e*f^5 - 40*B*b*d^14*e*f^5 + 59*C*b*c^2*d^12*f^6 - 56*C*a
*c*d^13*f^6 - 56*B*b*c*d^13*f^6 + 48*B*a*d^14*f^6 + 48*A*b*d^14*f^6)/(d^16
*f^7)) - 3*(35*C*b*d^15*e^3*f^3 + 15*C*b*c*d^14*e^2*f^4 - 40*C*a*d^15*e^2*
f^4 - 40*B*b*d^15*e^2*f^4 + 9*C*b*c^2*d^13*e*f^5 - 16*C*a*c*d^14*e*f^5 - 1
6*B*b*c*d^14*e*f^5 + 48*B*a*d^15*e*f^5 + 48*A*b*d^15*e*f^5 + 5*C*b*c^3*d^1
2*f^6 - 8*C*a*c^2*d^13*f^6 - 8*B*b*c^2*d^13*f^6 + 16*B*a*c*d^14*f^6 + 16*A
*b*c*d^14*f^6 - 64*A*a*d^15*f^6)/(d^16*f^7))*sqrt(d*x + c) - 3*(35*C*b*d^4
*e^4 - 20*C*b*c*d^3*e^3*f - 40*C*a*d^4*e^3*f - 40*B*b*d^4*e^3*f - 6*C*b*c^
2*d^2*e^2*f^2 + 24*C*a*c*d^3*e^2*f^2 + 24*B*b*c*d^3*e^2*f^2 + 48*B*a*d^4*e
^2*f^2 + 48*A*b*d^4*e^2*f^2 - 4*C*b*c^3*d*e*f^3 + 8*C*a*c^2*d^2*e*f^3 + 8*
B*b*c^2*d^2*e*f^3 - 32*B*a*c*d^3*e*f^3 - 32*A*b*c*d^3*e*f^3 - 64*A*a*d^4*e
*f^3 - 5*C*b*c^4*f^4 + 8*C*a*c^3*d*f^4 + 8*B*b*c^3*d*f^4 - 16*B*a*c^2*d^2*
f^4 - 16*A*b*c^2*d^2*f^4 + 64*A*a*c*d^3*f^4)*log(abs(-sqrt(d*f)*sqrt(d*x +
 c) + sqrt(d^2*e + (d*x + c)*d*f - c*d*f)))/(sqrt(d*f)*d^3*f^4))*d/abs(d)
```

3.48.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \text{Hanged}$$

input
```
int(((a + b*x)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(e + f*x)^(1/2),x)
```

output
```
\text{Hanged}
```

3.49 $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$

3.49.1 Optimal result

Integrand size = 29, antiderivative size = 246

$$\begin{aligned}\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx &= \frac{(C(5d^2e^2+2cdef+c^2f^2)+2df(4Adf-B(3de+cf)))\sqrt{c+dx}\sqrt{e+fx}}{8d^2f^3}\\ &\quad-\frac{(5Cde+7cCf-6Bdf)(c+dx)^{3/2}\sqrt{e+fx}}{12d^2f^2}+\frac{C(c+dx)^{5/2}\sqrt{e+fx}}{3d^2f}\\ &\quad-\frac{(de-cf)\,(C(5d^2e^2+2cdef+c^2f^2)+2df(4Adf-B(3de+cf)))\,\text{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)}{8d^{5/2}f^{7/2}}\end{aligned}$$

output
```
-1/8*(-c*f+d*e)*(C*(c^2*f^2+2*c*d*e*f+5*d^2*e^2)+2*d*f*(4*A*d*f-B*(c*f+3*d
*e)))*arctanh(f^(1/2)*(d*x+c)^(1/2)/d^(1/2)/(f*x+e)^(1/2))/d^(5/2)/f^(7/2)
-1/12*(-6*B*d*f+7*C*c*f+5*C*d*e)*(d*x+c)^(3/2)*(f*x+e)^(1/2)/d^2/f^2+1/3*C
*(d*x+c)^(5/2)*(f*x+e)^(1/2)/d^2/f+1/8*(C*(c^2*f^2+2*c*d*e*f+5*d^2*e^2)+2*
d*f*(4*A*d*f-B*(c*f+3*d*e)))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/d^2/f^3
```

3.49.2 Mathematica [A] (verified)

Time = 3.62 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$$
$$= \frac{\sqrt{c+dx}\sqrt{e+fx}(6df(4Adf+B(-3de+cf+2dfx))+C(-3c^2f^2+2cdf(-2e+fx)+d^2(15e^2-10efx}{24d^2f^3}$$
$$+\frac{(de-cf)\left(C(5d^2e^2+2cdef+c^2f^2)+2df(4Adf-B(3de+cf))\right)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{f}\left(\sqrt{c-\frac{de}{f}}-\sqrt{c+dx}\right)}\right)}{4d^{5/2}f^{7/2}}$$

input
```
Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]
```

output
```
(Sqrt[c + d*x]*Sqrt[e + f*x]*(6*d*f*(4*A*d*f + B*(-3*d*e + c*f + 2*d*f*x))
 + C*(-3*c^2*f^2 + 2*c*d*f*(-2*e + f*x) + d^2*(15*e^2 - 10*e*f*x + 8*f^2*x
^2))))/(24*d^2*f^3) + ((d*e - c*f)*(C*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2) +
2*d*f*(4*A*d*f - B*(3*d*e + c*f)))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/(Sqrt[f
]*(Sqrt[c - (d*e)/f] - Sqrt[c + d*x]))])/(4*d^(5/2)*f^(7/2))
```

3.49.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.85, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1194, 27, 90, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{\sqrt{e+fx}}\,dx$$

$$\Big\downarrow \ 1194$$

$$\frac{\int -\frac{\sqrt{c+dx}\left(Cfc^2+5Cdec-6Ad^2f+d(5Cde+7cCf-6Bdf)x\right)}{2\sqrt{e+fx}}dx}{3d^2f}+\frac{C(c+dx)^{5/2}\sqrt{e+fx}}{3d^2f}$$

$$\Big\downarrow \ 27$$

$$\frac{C(c+dx)^{5/2}\sqrt{e+fx}}{3d^2f}-\frac{\int \frac{\sqrt{c+dx}\left(Cfc^2+5Cdec-6Ad^2f+d(5Cde+7cCf-6Bdf)x\right)}{\sqrt{e+fx}}dx}{6d^2f}$$

$$\Big\downarrow \; 90$$

$$\frac{C(c+dx)^{5/2}\sqrt{e+fx}}{3d^2f}-\frac{\frac{(c+dx)^{3/2}\sqrt{e+fx}(-6Bdf+7cCf+5Cde)}{2f}-\frac{3\left(2df(4Adf-B(cf+3de))+C\left(c^2f^2+2cdef+5d^2e^2\right)\right)\int\frac{\sqrt{c+dx}}{\sqrt{e+fx}}dx}{4f}}{6d^2f}$$

$$\Big\downarrow \; 60$$

$$\frac{C(c+dx)^{5/2}\sqrt{e+fx}}{3d^2f}-\frac{\frac{(c+dx)^{3/2}\sqrt{e+fx}(-6Bdf+7cCf+5Cde)}{2f}-\frac{3\left(2df(4Adf-B(cf+3de))+C\left(c^2f^2+2cdef+5d^2e^2\right)\right)\left(\frac{\sqrt{c+dx}\sqrt{e+fx}}{f}-\frac{(de-cf)\int\frac{1}{\sqrt{c+dx}\sqrt{e+fx}}dx}{2f}\right)}{4f}}{6d^2f}$$

$$\Big\downarrow \; 66$$

$$\frac{C(c+dx)^{5/2}\sqrt{e+fx}}{3d^2f}-\frac{\frac{(c+dx)^{3/2}\sqrt{e+fx}(-6Bdf+7cCf+5Cde)}{2f}-\frac{3\left(2df(4Adf-B(cf+3de))+C\left(c^2f^2+2cdef+5d^2e^2\right)\right)\left(\frac{\sqrt{c+dx}\sqrt{e+fx}}{f}-\frac{(de-cf)\int\frac{1}{d-\frac{f(c+dx)}{e+fx}}d\frac{\sqrt{c+dx}}{\sqrt{e+fx}}}{f}\right)}{4f}}{6d^2f}$$

$$\Big\downarrow \; 221$$

$$\frac{C(c+dx)^{5/2}\sqrt{e+fx}}{3d^2f}-\frac{\frac{(c+dx)^{3/2}\sqrt{e+fx}(-6Bdf+7cCf+5Cde)}{2f}-\frac{3\left(\frac{\sqrt{c+dx}\sqrt{e+fx}}{f}-\frac{(de-cf)\text{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)}{\sqrt{d}f^{3/2}}\right)\left(2df(4Adf-B(cf+3de))+C\left(c^2f^2+2cdef+5d^2e^2\right)\right)}{4f}}{6d^2f}$$

input
```
Int[(Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]
```

output
```
(C*(c + d*x)^(5/2)*Sqrt[e + f*x])/(3*d^2*f) - ((((5*C*d*e + 7*c*C*f - 6*B*d
*f)*(c + d*x)^(3/2)*Sqrt[e + f*x])/(2*f) - (3*(C*(5*d^2*e^2 + 2*c*d*e*f +
c^2*f^2) + 2*d*f*(4*A*d*f - B*(3*d*e + c*f)))*((Sqrt[c + d*x]*Sqrt[e + f*x
])/f - ((d*e - c*f)*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x]
)])/(Sqrt[d]*f^(3/2))))/(4*f))/(6*d^2*f)
```

3.49. $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$

3.49.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1194
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Simp[1/(g*e^(2*p)*(m + n + 2
*p + 1))   Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(
2*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)
*(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ
[p, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && NeQ[m + n + 2*p + 1, 0]
```

3.49. $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$

3.49.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 762 vs. 2(214) = 428.

Time = 1.67 (sec) , antiderivative size = 763, normalized size of antiderivative = 3.10

method	result
default	$\frac{\sqrt{dx+c}\,\sqrt{fx+e}\left(16C\,d^2 f^2 x^2 \sqrt{(dx+c)(fx+e)}\,\sqrt{df}+24A\ln\left(\frac{2dfx+2\sqrt{(dx+c)(fx+e)}\,\sqrt{df}+cf+de}{2\sqrt{df}}\right)c\,d^2 f^3-24A\ln\left(\frac{2dfx+2\sqrt{(dx+c)(fx+}}{2\sqrt{df}}\right.\right.}{}$

input
```
int((C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/48*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(16*C*d^2*f^2*x^2*((d*x+c)*(f*x+e))^(1/2)
*(d*f)^(1/2)+24*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*
f+d*e)/(d*f)^(1/2))*c*d^2*f^3-24*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/
2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*d^3*e*f^2-6*B*ln(1/2*(2*d*f*x+2*((d*x
+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c^2*d*f^3-12*B*ln(1/2
*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c*d^
2*e*f^2+18*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e
)/(d*f)^(1/2))*d^3*e^2*f+24*B*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*d^2*f^2*
x+3*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)
^(1/2))*c^3*f^3+3*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+
c*f+d*e)/(d*f)^(1/2))*c^2*d*e*f^2+9*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^
(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c*d^2*e^2*f-15*C*ln(1/2*(2*d*f*x+2
*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*d^3*e^3+4*C*((d
*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*c*d*f^2*x-20*C*((d*x+c)*(f*x+e))^(1/2)*(d
*f)^(1/2)*d^2*e*f*x+48*A*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*d^2*f^2+12*B*
((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*c*d*f^2-36*B*(d*f)^(1/2)*((d*x+c)*(f*x
+e))^(1/2)*d^2*e*f-6*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*c^2*f^2-8*C*((d
*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*c*d*e*f+30*C*(d*f)^(1/2)*((d*x+c)*(f*x+e)
)^(1/2)*d^2*e^2)/f^3/((d*x+c)*(f*x+e))^(1/2)/d^2/(d*f)^(1/2)
```

3.49.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 576, normalized size of antiderivative = 2.34

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$$

$$= \left[-\frac{3\left(5\,Cd^3e^3-3\,(Ccd^2+2\,Bd^3)e^2f-(Cc^2d-4\,Bcd^2-8\,Ad^3)ef^2-(Cc^3-2\,Bc^2d+8\,Acd^2)f^3\right)\sqrt{df}\,\mathrm{l}}{}\right.$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
[-1/96*(3*(5*C*d^3*e^3 - 3*(C*c*d^2 + 2*B*d^3)*e^2*f - (C*c^2*d - 4*B*c*d^
2 - 8*A*d^3)*e*f^2 - (C*c^3 - 2*B*c^2*d + 8*A*c*d^2)*f^3)*sqrt(d*f)*log(8*
d^2*f^2*x^2 + d^2*e^2 + 6*c*d*e*f + c^2*f^2 + 4*(2*d*f*x + d*e + c*f)*sqrt
(d*f)*sqrt(d*x + c)*sqrt(f*x + e) + 8*(d^2*e*f + c*d*f^2)*x) - 4*(8*C*d^3*
f^3*x^2 + 15*C*d^3*e^2*f - 2*(2*C*c*d^2 + 9*B*d^3)*e*f^2 - 3*(C*c^2*d - 2*
B*c*d^2 - 8*A*d^3)*f^3 - 2*(5*C*d^3*e*f^2 - (C*c*d^2 + 6*B*d^3)*f^3)*x)*sq
rt(d*x + c)*sqrt(f*x + e))/(d^3*f^4), 1/48*(3*(5*C*d^3*e^3 - 3*(C*c*d^2 +
2*B*d^3)*e^2*f - (C*c^2*d - 4*B*c*d^2 - 8*A*d^3)*e*f^2 - (C*c^3 - 2*B*c^2*
d + 8*A*c*d^2)*f^3)*sqrt(-d*f)*arctan(1/2*(2*d*f*x + d*e + c*f)*sqrt(-d*f)
*sqrt(d*x + c)*sqrt(f*x + e)/(d^2*f^2*x^2 + c*d*e*f + (d^2*e*f + c*d*f^2)*
x)) + 2*(8*C*d^3*f^3*x^2 + 15*C*d^3*e^2*f - 2*(2*C*c*d^2 + 9*B*d^3)*e*f^2
- 3*(C*c^2*d - 2*B*c*d^2 - 8*A*d^3)*f^3 - 2*(5*C*d^3*e*f^2 - (C*c*d^2 + 6*
B*d^3)*f^3)*x)*sqrt(d*x + c)*sqrt(f*x + e))/(d^3*f^4)]
```

3.49.6 Sympy [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \int \frac{\sqrt{c+dx}\,(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$$

input
```
integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

output
```
Integral(sqrt(c + d*x)*(A + B*x + C*x**2)/sqrt(e + f*x), x)
```

3.49.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

3.49.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$$
$$= \frac{\left(\sqrt{d^2e+(dx+c)df-cdf}\sqrt{dx+c}\left(2\,(dx+c)\left(\frac{4\,(dx+c)C}{d^3f}-\frac{5\,Cd^7ef^3+7\,Ccd^6f^4-6\,Bd^7f^4}{d^9f^5}\right)+\frac{3\,(5\,Cd^8e^2f^2+2\,Ccd^7ef^3}{}\right.\right.}{}$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
1/24*(sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*sqrt(d*x + c)*(2*(d*x + c)*(4*(d
*x + c)*C/(d^3*f) - (5*C*d^7*e*f^3 + 7*C*c*d^6*f^4 - 6*B*d^7*f^4)/(d^9*f^5
)) + 3*(5*C*d^8*e^2*f^2 + 2*C*c*d^7*e*f^3 - 6*B*d^8*e*f^3 + C*c^2*d^6*f^4
- 2*B*c*d^7*f^4 + 8*A*d^8*f^4)/(d^9*f^5)) + 3*(5*C*d^3*e^3 - 3*C*c*d^2*e^2
*f - 6*B*d^3*e^2*f - C*c^2*d*e*f^2 + 4*B*c*d^2*e*f^2 + 8*A*d^3*e*f^2 - C*c
^3*f^3 + 2*B*c^2*d*f^3 - 8*A*c*d^2*f^3)*log(abs(-sqrt(d*f)*sqrt(d*x + c) +
 sqrt(d^2*e + (d*x + c)*d*f - c*d*f)))/(sqrt(d*f)*d^2*f^3))*d/abs(d)
```

3.49.9 Mupad [B] (verification not implemented)

Time = 98.80 (sec) , antiderivative size = 1832, normalized size of antiderivative = 7.45

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/(e + f*x)^(1/2),x)
```

output
```
((((c + d*x)^(1/2) - c^(1/2))*(2*A*d^2*e + 2*A*c*d*f))/(f^3*((e + f*x)^(1/
2) - e^(1/2))) + ((2*A*c*f + 2*A*d*e)*((c + d*x)^(1/2) - c^(1/2))^3)/(f^2*
((e + f*x)^(1/2) - e^(1/2))^3) - (8*A*c^(1/2)*d*e^(1/2)*((c + d*x)^(1/2) -
 c^(1/2))^2)/(f^2*((e + f*x)^(1/2) - e^(1/2))^2))/(((c + d*x)^(1/2) - c^(1
/2))^4/((e + f*x)^(1/2) - e^(1/2))^4 + d^2/f^2 - (2*d*((c + d*x)^(1/2) - c
^(1/2))^2)/(f*((e + f*x)^(1/2) - e^(1/2))^2)) - ((((c + d*x)^(1/2) - c^(1/
2))*((C*c^3*d^3*f^3)/4 - (5*C*d^6*e^3)/4 + (C*c^2*d^4*e*f^2)/4 + (3*C*c*d^
5*e^2*f)/4))/(f^9*((e + f*x)^(1/2) - e^(1/2))) - (((c + d*x)^(1/2) - c^(1/
2))^5*((33*C*d^4*e^3)/2 + (19*C*c^3*d*f^3)/2 + (275*C*c^2*d^2*e*f^2)/2 + (
313*C*c*d^3*e^2*f)/2))/(f^7*((e + f*x)^(1/2) - e^(1/2))^5) - (((c + d*x)^(
1/2) - c^(1/2))^7*((19*C*c^3*f^3)/2 + (33*C*d^3*e^3)/2 + (313*C*c*d^2*e^2*
f)/2 + (275*C*c^2*d*e*f^2)/2))/(f^6*((e + f*x)^(1/2) - e^(1/2))^7) - (((c
+ d*x)^(1/2) - c^(1/2))^3*((17*C*c^3*d^2*f^3)/12 - (85*C*d^5*e^3)/12 + (91
*C*c^2*d^3*e*f^2)/4 + (17*C*c*d^4*e^2*f)/4))/(f^8*((e + f*x)^(1/2) - e^(1/
2))^3) + (((c + d*x)^(1/2) - c^(1/2))^11*((C*c^3*f^3)/4 - (5*C*d^3*e^3)/4
+ (3*C*c*d^2*e^2*f)/4 + (C*c^2*d*e*f^2)/4))/(d^2*f^4*((e + f*x)^(1/2) - e^
(1/2))^11) - (((c + d*x)^(1/2) - c^(1/2))^9*((17*C*c^3*f^3)/12 - (85*C*d^3
*e^3)/12 + (17*C*c*d^2*e^2*f)/4 + (91*C*c^2*d*e*f^2)/4))/(d*f^5*((e + f*x)
^(1/2) - e^(1/2))^9) + (c^(1/2)*e^(1/2)*((c + d*x)^(1/2) - c^(1/2))^8*(32*
C*c^2*f + 96*C*c*d*e))/(f^4*((e + f*x)^(1/2) - e^(1/2))^8) + (c^(1/2)*e...
```

3.49. $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$

3.50 $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)\sqrt{e+fx}}\,dx$

3.50.1 Optimal result

Integrand size = 36, antiderivative size = 290

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)\sqrt{e+fx}}\,dx$$
$$= -\frac{(4aCdf+b(3Cde+cCf-4Bdf))\sqrt{c+dx}\sqrt{e+fx}}{4b^2df^2}+\frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2bdf}$$
$$+\frac{(2bdf(4Abdf-aC(3de+cf))+(bde-bcf+2adf)(4aCdf+b(3Cde+cCf-4Bdf)))\text{arctanh}\Big(\frac{\sqrt{f}\sqrt{c-}}{\sqrt{d}\sqrt{e-}}}{4b^3d^{3/2}f^{5/2}}$$
$$-\frac{2(Ab^2-a(bB-aC))\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{bc-ad}\sqrt{e+fx}}\right)}{b^3\sqrt{be-af}}$$

output
```
1/4*(2*b*d*f*(4*A*b*d*f-a*C*(c*f+3*d*e))+(2*a*d*f-b*c*f+b*d*e)*(4*a*C*d*f+
b*(-4*B*d*f+C*c*f+3*C*d*e)))*arctanh(f^(1/2)*(d*x+c)^(1/2)/d^(1/2)/(f*x+e)
^(1/2))/b^3/d^(3/2)/f^(5/2)-2*(A*b^2-a*(B*b-C*a))*arctanh((-a*f+b*e)^(1/2)
*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2)/(f*x+e)^(1/2))*(-a*d+b*c)^(1/2)/b^3/(-a*f+
b*e)^(1/2)+1/2*C*(d*x+c)^(3/2)*(f*x+e)^(1/2)/b/d/f-1/4*(4*a*C*d*f+b*(-4*B*
d*f+C*c*f+3*C*d*e))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^2/d/f^2
```

3.50.2 Mathematica [A] (verified)

Time = 11.91 (sec) , antiderivative size = 465, normalized size of antiderivative = 1.60

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)\sqrt{e+fx}}\,dx$$

$$= \frac{\frac{8(Ab^2+a(-bB+aC))\sqrt{de-cf}\sqrt{\frac{d(e+fx)}{de-cf}}\operatorname{arcsinh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{de-cf}}\right)}{\sqrt{f}\sqrt{e+fx}} + \frac{4b(bCe-bBf+aCf)\sqrt{e+fx}\left(-\sqrt{f}\sqrt{de-cf}(c+dx)\sqrt{\frac{d(e+fx)}{de-cf}}+(de-cf)\sqrt{c+}\right.}{f^{5/2}\sqrt{de-cf}\sqrt{c+dx}\sqrt{\frac{d(e+fx)}{de-cf}}}}{}$$

input
```
Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)*Sqrt[e + f*x]),x]
```

output
```
((8*(A*b^2 + a*(-(b*B) + a*C))*Sqrt[d*e - c*f]*Sqrt[(d*(e + f*x))/(d*e - c
*f)]*ArcSinh[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[d*e - c*f]])/(Sqrt[f]*Sqrt[e + f
*x]) + (4*b*(b*C*e - b*B*f + a*C*f)*Sqrt[e + f*x]*(-(Sqrt[f]*Sqrt[d*e - c*
f]*(c + d*x)*Sqrt[(d*(e + f*x))/(d*e - c*f)]) + (d*e - c*f)*Sqrt[c + d*x]*
ArcSinh[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[d*e - c*f]]))/(f^(5/2)*Sqrt[d*e - c*f
]*Sqrt[c + d*x]*Sqrt[(d*(e + f*x))/(d*e - c*f)]) + (b^2*C*Sqrt[e + f*x]*(S
qrt[f]*Sqrt[c + d*x]*(c*f + d*(e + 2*f*x)) - ((d*e - c*f)^(3/2)*ArcSinh[(S
qrt[f]*Sqrt[c + d*x])/Sqrt[d*e - c*f]])/Sqrt[(d*(e + f*x))/(d*e - c*f)]))/
(d*f^(5/2)) - (8*(A*b^2 + a*(-(b*B) + a*C))*Sqrt[-(b*c) + a*d]*ArcTanh[(Sq
rt[-(b*e) + a*f]*Sqrt[c + d*x])/(Sqrt[-(b*c) + a*d]*Sqrt[e + f*x])])/Sqrt[
-(b*e) + a*f])/(4*b^3)
```

3.50.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2118, 27, 171, 27, 175, 66, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{(a+bx)\sqrt{e+fx}}\,dx$$

$$\Big\downarrow \; 2118$$

$$\frac{\int \frac{b\sqrt{c+dx}(4Abdf-aC(3de+cf)-(4aCdf+b(3Cde+cCf-4Bdf))x)}{2(a+bx)\sqrt{e+fx}}dx}{2b^2df}+\frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2bdf}$$

$\downarrow$ 27

$$\frac{\int \frac{\sqrt{c+dx}(4Abdf-aC(3de+cf)-(4aCdf+b(3Cde+cCf-4Bdf))x)}{(a+bx)\sqrt{e+fx}}dx}{4bdf}+\frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2bdf}$$

$\downarrow$ 171

$$\frac{\frac{\int \frac{2bcf(4Abdf-aC(3de+cf))+a(de+cf)(4aCdf+b(3Cde+cCf-4Bdf))+(2bdf(4Abdf-aC(3de+cf))+(bde-bcf+2adf)(4aCdf+b(3Cde+cCf-4Bdf)))x}{2(a+bx)\sqrt{c+dx}\sqrt{e+fx}}dx}{bf}-\sqrt{c-}}{4bdf}$$

$$\frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2bdf}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{2bcf(4Abdf-aC(3de+cf))+a(de+cf)(4aCdf+b(3Cde+cCf-4Bdf))+(2bdf(4Abdf-aC(3de+cf))+(bde-bcf+2adf)(4aCdf+b(3Cde+cCf-4Bdf)))x}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}dx}{2bf}-\sqrt{c-}}{4bdf}$$

$$\frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2bdf}$$

$\downarrow$ 175

$$\frac{\frac{\frac{8df^2(bc-ad)\left(Ab^2-a(bB-aC)\right)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}dx}{b}+\frac{(2bdf(4Abdf-aC(cf+3de))+(2adf-bcf+bde)(4aCdf+b(-4Bdf+cCf+3Cde)))\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}}dx}{b}}{2bf}-}{4bdf}$$

$$\frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2bdf}$$

$\downarrow$ 66

$$\frac{\frac{\frac{8df^2(bc-ad)\left(Ab^2-a(bB-aC)\right)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}dx}{b}+\frac{2(2bdf(4Abdf-aC(cf+3de))+(2adf-bcf+bde)(4aCdf+b(-4Bdf+cCf+3Cde)))\int \frac{1}{d-\frac{f(c+dx)}{e+fx}}d\frac{\sqrt{c+}}{\sqrt{e+}}}{b}}{2bf}}{4bdf}$$

$$\frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2bdf}$$

$\downarrow$ 104

$$\frac{\frac{\frac{16df^2(bc-ad)\left(Ab^2-a(bB-aC)\right)\int \frac{1}{-bc+ad+\frac{(be-af)(c+dx)}{e+fx}}d\frac{\sqrt{c+dx}}{\sqrt{e+fx}}}{b}+\frac{2(2bdf(4Abdf-aC(cf+3de))+(2adf-bcf+bde)(4aCdf+b(-4Bdf+cCf+3Cde)))\int \frac{1}{d-\frac{f(c}{e-}}}{b}}{2bf}}{4bdf}$$

$$\frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2bdf}$$

3.50. $\int \frac{\sqrt{c+dx}\,(A+Bx+Cx^2)}{(a+bx)\sqrt{e+fx}}\,dx$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)(2bdf(4Abdf-aC(cf+3de))+(2adf-bcf+bde)(4aCdf+b(-4Bdf+cCf+3Cde)))}{b\sqrt{d}\sqrt{f}}-\frac{16df^2\sqrt{bc-ad}\left(Ab^2-a(bB-aC)\right)\text{arctanh}\left(\frac{\sqrt{c+dx}\sqrt{\ldots}}{\sqrt{e+fx}\sqrt{\ldots}}\right)}{b\sqrt{be-af}}}{2bf}\ldots}{4bdf}$$
$$\frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2bdf}$$

input
```
Int[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)*Sqrt[e + f*x]),x]
```

output
```
(C*(c + d*x)^(3/2)*Sqrt[e + f*x])/(2*b*d*f) + (-(((4*a*C*d*f + b*(3*C*d*e
+ c*C*f - 4*B*d*f))*Sqrt[c + d*x]*Sqrt[e + f*x])/(b*f)) + ((2*(2*b*d*f*(4*
A*b*d*f - a*C*(3*d*e + c*f)) + (b*d*e - b*c*f + 2*a*d*f)*(4*a*C*d*f + b*(3
*C*d*e + c*C*f - 4*B*d*f)))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[
e + f*x])])/(b*Sqrt[d]*Sqrt[f]) - (16*(A*b^2 - a*(b*B - a*C))*d*Sqrt[b*c -
 a*d]*f^2*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[b*c - a*d]*Sqrt[e
+ f*x])])/(b*Sqrt[b*e - a*f]))/(2*b*f))/(4*b*d*f)
```

3.50.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

3.50.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1821 vs. $2(252) = 504$.

Time = 1.69 (sec) , antiderivative size = 1822, normalized size of antiderivative = 6.28

method	result	size
default	Expression too large to display	1822

input
```
int((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)/(f*x+e)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
1/8*(8*A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*
e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*(d*f
)^(1/2)*a*b^2*d^2*f^2-8*A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c
*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c
*e)/(b*x+a))*(d*f)^(1/2)*b^3*c*d*f^2+8*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e
))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c
*e)/b^2)^(1/2)*b^3*d^2*f^2-8*B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-
a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+
2*b*c*e)/(b*x+a))*(d*f)^(1/2)*a^2*b*d^2*f^2+8*B*ln((-2*a*d*f*x+b*c*f*x+b*d
*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/
2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*(d*f)^(1/2)*a*b^2*c*d*f^2-8*B*ln(1/2*(2
*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*((a^2*d
*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*a*b^2*d^2*f^2+4*B*ln(1/2*(2*d*f*x+2
*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*((a^2*d*f-a*b*c
*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b^3*c*d*f^2-4*B*ln(1/2*(2*d*f*x+2*((d*x+c)*
(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*((a^2*d*f-a*b*c*f-a*b*d*e
+b^2*c*e)/b^2)^(1/2)*b^3*d^2*e*f+8*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^
2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-
a*d*e+2*b*c*e)/(b*x+a))*(d*f)^(1/2)*a^3*d^2*f^2-8*C*ln((-2*a*d*f*x+b*c*f*x
+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+...
```

3.50.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)\sqrt{e+fx}}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)/(f*x+e)^(1/2),x, algorithm="
fricas")
```

output
```
Timed out
```

3.50.6 Sympy [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)\sqrt{e+fx}}\,dx = \int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)\sqrt{e+fx}}\,dx$$

input
```
integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)/(b*x+a)/(f*x+e)**(1/2),x)
```

output
```
Integral(sqrt(c + d*x)*(A + B*x + C*x**2)/((a + b*x)*sqrt(e + f*x)), x)
```

3.50.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)/(f*x+e)^(1/2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(((-(2*a*d*f)/b^2)>0)', see `assu
me?` for m
```

3.50.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)\sqrt{e+fx}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)/(f*x+e)^(1/2),x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.50.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)\sqrt{e+fx}}\,dx = \text{Hanged}$$

input
```
int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)),x)
```

output
```
\text{Hanged}
```

3.51 $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^2\sqrt{e+fx}}\,dx$

3.51.1 Optimal result

Integrand size = 36, antiderivative size = 364

$$\begin{aligned}\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^2\sqrt{e+fx}}\,dx &= \frac{(2a^2Cdf+b^2(cCe+Adf)-ab(Cde+cCf+Bdf))\sqrt{c+dx}\sqrt{e+fx}}{b^2(bc-ad)f(be-af)} \\ &- \frac{(Ab^2-a(bB-aC))(c+dx)^{3/2}\sqrt{e+fx}}{b(bc-ad)(be-af)(a+bx)} \\ &- \frac{(4aCdf+b(Cde-cCf-2Bdf))\text{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)}{b^3\sqrt{d}f^{3/2}} \\ &+ \frac{(4a^3Cdf-b^3(2Bce+Ade-Acf)+ab^2(4cCe+3Bde+Bcf)-a^2b(5Cde+3cCf+2Bdf))\,\text{arctanh}}{b^3\sqrt{bc-ad}(be-af)^{3/2}}\end{aligned}$$

output
```
-(4*a*C*d*f+b*(-2*B*d*f-C*c*f+C*d*e))*arctanh(f^(1/2)*(d*x+c)^(1/2)/d^(1/2
)/(f*x+e)^(1/2))/b^3/f^(3/2)/d^(1/2)+(4*a^3*C*d*f-b^3*(-A*c*f+A*d*e+2*B*c*
e)+a*b^2*(B*c*f+3*B*d*e+4*C*c*e)-a^2*b*(2*B*d*f+3*C*c*f+5*C*d*e))*arctanh(
(-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2)/(f*x+e)^(1/2))/b^3/(-a*f+b
*e)^(3/2)/(-a*d+b*c)^(1/2)-(A*b^2-a*(B*b-C*a))*(d*x+c)^(3/2)*(f*x+e)^(1/2)
/b/(-a*d+b*c)/(-a*f+b*e)/(b*x+a)+(2*a^2*C*d*f+b^2*(A*d*f+C*c*e)-a*b*(B*d*f
+C*c*f+C*d*e))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^2/(-a*d+b*c)/f/(-a*f+b*e)
```

3.51.2 Mathematica [A] (verified)

Time = 11.28 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^2\sqrt{e+fx}}\,dx$$

$$= \frac{-\frac{2b(Ab^2+a(-bB+aC))\sqrt{c+dx}\sqrt{e+fx}}{(be-af)(a+bx)} + \frac{4(bB-2aC)\sqrt{de-cf}\sqrt{\frac{d(e+fx)}{de-cf}}\operatorname{arcsinh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{de-cf}}\right)}{\sqrt{f}\sqrt{e+fx}} + \frac{2bC\sqrt{e+fx}\left(\sqrt{f}\sqrt{c+dx}-\frac{\sqrt{de-cf}\operatorname{arcsi}}{\sqrt{\frac{d(}{d}}}\right.}{f^{3/2}}}{2b}$$

input
```
Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^2*Sqrt[e + f*x]),x]
```

output
```
((-2*b*(A*b^2 + a*(-(b*B) + a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/((b*e - a*f
)*(a + b*x)) + (4*(b*B - 2*a*C)*Sqrt[d*e - c*f]*Sqrt[(d*(e + f*x))/(d*e -
c*f)]*ArcSinh[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[d*e - c*f]])/(Sqrt[f]*Sqrt[e +
f*x]) + (2*b*C*Sqrt[e + f*x]*(Sqrt[f]*Sqrt[c + d*x] - (Sqrt[d*e - c*f]*Arc
Sinh[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[d*e - c*f]])/Sqrt[(d*(e + f*x))/(d*e - c
*f)]))/f^(3/2) - (4*(b*B - 2*a*C)*Sqrt[-(b*c) + a*d]*ArcTanh[(Sqrt[-(b*e)
+ a*f]*Sqrt[c + d*x])/(Sqrt[-(b*c) + a*d]*Sqrt[e + f*x])])/Sqrt[-(b*e) + a
*f] + (2*b*(A*b^2 + a*(-(b*B) + a*C))*(-(d*e) + c*f)*ArcTanh[(Sqrt[-(b*e)
+ a*f]*Sqrt[c + d*x])/(Sqrt[-(b*c) + a*d]*Sqrt[e + f*x])])/(Sqrt[-(b*c) +
a*d]*(-(b*e) + a*f)^(3/2)))/(2*b^3)
```

3.51.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2116, 27, 171, 27, 175, 66, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}\,(A+Bx+Cx^2)}{(a+bx)^2\sqrt{e+fx}}\,dx$$

$$\Big\downarrow \ 2116$$

$$-\frac{\int -\frac{\sqrt{c+dx}\left(C(3de+cf)a^2-b(2cCe+3Bde+Bcf-2Adf)a+b^2(2Bce+Ade-Acf)+2b\left(\frac{2Cdfa^2}{b}-(Cde+cCf+Bdf)a+b(cCe+Adf)\right)x\right)}{2b(a+bx)\sqrt{e+fx}}dx}{(bc-ad)(be-af)}-\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b(a+bx)(bc-ad)(be-af)}$$

$\big\downarrow$ 27

$$\frac{\int \frac{\sqrt{c+dx}\left(C(3de+cf)a^2-b(2cCe+3Bde+Bcf-2Adf)a+b^2(2Bce+Ade-Acf)+2b\left(\frac{2Cdfa^2}{b}-(Cde+cCf+Bdf)a+b(cCe+Adf)\right)x\right)}{(a+bx)\sqrt{e+fx}}dx}{2b(bc-ad)(be-af)}-\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b(a+bx)(bc-ad)(be-af)}$$

$\big\downarrow$ 171

$$\frac{\frac{\int \frac{(bc-ad)\left(2Cf(de+cf)a^2-b(Bf(de+cf)+Ce(de+3cf))a+b^2f(2Bce+Ade-Acf)-(be-af)(4aCdf+b(Cde-cCf-2Bdf))x\right)}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}dx}{bf}+\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(\frac{2a^2Cdf}{b}-\cdots\right.}{\cdots}}{2b(bc-ad)(be-af)}-\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b(a+bx)(bc-ad)(be-af)}$$

$\big\downarrow$ 27

$$\frac{\frac{(bc-ad)\int \frac{2Cf(de+cf)a^2-b(Bf(de+cf)+Ce(de+3cf))a+b^2f(2Bce+Ade-Acf)-(be-af)(4aCdf+b(Cde-cCf-2Bdf))x}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}dx}{bf}+\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(\frac{2a^2Cdf}{b}-a\cdots\right.}{\cdots}}{2b(bc-ad)(be-af)}-\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b(a+bx)(bc-ad)(be-af)}$$

$\big\downarrow$ 175

$$\frac{\frac{(bc-ad)\left(-\frac{f\left(4a^3Cdf-a^2b(2Bdf+3cCf+5Cde)+ab^2(Bcf+3Bde+4cCe)-b^3(-Acf+Ade+2Bce)\right)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}dx}{b}-\frac{(be-af)(4aCdf+b(-2Bdf-cCf-\cdots}{b}\cdots\right.}{bf}\cdots}{2b(bc-ad)(be-af)}-\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b(a+bx)(bc-ad)(be-af)}$$

$\big\downarrow$ 66

3.51. $\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{(a+bx)^2\sqrt{e+fx}}\,dx$

$$(bc-ad)\left(-\frac{f\left(4a^3Cdf-a^2b(2Bdf+3cCf+5Cde)+ab^2(Bcf+3Bde+4cCe)-b^3(-Acf+Ade+2Bce)\right)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}dx}{b} - \frac{2(be-af)(4aCdf+b(-2Bdf-cC}{}\right.$$

$$bf$$

$$2b(bc-ad)(be-af)$$

$$\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b(a+bx)(bc-ad)(be-af)}$$

$$\Big\downarrow \; 104$$

$$(bc-ad)\left(-\frac{2f\left(4a^3Cdf-a^2b(2Bdf+3cCf+5Cde)+ab^2(Bcf+3Bde+4cCe)-b^3(-Acf+Ade+2Bce)\right)\int \frac{1}{-bc+ad+\frac{(be-af)(c+dx)}{e+fx}}d\frac{\sqrt{c+dx}}{\sqrt{e+fx}}}{b} - \frac{2(be-af)(4aCdf+b(}{}\right.$$

$$bf$$

$$2b(bc-ad)(be-af)$$

$$\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b(a+bx)(bc-ad)(be-af)}$$

$$\Big\downarrow \; 221$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(\frac{2a^2Cdf}{b}-a(Bdf+cCf+Cde)+b(Adf+cCe)\right)}{f} + \frac{(bc-ad)\left(\frac{2f\text{arctanh}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{bc-ad}}\right)\left(4a^3Cdf-a^2b(2Bdf+3cCf+5Cde)+ab^2(B}{b\sqrt{bc-ad}\sqrt{be-af}}\right.}{}$$

$$2b(bc-ad)(be-af)$$

$$\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b(a+bx)(bc-ad)(be-af)}$$

input

```
Int[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^2*Sqrt[e + f*x]),x]
```

output

```
-(((A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*Sqrt[e + f*x])/(b*(b*c - a*d)*(
b*e - a*f)*(a + b*x))) + ((2*((2*a^2*C*d*f)/b + b*(c*C*e + A*d*f) - a*(C*d
*e + c*C*f + B*d*f))*Sqrt[c + d*x]*Sqrt[e + f*x])/f + ((b*c - a*d)*((-2*(b
*e - a*f)*(4*a*C*d*f + b*(C*d*e - c*C*f - 2*B*d*f))*ArcTanh[(Sqrt[f]*Sqrt[
c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(b*Sqrt[d]*Sqrt[f]) + (2*f*(4*a^3*C*d*
f - b^3*(2*B*c*e + A*d*e - A*c*f) + a*b^2*(4*c*C*e + 3*B*d*e + B*c*f) - a^
2*b*(5*C*d*e + 3*c*C*f + 2*B*d*f))*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c + d*x])
/(Sqrt[b*c - a*d]*Sqrt[e + f*x])])/(b*Sqrt[b*c - a*d]*Sqrt[b*e - a*f])))/(
b*f))/(2*b*(b*c - a*d)*(b*e - a*f))
```

3.51. $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^2\sqrt{e+fx}}\,dx$

3.51.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.51. $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^2\sqrt{e+fx}}\,dx$

rule 2116
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Si
mp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*
(e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1
) - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x]
, x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && ILtQ[m,
-1]
```

3.51.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3669 vs. $2(332) = 664$.

Time = 1.70 (sec) , antiderivative size = 3670, normalized size of antiderivative = 10.08

method	result	size
default	Expression too large to display	3670

input
```
int((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^2/(f*x+e)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
-1/2*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(2*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^
(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^4*d*e*f*x*((a^2*d*f-a*b*c*f-a*b*
d*e+b^2*c*e)/b^2)^(1/2)-C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^
(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b^3*c*f^2*x*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*
e)/b^2)^(1/2)-A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e
+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a
))*b^4*d*e*f*x*(d*f)^(1/2)-2*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(
d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b^3*d*f^2*x*((a^2*d*f-a*b*c*f-a*b*d*e+b
^2*c*e)/b^2)^(1/2)+4*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-
a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)
/(b*x+a))*a^4*d*f^2*(d*f)^(1/2)-2*A*b^4*f*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*
e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)-3*C*ln((-2*a*d*f*x+b*c*f
*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e
))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^3*b*c*f^2*(d*f)^(1/2)-2*B*ln((-
2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*
((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^2*b^2*d*f^2*x*(d
*f)^(1/2)+B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2
*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a
*b^3*c*f^2*x*(d*f)^(1/2)+B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*
c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2...
```

3.51.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^2\sqrt{e+fx}}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^2/(f*x+e)^(1/2),x, algorithm
="fricas")
```

output
```
Timed out
```

3.51.6 Sympy [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^2\sqrt{e+fx}}\,dx = \int \frac{\sqrt{c+dx}\,(A+Bx+Cx^2)}{(a+bx)^2\,\sqrt{e+fx}}\,dx$$

input
```
integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)/(b*x+a)**2/(f*x+e)**(1/2),x)
```

output
```
Integral(sqrt(c + d*x)*(A + B*x + C*x**2)/((a + b*x)**2*sqrt(e + f*x)), x)
```

3.51.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^2\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^2/(f*x+e)^(1/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(((-(2*a*d*f)/b^2)>0)', see `assu
me?` for m
```

3.51.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1354 vs. $2(331) = 662$.

Time = 1.40 (sec) , antiderivative size = 1354, normalized size of antiderivative = 3.72

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^2\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^2/(f*x+e)^(1/2),x, algorithm
="giac")
```

output
```
(4*sqrt(d*f)*C*a*b^2*c*d^2*e - 2*sqrt(d*f)*B*b^3*c*d^2*e - 5*sqrt(d*f)*C*a
^2*b*d^3*e + 3*sqrt(d*f)*B*a*b^2*d^3*e - sqrt(d*f)*A*b^3*d^3*e - 3*sqrt(d*
f)*C*a^2*b*c*d^2*f + sqrt(d*f)*B*a*b^2*c*d^2*f + sqrt(d*f)*A*b^3*c*d^2*f +
 4*sqrt(d*f)*C*a^3*d^3*f - 2*sqrt(d*f)*B*a^2*b*d^3*f)*arctan(-1/2*(b*d^2*e
 + b*c*d*f - 2*a*d^2*f - (sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)
*d*f - c*d*f))^2*b)/(sqrt(-b^2*c*d*e*f + a*b*d^2*e*f + a*b*c*d*f^2 - a^2*d
^2*f^2)*d))/(sqrt(-b^2*c*d*e*f + a*b*d^2*e*f + a*b*c*d*f^2 - a^2*d^2*f^2)*
(b^4*e*abs(d) - a*b^3*f*abs(d))*d) - 2*(sqrt(d*f)*C*a^2*b*d^5*e^2 - sqrt(d
*f)*B*a*b^2*d^5*e^2 + sqrt(d*f)*A*b^3*d^5*e^2 - 2*sqrt(d*f)*C*a^2*b*c*d^4*
e*f + 2*sqrt(d*f)*B*a*b^2*c*d^4*e*f - 2*sqrt(d*f)*A*b^3*c*d^4*e*f + sqrt(d
*f)*C*a^2*b*c^2*d^3*f^2 - sqrt(d*f)*B*a*b^2*c^2*d^3*f^2 + sqrt(d*f)*A*b^3*
c^2*d^3*f^2 - sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*
d*f - c*d*f))^2*C*a^2*b*d^3*e + sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(
d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a*b^2*d^3*e - sqrt(d*f)*(sqrt(d*f)*sqr
t(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*b^3*d^3*e - sqrt(d*f
)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^2*
b*c*d^2*f + sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*
f - c*d*f))^2*B*a*b^2*c*d^2*f - sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(
d^2*e + (d*x + c)*d*f - c*d*f))^2*A*b^3*c*d^2*f + 2*sqrt(d*f)*(sqrt(d*f)*s
qrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^3*d^3*f - 2*s...
```

3.51.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^2\sqrt{e+fx}}\,dx = \text{Hanged}$$

input
```
int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)^2),x)
```

output
```
\text{Hanged}
```

3.52 $\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{(a+bx)^3\sqrt{e+fx}}\,dx$

3.52.1 Optimal result

Integrand size = 36, antiderivative size = 484

$$\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{(a+bx)^3\sqrt{e+fx}}\,dx$$

$$= \frac{(4a^3Cdf - a^2bC(7de+5cf) - b^3(4Bce - Ade - 3Acf) + ab^2(8cCe + 3Bde + Bcf - 4Adf))\sqrt{c+dx}\sqrt{e}}{4b^2(bc-ad)(be-af)^2(a+bx)}$$

$$- \frac{(Ab^2 - a(bB-aC))\,(c+dx)^{3/2}\sqrt{e+fx}}{2b(bc-ad)(be-af)(a+bx)^2} + \frac{2C\sqrt{d}\text{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)}{b^3\sqrt{f}}$$

$$- \frac{(8a^4Cd^2f^2 - 4a^3bCdf(5de+3cf) + 3a^2b^2C(5d^2e^2 + 10cdef + c^2f^2) - ab^3(d^2e(3Be-4Af) + c^2f(8C}{4b^3}$$

output
```
-1/4*(8*a^4*C*d^2*f^2-4*a^3*b*C*d*f*(3*c*f+5*d*e)+3*a^2*b^2*C*(c^2*f^2+10*
c*d*e*f+5*d^2*e^2)-a*b^3*(d^2*e*(-4*A*f+3*B*e)+c^2*f*(-B*f+8*C*e)+2*c*d*(2
*A*f^2-B*e*f+12*C*e^2))-b^4*(A*d^2*e^2-2*c*d*e*(-A*f+2*B*e)-c^2*(3*A*f^2-4
*B*e*f+8*C*e^2)))*arctanh((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2)/
(f*x+e)^(1/2))/b^3/(-a*d+b*c)^(3/2)/(-a*f+b*e)^(5/2)+2*C*arctanh(f^(1/2)*(
d*x+c)^(1/2)/d^(1/2)/(f*x+e)^(1/2))*d^(1/2)/b^3/f^(1/2)-1/2*(A*b^2-a*(B*b-
C*a))*(d*x+c)^(3/2)*(f*x+e)^(1/2)/b/(-a*d+b*c)/(-a*f+b*e)/(b*x+a)^2+1/4*(4
*a^3*C*d*f-a^2*b*C*(5*c*f+7*d*e)-b^3*(-3*A*c*f-A*d*e+4*B*c*e)+a*b^2*(-4*A*
d*f+B*c*f+3*B*d*e+8*C*c*e))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^2/(-a*d+b*c)/(-a
*f+b*e)^2/(b*x+a)
```

3.52.2 Mathematica [A] (verified)

Time = 13.01 (sec) , antiderivative size = 523, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^3\sqrt{e+fx}}\,dx =$$

$$-\frac{\frac{4b(bB-2aC)\sqrt{c+dx}\sqrt{e+fx}}{(be-af)(a+bx)}+\frac{2b^2(Ab^2+a(-bB+aC))(c+dx)^{3/2}\sqrt{e+fx}}{(bc-ad)(be-af)(a+bx)^2}-\frac{8C\sqrt{de-cf}\sqrt{\frac{d(e+fx)}{de-cf}}\operatorname{arcsinh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{de-cf}}\right)}{\sqrt{f}\sqrt{e+fx}}+\frac{8C\sqrt{-bc+a}}{}}{}$$

input
```
Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^3*Sqrt[e + f*x]),x]
```

output
```
-1/4*((4*b*(b*B - 2*a*C)*Sqrt[c + d*x]*Sqrt[e + f*x])/((b*e - a*f)*(a + b*
x)) + (2*b^2*(A*b^2 + a*(-(b*B) + a*C))*(c + d*x)^(3/2)*Sqrt[e + f*x])/((b
*c - a*d)*(b*e - a*f)*(a + b*x)^2) - (8*C*Sqrt[d*e - c*f]*Sqrt[(d*(e + f*x
))/(d*e - c*f)]*ArcSinh[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[d*e - c*f]])/(Sqrt[f]
*Sqrt[e + f*x]) + (8*C*Sqrt[-(b*c) + a*d]*ArcTanh[(Sqrt[-(b*e) + a*f]*Sqrt
[c + d*x])/(Sqrt[-(b*c) + a*d]*Sqrt[e + f*x])])/Sqrt[-(b*e) + a*f] - (4*b*
(b*B - 2*a*C)*(-(d*e) + c*f)*ArcTanh[(Sqrt[-(b*e) + a*f]*Sqrt[c + d*x])/(S
qrt[-(b*c) + a*d]*Sqrt[e + f*x])])/(Sqrt[-(b*c) + a*d]*(-(b*e) + a*f)^(3/2
)) + (b*(A*b^2 + a*(-(b*B) + a*C))*(b*d*e + 3*b*c*f - 4*a*d*f)*(Sqrt[-(b*c
) + a*d]*Sqrt[-(b*e) + a*f]*Sqrt[c + d*x]*Sqrt[e + f*x] - (d*e - c*f)*(a +
 b*x)*ArcTanh[(Sqrt[-(b*e) + a*f]*Sqrt[c + d*x])/(Sqrt[-(b*c) + a*d]*Sqrt[
e + f*x])]))/((-(b*c) + a*d)^(3/2)*(-(b*e) + a*f)^(5/2)*(a + b*x)))/b^3
```

3.52.3 Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 533, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2116, 27, 166, 27, 175, 66, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^3\sqrt{e+fx}}\,dx$$

$$\Big\downarrow \text{2116}$$

$$-\frac{\int -\frac{\sqrt{c+dx}(C(3de+cf)a^2-b(4cCe+3Bde+Bcf-4Adf)a+b^2(4Bce-Ade-3Acf)+4C(bc-ad)(be-af)x)}{2b(a+bx)^2\sqrt{e+fx}}dx}{2(bc-ad)(be-af)}-\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{2b(a+bx)^2(bc-ad)(be-af)}$$

$\downarrow$ 27

$$\frac{\int \frac{\sqrt{c+dx}(C(3de+cf)a^2-b(4cCe+3Bde+Bcf-4Adf)a+b^2(4Bce-A(de+3cf))+4C(bc-ad)(be-af)x)}{(a+bx)^2\sqrt{e+fx}}dx}{4b(bc-ad)(be-af)}-\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{2b(a+bx)^2(bc-ad)(be-af)}$$

$\downarrow$ 166

$$\frac{\frac{\int -\frac{4Cdf(de+cf)a^3-bC\left(7d^2e^2+14cdfe+3c^2f^2\right)a^2+b^2\left(f(8Ce-Bf)c^2+2d\left(8Ce^2-Bfe+2Af^2\right)c+d^2e(3Be-4Af)\right)a+b^3\left(-\left(\left(8Ce^2-4Bfe+3Af^2\right)c^2\right)-2de(2Be-\cdots\right.}{2(a+bx)\sqrt{c+dx}\sqrt{e+fx}}}{b(be-af)}\cdots}{4b(bc-ad\cdots}$$

$$\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{2b(a+bx)^2(bc-ad)(be-af)}$$

$\downarrow$ 27

$$\frac{\frac{\sqrt{c+dx}\sqrt{e+fx}(4a^3Cdf-a^2bC(5cf+7de)+ab^2(-4Adf+Bcf+3Bde+8cCe)-b^3(-3Acf-Ade+4Bce))}{b(a+bx)(be-af)}-\frac{\int \frac{4Cdf(de+cf)a^3-bC\left(7d^2e^2+14cdfe+3c^2\cdots\right.}{\cdots}}{\cdots}}{4b(bc-ad)\cdots}$$

$$\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{2b(a+bx)^2(bc-ad)(be-af)}$$

$\downarrow$ 175

$$\frac{\frac{\sqrt{c+dx}\sqrt{e+fx}(4a^3Cdf-a^2bC(5cf+7de)+ab^2(-4Adf+Bcf+3Bde+8cCe)-b^3(-3Acf-Ade+4Bce))}{b(a+bx)(be-af)}-\frac{-\left(8a^4Cd^2f^2-4a^3bCdf(3cf+5de)+3a^2b^2\cdots\right.}{\cdots}}{\cdots}$$

$$\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{2b(a+bx)^2(bc-ad)(be-af)}$$

$\downarrow$ 66

$$\frac{\frac{\sqrt{c+dx}\sqrt{e+fx}(4a^3Cdf-a^2bC(5cf+7de)+ab^2(-4Adf+Bcf+3Bde+8cCe)-b^3(-3Acf-Ade+4Bce))}{b(a+bx)(be-af)}-\frac{-\left(8a^4Cd^2f^2-4a^3bCdf(3cf+5de)+3a^2b^2\cdots\right.}{\cdots}}{\cdots}$$

$$\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{2b(a+bx)^2(bc-ad)(be-af)}$$

3.52. $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^3\sqrt{e+fx}}\,dx$

$\Big\downarrow$ 104

$$\frac{\frac{\sqrt{c+dx}\sqrt{e+fx}(4a^3Cdf-a^2bC(5cf+7de)+ab^2(-4Adf+Bcf+3Bde+8cCe)-b^3(-3Acf-Ade+4Bce))}{b(a+bx)(be-af)}-\frac{-\frac{2\left(8a^4Cd^2f^2-4a^3bCdf(3cf+5de)+3a^2b^2\cdots\right.}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{2b(a+bx)^2(bc-ad)(be-af)}$$

$\Big\downarrow$ 221

$$\frac{\frac{\sqrt{c+dx}\sqrt{e+fx}(4a^3Cdf-a^2bC(5cf+7de)+ab^2(-4Adf+Bcf+3Bde+8cCe)-b^3(-3Acf-Ade+4Bce))}{b(a+bx)(be-af)}-\frac{2\text{arctanh}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{bc-ad}}\right)\left(8a^4Cd^2f^2\cdots\right.}{\cdots}}{\cdots}$$

$$\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{2b(a+bx)^2(bc-ad)(be-af)}$$

input
```
Int[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^3*Sqrt[e + f*x]),x]
```

output
```
-1/2*((A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*Sqrt[e + f*x])/(b*(b*c - a*d
)*(b*e - a*f)*(a + b*x)^2) + (((4*a^3*C*d*f - a^2*b*C*(7*d*e + 5*c*f) - b^
3*(4*B*c*e - A*d*e - 3*A*c*f) + a*b^2*(8*c*C*e + 3*B*d*e + B*c*f - 4*A*d*f
))*Sqrt[c + d*x]*Sqrt[e + f*x])/(b*(b*e - a*f)*(a + b*x)) - ((-16*C*Sqrt[d
]*(b*c - a*d)*(b*e - a*f)^2*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[
e + f*x])])/(b*Sqrt[f]) + (2*(8*a^4*C*d^2*f^2 - 4*a^3*b*C*d*f*(5*d*e + 3*c
*f) + 3*a^2*b^2*C*(5*d^2*e^2 + 10*c*d*e*f + c^2*f^2) - a*b^3*(d^2*e*(3*B*e
 - 4*A*f) + c^2*f*(8*C*e - B*f) + 2*c*d*(12*C*e^2 - B*e*f + 2*A*f^2)) - b^
4*(A*d^2*e^2 - 2*c*d*e*(2*B*e - A*f) - c^2*(8*C*e^2 - 4*B*e*f + 3*A*f^2)))
*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x])])
/(b*Sqrt[b*c - a*d]*Sqrt[b*e - a*f]))/(2*b*(b*e - a*f)))/(4*b*(b*c - a*d)*
(b*e - a*f))
```

3.52. $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^3\sqrt{e+fx}}\,dx$

3.52.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2116
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Si
mp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*
(e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1
) - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x]
, x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && ILtQ[m,
-1]
```

3.52.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 9099 vs. $2(446) = 892$.

Time = 1.70 (sec) , antiderivative size = 9100, normalized size of antiderivative = 18.80

method	result	size
default	Expression too large to display	9100

input
```
int((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^3/(f*x+e)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
result too large to display
```

3.52.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^3\sqrt{e+fx}}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^3/(f*x+e)^(1/2),x, algorithm
="fricas")
```

output
```
Timed out
```

3.52.6 Sympy [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^3\sqrt{e+fx}}\,dx = \int \frac{\sqrt{c+dx}\,(A+Bx+Cx^2)}{(a+bx)^3\,\sqrt{e+fx}}\,dx$$

input
```
integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)/(b*x+a)**3/(f*x+e)**(1/2),x)
```

output
```
Integral(sqrt(c + d*x)*(A + B*x + C*x**2)/((a + b*x)**3*sqrt(e + f*x)), x)
```

3.52.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^3\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^3/(f*x+e)^(1/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(((-(2*a*d*f)/b^2)>0)', see `assu
me?` for m
```

3.52.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7922 vs. $2(445) = 890$.

Time = 23.15 (sec) , antiderivative size = 7922, normalized size of antiderivative = 16.37

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^3\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^3/(f*x+e)^(1/2),x, algorithm
="giac")
```

output
```
-1/4*(8*sqrt(d*f)*C*b^4*c^2*d^2*e^2 - 24*sqrt(d*f)*C*a*b^3*c*d^3*e^2 + 4*s
qrt(d*f)*B*b^4*c*d^3*e^2 + 15*sqrt(d*f)*C*a^2*b^2*d^4*e^2 - 3*sqrt(d*f)*B*
a*b^3*d^4*e^2 - sqrt(d*f)*A*b^4*d^4*e^2 - 8*sqrt(d*f)*C*a*b^3*c^2*d^2*e*f
- 4*sqrt(d*f)*B*b^4*c^2*d^2*e*f + 30*sqrt(d*f)*C*a^2*b^2*c*d^3*e*f + 2*sqr
t(d*f)*B*a*b^3*c*d^3*e*f - 2*sqrt(d*f)*A*b^4*c*d^3*e*f - 20*sqrt(d*f)*C*a^
3*b*d^4*e*f + 4*sqrt(d*f)*A*a*b^3*d^4*e*f + 3*sqrt(d*f)*C*a^2*b^2*c^2*d^2*
f^2 + sqrt(d*f)*B*a*b^3*c^2*d^2*f^2 + 3*sqrt(d*f)*A*b^4*c^2*d^2*f^2 - 12*s
qrt(d*f)*C*a^3*b*c*d^3*f^2 - 4*sqrt(d*f)*A*a*b^3*c*d^3*f^2 + 8*sqrt(d*f)*C
*a^4*d^4*f^2)*arctan(-1/2*(b*d^2*e + b*c*d*f - 2*a*d^2*f - (sqrt(d*f)*sqrt
(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*b)/(sqrt(-b^2*c*d*e*f +
 a*b*d^2*e*f + a*b*c*d*f^2 - a^2*d^2*f^2)*d))/((b^6*c*e^2*abs(d) - a*b^5*d
*e^2*abs(d) - 2*a*b^5*c*e*f*abs(d) + 2*a^2*b^4*d*e*f*abs(d) + a^2*b^4*c*f^
2*abs(d) - a^3*b^3*d*f^2*abs(d))*sqrt(-b^2*c*d*e*f + a*b*d^2*e*f + a*b*c*d
*f^2 - a^2*d^2*f^2)*d) - sqrt(d*f)*C*d*log((sqrt(d*f)*sqrt(d*x + c) - sqrt
(d^2*e + (d*x + c)*d*f - c*d*f))^2)/(b^3*f*abs(d)) + 1/2*(8*sqrt(d*f)*C*a*
b^4*c*d^9*e^5 - 4*sqrt(d*f)*B*b^5*c*d^9*e^5 - 9*sqrt(d*f)*C*a^2*b^3*d^10*e
^5 + 5*sqrt(d*f)*B*a*b^4*d^10*e^5 - sqrt(d*f)*A*b^5*d^10*e^5 - 32*sqrt(d*f
)*C*a*b^4*c^2*d^8*e^4*f + 16*sqrt(d*f)*B*b^5*c^2*d^8*e^4*f + 31*sqrt(d*f)*
C*a^2*b^3*c*d^9*e^4*f - 19*sqrt(d*f)*B*a*b^4*c*d^9*e^4*f + 7*sqrt(d*f)*A*b
^5*c*d^9*e^4*f + 6*sqrt(d*f)*C*a^3*b^2*d^10*e^4*f - 2*sqrt(d*f)*B*a^2*b...
```

3.52.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^3\sqrt{e+fx}}\,dx = \text{Hanged}$$

input
```
int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)^3),x)
```

output
```
\text{Hanged}
```

3.53 $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^4\sqrt{e+fx}}\,dx$

3.53.1 Optimal result

Integrand size = 36, antiderivative size = 685

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^4\sqrt{e+fx}}\,dx$$

$$= \frac{(4a^3Cdf - b^3(6Bce - 3Ade - 5Acf) + ab^2(12cCe + 3Bde + Bcf - 8Adf) - a^2b(9Cde + 7cCf - 2Bdf)}{12b^2(bc-ad)(be-af)^2(a+bx)^2}$$

$$- \frac{(8a^4Cd^2f^2 - 2a^3bdf(13Cde + 7cCf - 2Bdf) - b^4(3Ad^2e^2 - 2cde(3Be - 2Af) - 3c^2(8Ce^2 - 6Bef +}{}$$

$$- \frac{(Ab^2 - a(bB - aC))(c+dx)^{3/2}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^3}$$

$$- \frac{(de-cf)(b^2(Ad^2e^2 - 2cde(Be-Af) + c^2(8Ce^2 - 6Bef + 5Af^2)) + ab(d^2e(Be-4Af) - c^2f(4Ce -}{8(b}$$

output
```
-1/8*(-c*f+d*e)*(b^2*(A*d^2*e^2-2*c*d*e*(-A*f+B*e)+c^2*(5*A*f^2-6*B*e*f+8*
C*e^2))+a*b*(d^2*e*(-4*A*f+B*e)-c^2*f*(-B*f+4*C*e)-2*c*d*(6*A*f^2-7*B*e*f+
6*C*e^2))-a^2*(2*d*f*(-4*A*d*f+B*c*f+3*B*d*e)-C*(c^2*f^2+2*c*d*e*f+5*d^2*e
^2)))*arctanh((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2)/(f*x+e)^(1/2
))/(-a*d+b*c)^(5/2)/(-a*f+b*e)^(7/2)-1/3*(A*b^2-a*(B*b-C*a))*(d*x+c)^(3/2)
*(f*x+e)^(1/2)/b/(-a*d+b*c)/(-a*f+b*e)/(b*x+a)^3+1/12*(4*a^3*C*d*f-b^3*(-5
*A*c*f-3*A*d*e+6*B*c*e)+a*b^2*(-8*A*d*f+B*c*f+3*B*d*e+12*C*c*e)-a^2*b*(-2*
B*d*f+7*C*c*f+9*C*d*e))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^2/(-a*d+b*c)/(-a*f+b
*e)^2/(b*x+a)^2-1/24*(8*a^4*C*d^2*f^2-2*a^3*b*d*f*(-2*B*d*f+7*C*c*f+13*C*d
*e)-b^4*(3*A*d^2*e^2-2*c*d*e*(-2*A*f+3*B*e)-3*c^2*(5*A*f^2-6*B*e*f+8*C*e^2
))-a*b^3*(d^2*e*(-10*A*f+3*B*e)+3*c^2*f*(-B*f+4*C*e)+2*c*d*(13*A*f^2-14*B*
e*f+30*C*e^2))-a^2*b^2*(4*d*f*(-2*A*d*f+B*c*f+4*B*d*e)-C*(3*c^2*f^2+44*c*d
*e*f+33*d^2*e^2)))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^2/(-a*d+b*c)^2/(-a*f+b*e)
^3/(b*x+a)
```

3.53.2 Mathematica [A] (verified)

Time = 14.91 (sec) , antiderivative size = 657, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^4\sqrt{e+fx}}\,dx =$$

$$-\frac{\frac{12C\sqrt{c+dx}\sqrt{e+fx}}{(be-af)(a+bx)}+\frac{4b\left(Ab^2+a(-bB+aC)\right)(c+dx)^{3/2}\sqrt{e+fx}}{(bc-ad)(be-af)(a+bx)^3}+\frac{6b(bB-2aC)(c+dx)^{3/2}\sqrt{e+fx}}{(bc-ad)(be-af)(a+bx)^2}-\frac{12C(-de+cf)\text{arctanh}\left(\frac{\sqrt{-be+af}}{\sqrt{-bc+ad}}\right.}{\sqrt{-bc+ad}(-be+af)^{3/2}}}{}$$

input
```
Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^4*Sqrt[e + f*x]),x]
```

output
```
-1/12*((12*C*Sqrt[c + d*x]*Sqrt[e + f*x])/((b*e - a*f)*(a + b*x)) + (4*b*(
A*b^2 + a*(-(b*B) + a*C))*(c + d*x)^(3/2)*Sqrt[e + f*x])/((b*c - a*d)*(b*e
 - a*f)*(a + b*x)^3) + (6*b*(b*B - 2*a*C)*(c + d*x)^(3/2)*Sqrt[e + f*x])/(
(b*c - a*d)*(b*e - a*f)*(a + b*x)^2) - (12*C*(-(d*e) + c*f)*ArcTanh[(Sqrt[
-(b*e) + a*f]*Sqrt[c + d*x])/(Sqrt[-(b*c) + a*d]*Sqrt[e + f*x])])/(Sqrt[-(
b*c) + a*d]*(-(b*e) + a*f)^(3/2)) + (3*(b*B - 2*a*C)*(b*d*e + 3*b*c*f - 4*
a*d*f)*(Sqrt[-(b*c) + a*d]*Sqrt[-(b*e) + a*f]*Sqrt[c + d*x]*Sqrt[e + f*x]
- (d*e - c*f)*(a + b*x)*ArcTanh[(Sqrt[-(b*e) + a*f]*Sqrt[c + d*x])/(Sqrt[-
(b*c) + a*d]*Sqrt[e + f*x])]))/((-(b*c) + a*d)^(3/2)*(-(b*e) + a*f)^(5/2)*
(a + b*x)) - ((A*b^2 + a*(-(b*B) + a*C))*((2*b*(3*b*d*e + 5*b*c*f - 8*a*d*
f)*(c + d*x)^(3/2)*Sqrt[e + f*x])/(a + b*x)^2 + 3*(8*a^2*d^2*f^2 - 4*a*b*d
*f*(d*e + 3*c*f) + b^2*(d^2*e^2 + 2*c*d*e*f + 5*c^2*f^2))*((Sqrt[c + d*x]*
Sqrt[e + f*x])/((-(b*e) + a*f)*(a + b*x)) + ((-(d*e) + c*f)*ArcTanh[(Sqrt[
-(b*e) + a*f]*Sqrt[c + d*x])/(Sqrt[-(b*c) + a*d]*Sqrt[e + f*x])])/(Sqrt[-(
b*c) + a*d]*(-(b*e) + a*f)^(3/2)))))/(2*(b*c - a*d)^2*(b*e - a*f)^2))/b^2
```

3.53.3 Rubi [A] (verified)

Time = 1.59 (sec) , antiderivative size = 719, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2116, 27, 166, 27, 168, 27, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{(a+bx)^4\sqrt{e+fx}}\,dx$$

$$\Big\downarrow \ 2116$$

$$-\frac{\int -\frac{\sqrt{c+dx}\left(C(3de+cf)a^2-b(6cCe+3Bde+Bcf-6Adf)a+b^2(6Bce-3Ade-5Acf)+2b\left(\frac{2Cdfa^2}{b}-3Cdea-3cCfa+Bdfa+3bcCe-Abdf\right)x\right)}{2b(a+bx)^3\sqrt{e+fx}}dx}{3(bc-ad)(be-af)}$$
$$\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^3(bc-ad)(be-af)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\sqrt{c+dx}\left(C(3de+cf)a^2-b(6cCe+3Bde+Bcf-6Adf)a+b^2(6Bce-3Ade-5Acf)+2b\left(\frac{2Cdfa^2}{b}+Bdfa-3C(de+cf)a+b(3cCe-Adf)\right)x\right)}{(a+bx)^3\sqrt{e+fx}}dx}{6b(bc-ad)(be-af)}-\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^3(bc-ad)(be-af)}$$

$\downarrow$ 166

$$\frac{\int -\frac{4Cdf(de+cf)a^3+b\left(2Bdf(de+cf)-C\left(9d^2e^2+20cdfe+3c^2f^2\right)\right)a^2+b^2\left(3f(4Ce-Bf)c^2+4d\left(9Ce^2-4Bfe+4Af^2\right)c+d^2e(3Be-8Af)\right)a+b^3\left(-3\left(8Ce^2-6Bfe+5\cdots\right.\right.}{2(a+bx)^2\cdots}}{2b(b\cdots}$$

$$\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^3(bc-ad)(be-af)}$$

$\downarrow$ 27

$$\frac{\frac{\sqrt{c+dx}\sqrt{e+fx}(4a^3Cdf-a^2b(-2Bdf+7cCf+9Cde)+ab^2(-8Adf+Bcf+3Bde+12cCe)-b^3(-5Acf-3Ade+6Bce))}{2b(a+bx)^2(be-af)}-\frac{\int \frac{4Cdf(de+cf)a^3+b\left(2Bdf(\cdots\right.}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^3(bc-ad)(be-af)}$$

$\downarrow$ 168

$$\frac{\frac{\sqrt{c+dx}\sqrt{e+fx}(4a^3Cdf-a^2b(-2Bdf+7cCf+9Cde)+ab^2(-8Adf+Bcf+3Bde+12cCe)-b^3(-5Acf-3Ade+6Bce))}{2b(a+bx)^2(be-af)}-\frac{\sqrt{c+dx}\sqrt{e+fx}\left(8a^4Cd^2f^2-\cdots\right.}{\cdots}}{\cdots}$$

$$\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^3(bc-ad)(be-af)}$$

$\downarrow$ 27

$$\frac{\frac{\sqrt{c+dx}\sqrt{e+fx}(4a^3Cdf-a^2b(-2Bdf+7cCf+9Cde)+ab^2(-8Adf+Bcf+3Bde+12cCe)-b^3(-5Acf-3Ade+6Bce))}{2b(a+bx)^2(be-af)}-\frac{3b^2(de-cf)\left(a^2\left(2df(-4Adf-\cdots\right.\right.}{\cdots}}{\cdots}$$

$$\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^3(bc-ad)(be-af)}$$

$\downarrow$ 104

3.53. $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^4\sqrt{e+fx}}\,dx$

$$\frac{\sqrt{c+dx}\sqrt{e+fx}(4a^3Cdf-a^2b(-2Bdf+7cCf+9Cde)+ab^2(-8Adf+Bcf+3Bde+12cCe)-b^3(-5Acf-3Ade+6Bce))}{2b(a+bx)^2(be-af)} - \frac{3b^2(de-cf)\left(a^2\left(2df(-4Adf\right.\right.}{}$$

$$\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^3(bc-ad)(be-af)}$$

$\downarrow$ 221

$$\frac{\sqrt{c+dx}\sqrt{e+fx}(4a^3Cdf-a^2b(-2Bdf+7cCf+9Cde)+ab^2(-8Adf+Bcf+3Bde+12cCe)-b^3(-5Acf-3Ade+6Bce))}{2b(a+bx)^2(be-af)} - \frac{\sqrt{c+dx}\sqrt{e+fx}\left(8a^4Cd^2f^2-\right.}{}$$

$$\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^3(bc-ad)(be-af)}$$

input
```
Int[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^4*Sqrt[e + f*x]),x]
```

output
```
-1/3*((A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*Sqrt[e + f*x])/(b*(b*c - a*d
)*(b*e - a*f)*(a + b*x)^3) + (((4*a^3*C*d*f - b^3*(6*B*c*e - 3*A*d*e - 5*A
*c*f) + a*b^2*(12*c*C*e + 3*B*d*e + B*c*f - 8*A*d*f) - a^2*b*(9*C*d*e + 7*
c*C*f - 2*B*d*f))*Sqrt[c + d*x]*Sqrt[e + f*x])/(2*b*(b*e - a*f)*(a + b*x)^
2) - (((8*a^4*C*d^2*f^2 - 2*a^3*b*d*f*(13*C*d*e + 7*c*C*f - 2*B*d*f) - b^4
*(3*A*d^2*e^2 - 2*c*d*e*(3*B*e - 2*A*f) - 3*c^2*(8*C*e^2 - 6*B*e*f + 5*A*f
^2)) - a*b^3*(d^2*e*(3*B*e - 10*A*f) + 3*c^2*f*(4*C*e - B*f) + 2*c*d*(30*C
*e^2 - 14*B*e*f + 13*A*f^2)) - a^2*b^2*(4*d*f*(4*B*d*e + B*c*f - 2*A*d*f)
- C*(33*d^2*e^2 + 44*c*d*e*f + 3*c^2*f^2)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(
(b*c - a*d)*(b*e - a*f)*(a + b*x)) - (3*b^2*(d*e - c*f)*(a^2*(2*d*f*(3*B*d
*e + B*c*f - 4*A*d*f) - C*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2)) - a*b*(d^2*e*
(B*e - 4*A*f) - c^2*f*(4*C*e - B*f) - 2*c*d*(6*C*e^2 - f*(7*B*e - 6*A*f)))
 - b^2*(A*d^2*e^2 - 2*c*d*e*(B*e - A*f) + c^2*(8*C*e^2 - f*(6*B*e - 5*A*f)
)))*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x]
)])/((b*c - a*d)^(3/2)*(b*e - a*f)^(3/2)))/(4*b*(b*e - a*f)))/(6*b*(b*c -
a*d)*(b*e - a*f))
```

3.53. $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^4\sqrt{e+fx}}\,dx$

3.53.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2116
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Si
mp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*
(e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1
) - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x]
, x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && ILtQ[m,
-1]
```

3.53.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 15989 vs. $2(653) = 1306$.

Time = 1.71 (sec) , antiderivative size = 15990, normalized size of antiderivative = 23.34

method	result	size
default	Expression too large to display	15990

input
```
int((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^4/(f*x+e)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
result too large to display
```

3.53.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^4\sqrt{e+fx}}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^4/(f*x+e)^(1/2),x, algorithm
="fricas")
```

output
```
Timed out
```

3.53.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^4\sqrt{e+fx}}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)/(b*x+a)**4/(f*x+e)**(1/2),x)
```

output
```
Timed out
```

3.53.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^4\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^4/(f*x+e)^(1/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((a*d-b*c)>0)', see `assume?` for
 more deta
```

3.53.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25338 vs. $2(653) = 1306$.

Time = 60.94 (sec) , antiderivative size = 25338, normalized size of antiderivative = 36.99

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^4\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^4/(f*x+e)^(1/2),x, algorithm
="giac")
```

output
```
-1/8*(8*sqrt(d*f)*C*b^2*c^2*d^3*e^3 - 12*sqrt(d*f)*C*a*b*c*d^4*e^3 - 2*sqr
t(d*f)*B*b^2*c*d^4*e^3 + 5*sqrt(d*f)*C*a^2*d^5*e^3 + sqrt(d*f)*B*a*b*d^5*e
^3 + sqrt(d*f)*A*b^2*d^5*e^3 - 8*sqrt(d*f)*C*b^2*c^3*d^2*e^2*f + 8*sqrt(d*
f)*C*a*b*c^2*d^3*e^2*f - 4*sqrt(d*f)*B*b^2*c^2*d^3*e^2*f - 3*sqrt(d*f)*C*a
^2*c*d^4*e^2*f + 13*sqrt(d*f)*B*a*b*c*d^4*e^2*f + sqrt(d*f)*A*b^2*c*d^4*e^
2*f - 6*sqrt(d*f)*B*a^2*d^5*e^2*f - 4*sqrt(d*f)*A*a*b*d^5*e^2*f + 4*sqrt(d
*f)*C*a*b*c^3*d^2*e*f^2 + 6*sqrt(d*f)*B*b^2*c^3*d^2*e*f^2 - sqrt(d*f)*C*a^
2*c^2*d^3*e*f^2 - 13*sqrt(d*f)*B*a*b*c^2*d^3*e*f^2 + 3*sqrt(d*f)*A*b^2*c^2
*d^3*e*f^2 + 4*sqrt(d*f)*B*a^2*c*d^4*e*f^2 - 8*sqrt(d*f)*A*a*b*c*d^4*e*f^2
 + 8*sqrt(d*f)*A*a^2*d^5*e*f^2 - sqrt(d*f)*C*a^2*c^3*d^2*f^3 - sqrt(d*f)*B
*a*b*c^3*d^2*f^3 - 5*sqrt(d*f)*A*b^2*c^3*d^2*f^3 + 2*sqrt(d*f)*B*a^2*c^2*d
^3*f^3 + 12*sqrt(d*f)*A*a*b*c^2*d^3*f^3 - 8*sqrt(d*f)*A*a^2*c*d^4*f^3)*arc
tan(-1/2*(b*d^2*e + b*c*d*f - 2*a*d^2*f - (sqrt(d*f)*sqrt(d*x + c) - sqrt(
d^2*e + (d*x + c)*d*f - c*d*f))^2*b)/(sqrt(-b^2*c*d*e*f + a*b*d^2*e*f + a*
b*c*d*f^2 - a^2*d^2*f^2)*d))/((b^5*c^2*e^3*abs(d) - 2*a*b^4*c*d*e^3*abs(d)
 + a^2*b^3*d^2*e^3*abs(d) - 3*a*b^4*c^2*e^2*f*abs(d) + 6*a^2*b^3*c*d*e^2*f
*abs(d) - 3*a^3*b^2*d^2*e^2*f*abs(d) + 3*a^2*b^3*c^2*e*f^2*abs(d) - 6*a^3*
b^2*c*d*e*f^2*abs(d) + 3*a^4*b*d^2*e*f^2*abs(d) - a^3*b^2*c^2*f^3*abs(d) +
 2*a^4*b*c*d*f^3*abs(d) - a^5*d^2*f^3*abs(d))*sqrt(-b^2*c*d*e*f + a*b*d^2*
e*f + a*b*c*d*f^2 - a^2*d^2*f^2)*d) - 1/12*(24*sqrt(d*f)*C*b^7*c^2*d^13...
```

3.53.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^4\sqrt{e+fx}}\,dx = \text{Hanged}$$

input
```
int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)^4),x)
```

output
```
\text{Hanged}
```

3.54 $\int \frac{(a+bx)^2(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$

3.54.1 Optimal result

Integrand size = 36, antiderivative size = 718

$$\int \frac{(a+bx)^2\,(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

$$= -\frac{(2aCdf-b(8Bdf-7C(de+cf)))(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}{24bd^2f^2}$$

$$+\frac{C(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}{4bdf}$$

$$-\frac{\sqrt{c+dx}\sqrt{e+fx}(32a^3Cd^3f^3-8a^2bd^2f^2(16Bdf-11C(de+cf))-16ab^2df(C(15d^2e^2+14cdef+15c}{}$$

$$+\frac{(16a^2d^2f^2(C(3d^2e^2+2cdef+3c^2f^2)+4df(2Adf-B(de+cf)))-16abdf(C(5d^3e^3+3cd^2e^2f+3c^2d}{}$$

output
```
1/64*(16*a^2*d^2*f^2*(C*(3*c^2*f^2+2*c*d*e*f+3*d^2*e^2)+4*d*f*(2*A*d*f-B*(
c*f+d*e)))-16*a*b*d*f*(C*(5*c^3*f^3+3*c^2*d*e*f^2+3*c*d^2*e^2*f+5*d^3*e^3)
+2*d*f*(4*A*d*f*(c*f+d*e)-B*(3*c^2*f^2+2*c*d*e*f+3*d^2*e^2)))+b^2*(C*(35*c
^4*f^4+20*c^3*d*e*f^3+18*c^2*d^2*e^2*f^2+20*c*d^3*e^3*f+35*d^4*e^4)+8*d*f*
(2*A*d*f*(3*c^2*f^2+2*c*d*e*f+3*d^2*e^2)-B*(5*c^3*f^3+3*c^2*d*e*f^2+3*c*d^
2*e^2*f+5*d^3*e^3))))*arctanh(f^(1/2)*(d*x+c)^(1/2)/d^(1/2)/(f*x+e)^(1/2))
/d^(9/2)/f^(9/2)-1/24*(2*a*C*d*f-b*(8*B*d*f-7*C*(c*f+d*e)))*(b*x+a)^2*(d*x
+c)^(1/2)*(f*x+e)^(1/2)/b/d^2/f^2+1/4*C*(b*x+a)^3*(d*x+c)^(1/2)*(f*x+e)^(1
/2)/b/d/f-1/192*(32*a^3*C*d^3*f^3-8*a^2*b*d^2*f^2*(16*B*d*f-11*C*(c*f+d*e)
)-16*a*b^2*d*f*(C*(15*c^2*f^2+14*c*d*e*f+15*d^2*e^2)+6*d*f*(4*A*d*f-3*B*(c
*f+d*e)))+b^3*(5*C*(21*c^3*f^3+19*c^2*d*e*f^2+19*c*d^2*e^2*f+21*d^3*e^3)+8
*d*f*(18*A*d*f*(c*f+d*e)-B*(15*c^2*f^2+14*c*d*e*f+15*d^2*e^2)))+2*b*d*f*(6
*b*d*f*(-8*A*b*d*f+C*a*c*f+C*a*d*e+6*C*b*c*e)+(4*a*d*f-5*b*(c*f+d*e))*(2*a
*C*d*f-b*(8*B*d*f-7*C*(c*f+d*e))))*x)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/d^4/f^
4
```

3.54.2 Mathematica [A] (verified)

Time = 2.22 (sec) , antiderivative size = 632, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$$
$$= \frac{\sqrt{d}\sqrt{f}\sqrt{c+dx}\sqrt{e+fx}(48a^2d^2f^2(4Bdf+C(-3de-3cf+2dfx))+16abdf(6df(4Adf+B(-3de-3cf$$

input
```
Integrate[((a + b*x)^2*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

output

```
(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x]*Sqrt[e + f*x]*(48*a^2*d^2*f^2*(4*B*d*f + C*
(-3*d*e - 3*c*f + 2*d*f*x)) + 16*a*b*d*f*(6*d*f*(4*A*d*f + B*(-3*d*e - 3*c
*f + 2*d*f*x)) + C*(15*c^2*f^2 + 2*c*d*f*(7*e - 5*f*x) + d^2*(15*e^2 - 10*
e*f*x + 8*f^2*x^2))) + b^2*(-(C*(105*c^3*f^3 + 5*c^2*d*f^2*(19*e - 14*f*x)
 + c*d^2*f*(95*e^2 - 68*e*f*x + 56*f^2*x^2) + d^3*(105*e^3 - 70*e^2*f*x +
56*e*f^2*x^2 - 48*f^3*x^3))) + 8*d*f*(6*A*d*f*(-3*d*e - 3*c*f + 2*d*f*x) +
 B*(15*c^2*f^2 + 2*c*d*f*(7*e - 5*f*x) + d^2*(15*e^2 - 10*e*f*x + 8*f^2*x^
2))))) + 3*(16*a^2*d^2*f^2*(C*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d*f*
(2*A*d*f - B*(d*e + c*f))) - 16*a*b*d*f*(C*(5*d^3*e^3 + 3*c*d^2*e^2*f + 3*
c^2*d*e*f^2 + 5*c^3*f^3) + 2*d*f*(4*A*d*f*(d*e + c*f) - B*(3*d^2*e^2 + 2*c
*d*e*f + 3*c^2*f^2))) + b^2*(C*(35*d^4*e^4 + 20*c*d^3*e^3*f + 18*c^2*d^2*e
^2*f^2 + 20*c^3*d*e*f^3 + 35*c^4*f^4) + 8*d*f*(2*A*d*f*(3*d^2*e^2 + 2*c*d*
e*f + 3*c^2*f^2) - B*(5*d^3*e^3 + 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 + 5*c^3*f^
3))))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/(Sqrt[f]*Sqrt[c + d*x])])/(192*d^(9/
2)*f^(9/2))
```

3.54.3 Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 735, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2118, 27, 170, 27, 164, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2 \left(A+Bx+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}}dx$$

$$\Big\downarrow \text{2118}$$

$$\frac{\int -\frac{b(a+bx)^2(6bcCe+aCde+acCf-8Abdf-(8bBdf-2aCdf-7bC(de+cf))x)}{2\sqrt{c+dx}\sqrt{e+fx}}dx}{4b^2df}+\frac{C(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}{4bdf}$$

$$\Big\downarrow \text{27}$$

$$\frac{C(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}{4bdf}-\frac{\int \frac{(a+bx)^2(6bcCe+aCde+acCf-8Abdf-(8bBdf-2aCdf-7bC(de+cf))x)}{\sqrt{c+dx}\sqrt{e+fx}}dx}{8bdf}$$

$$\Big\downarrow \text{170}$$

$$\frac{C(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}{4bdf}-$$

$$\frac{\frac{\int \frac{(a+bx)(6adf(6bcCe+aCde+acCf-8Abdf)+(4bce+ade+acf)(8bBdf-2aCdf-7bC(de+cf))+(6bdf(6bcCe+aCde+acCf-8Abdf)-(4adf-5b(de+cf))(8bBdf-2aC}{2\sqrt{c+dx}\sqrt{e+fx}}}{3df}}{8bdf}$$

$\downarrow$ 27

$$\frac{C(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}{4bdf}-$$

$$\int \frac{(a+bx)(6adf(6bcCe+aCde+acCf-8Abdf)+(4bce+ade+acf)(8bBdf-2aCdf-7bC(de+cf))+(6bdf(6bcCe+aCde+acCf-8Abdf)-(4adf-5b(de+cf))(8bBdf-2aC}{\sqrt{c+dx}\sqrt{e+fx}}$$

$$6df$$

$$8bdf$$

$\downarrow$ 164

$$\frac{C(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}{4bdf}-$$

$$\sqrt{c+dx}\sqrt{e+fx}\Big(32a^3Cd^3f^3-8a^2bd^2f^2(16Bdf-11C(cf+de))-16ab^2df\Big(6df(4Adf-3B(cf+de))+C\Big(15c^2f^2+14cdef+15d^2e^2\Big)\Big)+2bdfx(6bdf(acCf+aCde-8Abdf+$$

$$4d$$

$\downarrow$ 66

$$\frac{C(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}{4bdf}-$$

$$\sqrt{c+dx}\sqrt{e+fx}\Big(32a^3Cd^3f^3-8a^2bd^2f^2(16Bdf-11C(cf+de))-16ab^2df\Big(6df(4Adf-3B(cf+de))+C\Big(15c^2f^2+14cdef+15d^2e^2\Big)\Big)+2bdfx(6bdf(acCf+aCde-8Abdf+$$

$$4d$$

$\downarrow$ 221

$$\frac{C(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}{4bdf}-$$

$$\sqrt{c+dx}\sqrt{e+fx}\Big(32a^3Cd^3f^3-8a^2bd^2f^2(16Bdf-11C(cf+de))-16ab^2df\Big(6df(4Adf-3B(cf+de))+C\Big(15c^2f^2+14cdef+15d^2e^2\Big)\Big)+2bdfx(6bdf(acCf+aCde-8Abdf+$$

$$4d$$

input

```
Int[((a + b*x)^2*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

3.54. $\int \frac{(a+bx)^2(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$

output
```
(C*(a + b*x)^3*Sqrt[c + d*x]*Sqrt[e + f*x])/(4*b*d*f) - (-1/3*((8*b*B*d*f
- 2*a*C*d*f - 7*b*C*(d*e + c*f))*(a + b*x)^2*Sqrt[c + d*x]*Sqrt[e + f*x])/
(d*f) + ((Sqrt[c + d*x]*Sqrt[e + f*x]*(32*a^3*C*d^3*f^3 - 8*a^2*b*d^2*f^2*
(16*B*d*f - 11*C*(d*e + c*f)) - 16*a*b^2*d*f*(C*(15*d^2*e^2 + 14*c*d*e*f +
 15*c^2*f^2) + 6*d*f*(4*A*d*f - 3*B*(d*e + c*f))) + b^3*(5*C*(21*d^3*e^3 +
 19*c*d^2*e^2*f + 19*c^2*d*e*f^2 + 21*c^3*f^3) + 8*d*f*(18*A*d*f*(d*e + c*
f) - B*(15*d^2*e^2 + 14*c*d*e*f + 15*c^2*f^2))) + 2*b*d*f*(6*b*d*f*(6*b*c*
C*e + a*C*d*e + a*c*C*f - 8*A*b*d*f) - (4*a*d*f - 5*b*(d*e + c*f))*(8*b*B*
d*f - 2*a*C*d*f - 7*b*C*(d*e + c*f)))*x))/(4*d^2*f^2) - (3*b*(16*a^2*d^2*f
^2*(C*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d*f*(2*A*d*f - B*(d*e + c*f)
)) - 16*a*b*d*f*(C*(5*d^3*e^3 + 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 + 5*c^3*f^3)
 + 2*d*f*(4*A*d*f*(d*e + c*f) - B*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2))) +
b^2*(C*(35*d^4*e^4 + 20*c*d^3*e^3*f + 18*c^2*d^2*e^2*f^2 + 20*c^3*d*e*f^3
+ 35*c^4*f^4) + 8*d*f*(2*A*d*f*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) - B*(5*
d^3*e^3 + 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 + 5*c^3*f^3))))*ArcTanh[(Sqrt[f]*S
qrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(4*d^(5/2)*f^(5/2)))/(6*d*f))/(8*b
*d*f)
```

3.54.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

3.54.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2527 vs. $2(686) = 1372$.

Time = 1.68 (sec) , antiderivative size = 2528, normalized size of antiderivative = 3.52

method	result	size
default	Expression too large to display	2528

input
```
int((b*x+a)^2*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
1/384*(-112*C*b^2*d^3*e*f^2*x^2*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)-72*B*l
n(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))
*b^2*c^2*d^2*e*f^3-72*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1
/2)+c*f+d*e)/(d*f)^(1/2))*b^2*c*d^3*e^2*f^2+60*C*ln(1/2*(2*d*f*x+2*((d*x+c
)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*c*d^3*e^3*f+54*C*ln
(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*
b^2*c^2*d^2*e^2*f^2-320*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*a*b*c*d^2*f^
3*x-320*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*a*b*d^3*e*f^2*x+136*C*((d*x+
c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b^2*c*d^2*e*f^2*x+144*A*ln(1/2*(2*d*f*x+2*((
d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*d^4*e^2*f^2-19
2*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(
1/2))*a^2*c*d^3*f^4+144*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^
(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*c^2*d^2*f^4-288*A*(d*f)^(1/2)*((d*x+c)*(f*
x+e))^(1/2)*b^2*c*d^2*f^3+448*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*b*c*
d^2*e*f^2+192*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*a^2*d^3*f^3*x+96*C*ln(
1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a
^2*c*d^3*e*f^3+105*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)
+c*f+d*e)/(d*f)^(1/2))*b^2*c^4*f^4+105*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e
))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*d^4*e^4-288*A*(d*f)^(1/2)*(
(d*x+c)*(f*x+e))^(1/2)*b^2*d^3*e*f^2+384*B*(d*f)^(1/2)*((d*x+c)*(f*x+e)...
```

3.54.5 Fricas [A] (verification not implemented)

Time = 1.12 (sec) , antiderivative size = 1436, normalized size of antiderivative = 2.00

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm
="fricas")
```

output
```
[1/768*(3*(35*C*b^2*d^4*e^4 + 20*(C*b^2*c*d^3 - 2*(2*C*a*b + B*b^2)*d^4)*e
^3*f + 6*(3*C*b^2*c^2*d^2 - 4*(2*C*a*b + B*b^2)*c*d^3 + 8*(C*a^2 + 2*B*a*b
 + A*b^2)*d^4)*e^2*f^2 + 4*(5*C*b^2*c^3*d - 6*(2*C*a*b + B*b^2)*c^2*d^2 +
8*(C*a^2 + 2*B*a*b + A*b^2)*c*d^3 - 16*(B*a^2 + 2*A*a*b)*d^4)*e*f^3 + (35*
C*b^2*c^4 + 128*A*a^2*d^4 - 40*(2*C*a*b + B*b^2)*c^3*d + 48*(C*a^2 + 2*B*a
*b + A*b^2)*c^2*d^2 - 64*(B*a^2 + 2*A*a*b)*c*d^3)*f^4)*sqrt(d*f)*log(8*d^2
*f^2*x^2 + d^2*e^2 + 6*c*d*e*f + c^2*f^2 + 4*(2*d*f*x + d*e + c*f)*sqrt(d*
f)*sqrt(d*x + c)*sqrt(f*x + e) + 8*(d^2*e*f + c*d*f^2)*x) + 4*(48*C*b^2*d^
4*f^4*x^3 - 105*C*b^2*d^4*e^3*f - 5*(19*C*b^2*c*d^3 - 24*(2*C*a*b + B*b^2)
*d^4)*e^2*f^2 - (95*C*b^2*c^2*d^2 - 112*(2*C*a*b + B*b^2)*c*d^3 + 144*(C*a
^2 + 2*B*a*b + A*b^2)*d^4)*e*f^3 - 3*(35*C*b^2*c^3*d - 40*(2*C*a*b + B*b^2
)*c^2*d^2 + 48*(C*a^2 + 2*B*a*b + A*b^2)*c*d^3 - 64*(B*a^2 + 2*A*a*b)*d^4)
*f^4 - 8*(7*C*b^2*d^4*e*f^3 + (7*C*b^2*c*d^3 - 8*(2*C*a*b + B*b^2)*d^4)*f^
4)*x^2 + 2*(35*C*b^2*d^4*e^2*f^2 + 2*(17*C*b^2*c*d^3 - 20*(2*C*a*b + B*b^2
)*d^4)*e*f^3 + (35*C*b^2*c^2*d^2 - 40*(2*C*a*b + B*b^2)*c*d^3 + 48*(C*a^2
+ 2*B*a*b + A*b^2)*d^4)*f^4)*x)*sqrt(d*x + c)*sqrt(f*x + e))/(d^5*f^5), -1
/384*(3*(35*C*b^2*d^4*e^4 + 20*(C*b^2*c*d^3 - 2*(2*C*a*b + B*b^2)*d^4)*e^3
*f + 6*(3*C*b^2*c^2*d^2 - 4*(2*C*a*b + B*b^2)*c*d^3 + 8*(C*a^2 + 2*B*a*b +
 A*b^2)*d^4)*e^2*f^2 + 4*(5*C*b^2*c^3*d - 6*(2*C*a*b + B*b^2)*c^2*d^2 + 8*
(C*a^2 + 2*B*a*b + A*b^2)*c*d^3 - 16*(B*a^2 + 2*A*a*b)*d^4)*e*f^3 + (35...
```

3.54.6 Sympy [F]

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{(a+bx)^2 (A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

input
```
integrate((b*x+a)**2*(C*x**2+B*x+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

output
```
Integral((a + b*x)**2*(A + B*x + C*x**2)/(sqrt(c + d*x)*sqrt(e + f*x)), x)
```

3.54.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^2*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

3.54.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 946, normalized size of antiderivative = 1.32

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}} \, dx$$
$$= \frac{\left(\sqrt{d^2e+(dx+c)df-cdf}\left(2\,(dx+c)\left(4\,(dx+c)\left(\frac{6\,(dx+c)Cb^2}{d^5f} - \frac{7\,Cb^2d^{20}ef^5+25\,Cb^2cd^{19}f^6-16\,Cabd^{20}f^6-8\,Bb^2d^{20}f^6}{d^{24}f^7}\right.\right.\right.\right.}{}$$

input
```
integrate((b*x+a)^2*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm
="giac")
```

output
```
1/192*(sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*(2*(d*x + c)*(4*(d*x + c)*(6*(d
*x + c)*C*b^2/(d^5*f) - (7*C*b^2*d^20*e*f^5 + 25*C*b^2*c*d^19*f^6 - 16*C*a
*b*d^20*f^6 - 8*B*b^2*d^20*f^6)/(d^24*f^7)) + (35*C*b^2*d^21*e^2*f^4 + 90*
C*b^2*c*d^20*e*f^5 - 80*C*a*b*d^21*e*f^5 - 40*B*b^2*d^21*e*f^5 + 163*C*b^2
*c^2*d^19*f^6 - 208*C*a*b*c*d^20*f^6 - 104*B*b^2*c*d^20*f^6 + 48*C*a^2*d^2
1*f^6 + 96*B*a*b*d^21*f^6 + 48*A*b^2*d^21*f^6)/(d^24*f^7)) - 3*(35*C*b^2*d
^22*e^3*f^3 + 55*C*b^2*c*d^21*e^2*f^4 - 80*C*a*b*d^22*e^2*f^4 - 40*B*b^2*d
^22*e^2*f^4 + 73*C*b^2*c^2*d^20*e*f^5 - 128*C*a*b*c*d^21*e*f^5 - 64*B*b^2*
c*d^21*e*f^5 + 48*C*a^2*d^22*e*f^5 + 96*B*a*b*d^22*e*f^5 + 48*A*b^2*d^22*e
*f^5 + 93*C*b^2*c^3*d^19*f^6 - 176*C*a*b*c^2*d^20*f^6 - 88*B*b^2*c^2*d^20*
f^6 + 80*C*a^2*c*d^21*f^6 + 160*B*a*b*c*d^21*f^6 + 80*A*b^2*c*d^21*f^6 - 6
4*B*a^2*d^22*f^6 - 128*A*a*b*d^22*f^6)/(d^24*f^7))*sqrt(d*x + c) - 3*(35*C
*b^2*d^4*e^4 + 20*C*b^2*c*d^3*e^3*f - 80*C*a*b*d^4*e^3*f - 40*B*b^2*d^4*e^
3*f + 18*C*b^2*c^2*d^2*e^2*f^2 - 48*C*a*b*c*d^3*e^2*f^2 - 24*B*b^2*c*d^3*e
^2*f^2 + 48*C*a^2*d^4*e^2*f^2 + 96*B*a*b*d^4*e^2*f^2 + 48*A*b^2*d^4*e^2*f^
2 + 20*C*b^2*c^3*d*e*f^3 - 48*C*a*b*c^2*d^2*e*f^3 - 24*B*b^2*c^2*d^2*e*f^3
 + 32*C*a^2*c*d^3*e*f^3 + 64*B*a*b*c*d^3*e*f^3 + 32*A*b^2*c*d^3*e*f^3 - 64
*B*a^2*d^4*e*f^3 - 128*A*a*b*d^4*e*f^3 + 35*C*b^2*c^4*f^4 - 80*C*a*b*c^3*d
*f^4 - 40*B*b^2*c^3*d*f^4 + 48*C*a^2*c^2*d^2*f^4 + 96*B*a*b*c^2*d^2*f^4 +
48*A*b^2*c^2*d^2*f^4 - 64*B*a^2*c*d^3*f^4 - 128*A*a*b*c*d^3*f^4 + 128*A...
```

3.54.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Hanged}$$

input
```
int(((a + b*x)^2*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(c + d*x)^(1/2)),x)
```

output
```
\text{Hanged}
```

3.55 $\int \frac{(a+bx)(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$

3.55.1 Optimal result

Integrand size = 34, antiderivative size = 371

$$\int \frac{(a+bx)\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \frac{C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}{3bdf}$$
$$-\frac{\sqrt{c+dx}\sqrt{e+fx}(8a^2Cd^2f^2-6abdf(4Bdf-3C(de+cf))-b^2(C(15d^2e^2+14cdef+15c^2f^2)+6df(4}{24bd^3f^3}$$
$$+\frac{(2adf(C(3d^2e^2+2cdef+3c^2f^2)+4df(2Adf-B(de+cf)))-b(C(5d^3e^3+3cd^2e^2f+3c^2def^2+5c^3}{8d^{7/2}f^{7/2}}$$

output
```
1/8*(2*a*d*f*(C*(3*c^2*f^2+2*c*d*e*f+3*d^2*e^2)+4*d*f*(2*A*d*f-B*(c*f+d*e)
))-b*(C*(5*c^3*f^3+3*c^2*d*e*f^2+3*c*d^2*e^2*f+5*d^3*e^3)+2*d*f*(4*A*d*f*(
c*f+d*e)-B*(3*c^2*f^2+2*c*d*e*f+3*d^2*e^2))))*arctanh(f^(1/2)*(d*x+c)^(1/2
)/d^(1/2)/(f*x+e)^(1/2))/d^(7/2)/f^(7/2)+1/3*C*(b*x+a)^2*(d*x+c)^(1/2)*(f*
x+e)^(1/2)/b/d/f-1/24*(8*a^2*C*d^2*f^2-6*a*b*d*f*(4*B*d*f-3*C*(c*f+d*e))-b
^2*(C*(15*c^2*f^2+14*c*d*e*f+15*d^2*e^2)+6*d*f*(4*A*d*f-3*B*(c*f+d*e)))+2*
b*d*f*(2*a*C*d*f-b*(6*B*d*f-5*C*(c*f+d*e)))*x)*(d*x+c)^(1/2)*(f*x+e)^(1/2)
/b/d^3/f^3
```

3.55.2 Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 314, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$$
$$=\frac{\sqrt{c+dx}\sqrt{e+fx}(6adf(4Bdf+C(-3de-3cf+2dfx))+b(6df(4Adf+B(-3de-3cf+2dfx))+C(15}{24d^3f^3}$$
$$-\frac{(-2adf(C(3d^2e^2+2cdef+3c^2f^2)+4df(2Adf-B(de+cf)))+b(C(5d^3e^3+3cd^2e^2f+3c^2def^2+5c}{8d^{7/2}f^{7/2}}$$

input
```
Integrate[((a + b*x)*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

output
```
(Sqrt[c + d*x]*Sqrt[e + f*x]*(6*a*d*f*(4*B*d*f + C*(-3*d*e - 3*c*f + 2*d*f
*x)) + b*(6*d*f*(4*A*d*f + B*(-3*d*e - 3*c*f + 2*d*f*x)) + C*(15*c^2*f^2 +
 2*c*d*f*(7*e - 5*f*x) + d^2*(15*e^2 - 10*e*f*x + 8*f^2*x^2)))))/(24*d^3*f
^3) - ((-2*a*d*f*(C*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d*f*(2*A*d*f -
 B*(d*e + c*f))) + b*(C*(5*d^3*e^3 + 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 + 5*c^3
*f^3) + 2*d*f*(4*A*d*f*(d*e + c*f) - B*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2)
)))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/(Sqrt[f]*Sqrt[c + d*x])])/(8*d^(7/2)*f
^(7/2))
```

3.55.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {2118, 27, 164, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

$$\Big\downarrow \ 2118$$

$$\frac{\int -\frac{b(a+bx)(4bcCe+aCde+acCf-6Abdf-(6bBdf-2aCdf-5bC(de+cf))x)}{2\sqrt{c+dx}\sqrt{e+fx}}dx}{3b^2df}+\frac{C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}{3bdf}$$

$$\Big\downarrow \ 27$$

$$\frac{C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}{3bdf}-\frac{\int\frac{(a+bx)(4bcCe+aCde+acCf-6Abdf-(6bBdf-2aCdf-5bC(de+cf))x)}{\sqrt{c+dx}\sqrt{e+fx}}dx}{6bdf}$$

$\downarrow$ 164

$$\frac{C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}{3bdf}-$$
$$\frac{\sqrt{c+dx}\sqrt{e+fx}(8a^2Cd^2f^2-2bdfx(-2aCdf+6bBdf-5bC(cf+de))-6abdf(4Bdf-3C(cf+de))-(b^2(6df(4Adf-3B(cf+de))+C(15c^2f^2+14cdef+}{4d^2f^2}$$

$\downarrow$ 66

$$\frac{C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}{3bdf}-$$
$$\frac{\sqrt{c+dx}\sqrt{e+fx}(8a^2Cd^2f^2-2bdfx(-2aCdf+6bBdf-5bC(cf+de))-6abdf(4Bdf-3C(cf+de))-(b^2(6df(4Adf-3B(cf+de))+C(15c^2f^2+14cdef+}{4d^2f^2}$$

$\downarrow$ 221

$$\frac{C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}{3bdf}-$$
$$\frac{\sqrt{c+dx}\sqrt{e+fx}(8a^2Cd^2f^2-2bdfx(-2aCdf+6bBdf-5bC(cf+de))-6abdf(4Bdf-3C(cf+de))-(b^2(6df(4Adf-3B(cf+de))+C(15c^2f^2+14cdef+}{4d^2f^2}$$

input
```
Int[((a + b*x)*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

output
```
(C*(a + b*x)^2*Sqrt[c + d*x]*Sqrt[e + f*x])/(3*b*d*f) - ((Sqrt[c + d*x]*Sq
rt[e + f*x]*(8*a^2*C*d^2*f^2 - 6*a*b*d*f*(4*B*d*f - 3*C*(d*e + c*f)) - b^2
*(C*(15*d^2*e^2 + 14*c*d*e*f + 15*c^2*f^2) + 6*d*f*(4*A*d*f - 3*B*(d*e + c
*f))) - 2*b*d*f*(6*b*B*d*f - 2*a*C*d*f - 5*b*C*(d*e + c*f))*x))/(4*d^2*f^2
) - (3*b*(2*a*d*f*(C*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d*f*(2*A*d*f
- B*(d*e + c*f))) - b*(C*(5*d^3*e^3 + 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 + 5*c^
3*f^3) + 2*d*f*(4*A*d*f*(d*e + c*f) - B*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2
))))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(4*d^(5/2)*
f^(5/2)))/(6*b*d*f)
```

3.55. $\int \frac{(a+bx)(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$

3.55.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

3.55.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1198 vs. 2(345) = 690.

Time = 1.68 (sec) , antiderivative size = 1199, normalized size of antiderivative = 3.23

method	result	size
default	Expression too large to display	1199

input
```
int((b*x+a)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
1/48*(-9*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/
(d*f)^(1/2))*b*c*d^2*e^2*f+12*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*
(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c*d^2*e*f^2-24*A*ln(1/2*(2*d*f*x+2*((d
*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c*d^2*f^3-24*A*ln
(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*
b*d^3*e*f^2+24*B*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b*d^2*f^2*x-20*C*((d*
x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b*d^2*e*f*x-36*C*(d*f)^(1/2)*((d*x+c)*(f*x
+e))^(1/2)*a*d^2*e*f-15*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^
(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c^3*f^3-15*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x
+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*d^3*e^3+12*B*ln(1/2*(2*d*f*
x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c*d^2*e*f^
2+24*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*a*d^2*f^2*x+18*C*ln(1/2*(2*d*f*
x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*d^3*e^2*f+
48*A*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b*d^2*f^2+48*B*(d*f)^(1/2)*((d*x+
c)*(f*x+e))^(1/2)*a*d^2*f^2+48*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)
*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*d^3*f^3+30*C*(d*f)^(1/2)*((d*x+c)*(f*
x+e))^(1/2)*b*c^2*f^2+30*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b*d^2*e^2-2
4*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(
1/2))*a*c*d^2*f^3-24*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/
2)+c*f+d*e)/(d*f)^(1/2))*a*d^3*e*f^2+18*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f...
```

3.55.5 Fricas [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 720, normalized size of antiderivative = 1.94

$$\int \frac{(a+bx)(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$$
$$= \left[-\frac{3\,(5\,Cbd^3e^3+3\,(Cbcd^2-2\,(Ca+Bb)d^3)e^2f+(3\,Cbc^2d-4\,(Ca+Bb)cd^2+8\,(Ba+Ab)d^3)ef^2+(5\right.$$

input
```
integrate((b*x+a)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="
fricas")
```

output
```
[-1/96*(3*(5*C*b*d^3*e^3 + 3*(C*b*c*d^2 - 2*(C*a + B*b)*d^3)*e^2*f + (3*C*
b*c^2*d - 4*(C*a + B*b)*c*d^2 + 8*(B*a + A*b)*d^3)*e*f^2 + (5*C*b*c^3 - 16
*A*a*d^3 - 6*(C*a + B*b)*c^2*d + 8*(B*a + A*b)*c*d^2)*f^3)*sqrt(d*f)*log(8
*d^2*f^2*x^2 + d^2*e^2 + 6*c*d*e*f + c^2*f^2 + 4*(2*d*f*x + d*e + c*f)*sqr
t(d*f)*sqrt(d*x + c)*sqrt(f*x + e) + 8*(d^2*e*f + c*d*f^2)*x) - 4*(8*C*b*d
^3*f^3*x^2 + 15*C*b*d^3*e^2*f + 2*(7*C*b*c*d^2 - 9*(C*a + B*b)*d^3)*e*f^2
+ 3*(5*C*b*c^2*d - 6*(C*a + B*b)*c*d^2 + 8*(B*a + A*b)*d^3)*f^3 - 2*(5*C*b
*d^3*e*f^2 + (5*C*b*c*d^2 - 6*(C*a + B*b)*d^3)*f^3)*x)*sqrt(d*x + c)*sqrt(
f*x + e))/(d^4*f^4), 1/48*(3*(5*C*b*d^3*e^3 + 3*(C*b*c*d^2 - 2*(C*a + B*b)
*d^3)*e^2*f + (3*C*b*c^2*d - 4*(C*a + B*b)*c*d^2 + 8*(B*a + A*b)*d^3)*e*f^
2 + (5*C*b*c^3 - 16*A*a*d^3 - 6*(C*a + B*b)*c^2*d + 8*(B*a + A*b)*c*d^2)*f
^3)*sqrt(-d*f)*arctan(1/2*(2*d*f*x + d*e + c*f)*sqrt(-d*f)*sqrt(d*x + c)*s
qrt(f*x + e)/(d^2*f^2*x^2 + c*d*e*f + (d^2*e*f + c*d*f^2)*x)) + 2*(8*C*b*d
^3*f^3*x^2 + 15*C*b*d^3*e^2*f + 2*(7*C*b*c*d^2 - 9*(C*a + B*b)*d^3)*e*f^2
+ 3*(5*C*b*c^2*d - 6*(C*a + B*b)*c*d^2 + 8*(B*a + A*b)*d^3)*f^3 - 2*(5*C*b
*d^3*e*f^2 + (5*C*b*c*d^2 - 6*(C*a + B*b)*d^3)*f^3)*x)*sqrt(d*x + c)*sqrt(
f*x + e))/(d^4*f^4)]
```

3.55.6 Sympy [F]

$$\int \frac{(a+bx)\,(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{(a+bx)\,(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

input
```
integrate((b*x+a)*(C*x**2+B*x+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

output
```
Integral((a + b*x)*(A + B*x + C*x**2)/(sqrt(c + d*x)*sqrt(e + f*x)), x)
```

3.55.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

3.55.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx)(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$$
$$= \frac{\left(\sqrt{d^2e+(dx+c)df-cdf}\sqrt{dx+c}\left(2\,(dx+c)\left(\frac{4\,(dx+c)Cb}{d^4f}-\frac{5\,Cbd^{12}ef^3+13\,Cbcd^{11}f^4-6\,Cad^{12}f^4-6\,Bbd^{12}f^4}{d^{15}f^5}\right)+\frac{3\,(5\cdots}{}\right.\right.}{}$$

input
```
integrate((b*x+a)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="
giac")
```

output
```
1/24*(sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*sqrt(d*x + c)*(2*(d*x + c)*(4*(d
*x + c)*C*b/(d^4*f) - (5*C*b*d^12*e*f^3 + 13*C*b*c*d^11*f^4 - 6*C*a*d^12*f
^4 - 6*B*b*d^12*f^4)/(d^15*f^5)) + 3*(5*C*b*d^13*e^2*f^2 + 8*C*b*c*d^12*e*
f^3 - 6*C*a*d^13*e*f^3 - 6*B*b*d^13*e*f^3 + 11*C*b*c^2*d^11*f^4 - 10*C*a*c
*d^12*f^4 - 10*B*b*c*d^12*f^4 + 8*B*a*d^13*f^4 + 8*A*b*d^13*f^4)/(d^15*f^5
)) + 3*(5*C*b*d^3*e^3 + 3*C*b*c*d^2*e^2*f - 6*C*a*d^3*e^2*f - 6*B*b*d^3*e^
2*f + 3*C*b*c^2*d*e*f^2 - 4*C*a*c*d^2*e*f^2 - 4*B*b*c*d^2*e*f^2 + 8*B*a*d^
3*e*f^2 + 8*A*b*d^3*e*f^2 + 5*C*b*c^3*f^3 - 6*C*a*c^2*d*f^3 - 6*B*b*c^2*d*
f^3 + 8*B*a*c*d^2*f^3 + 8*A*b*c*d^2*f^3 - 16*A*a*d^3*f^3)*log(abs(-sqrt(d*
f)*sqrt(d*x + c) + sqrt(d^2*e + (d*x + c)*d*f - c*d*f)))/(sqrt(d*f)*d^3*f^
3))*d/abs(d)
```

3.55.9 Mupad [B] (verification not implemented)

Time = 140.16 (sec) , antiderivative size = 2621, normalized size of antiderivative = 7.06

$$\int \frac{(a+bx)\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
int(((a + b*x)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(c + d*x)^(1/2)),x)
```

output
```
((((c + d*x)^(1/2) - c^(1/2))*(2*A*b*c*f + 2*A*b*d*e))/(f^3*((e + f*x)^(1/
2) - e^(1/2))) + (((c + d*x)^(1/2) - c^(1/2))^3*(2*A*b*c*f + 2*A*b*d*e))/(
d*f^2*((e + f*x)^(1/2) - e^(1/2))^3) - (8*A*b*c^(1/2)*e^(1/2)*((c + d*x)^(
1/2) - c^(1/2))^2)/(f^2*((e + f*x)^(1/2) - e^(1/2))^2))/(((c + d*x)^(1/2)
- c^(1/2))^4/((e + f*x)^(1/2) - e^(1/2))^4 + d^2/f^2 - (2*d*((c + d*x)^(1/
2) - c^(1/2))^2)/(f*((e + f*x)^(1/2) - e^(1/2))^2)) - ((((c + d*x)^(1/2) -
 c^(1/2))*((3*C*a*d^3*e^2)/2 + (3*C*a*c^2*d*f^2)/2 + C*a*c*d^2*e*f))/(f^6*
((e + f*x)^(1/2) - e^(1/2))) - (((c + d*x)^(1/2) - c^(1/2))^3*((11*C*a*c^2
*f^2)/2 + (11*C*a*d^2*e^2)/2 + 25*C*a*c*d*e*f))/(f^5*((e + f*x)^(1/2) - e^
(1/2))^3) + (((c + d*x)^(1/2) - c^(1/2))^7*((3*C*a*c^2*f^2)/2 + (3*C*a*d^2
*e^2)/2 + C*a*c*d*e*f))/(d^2*f^3*((e + f*x)^(1/2) - e^(1/2))^7) - (((c + d
*x)^(1/2) - c^(1/2))^5*((11*C*a*c^2*f^2)/2 + (11*C*a*d^2*e^2)/2 + 25*C*a*c
*d*e*f))/(d*f^4*((e + f*x)^(1/2) - e^(1/2))^5) + (c^(1/2)*e^(1/2)*((c + d*
x)^(1/2) - c^(1/2))^4*(32*C*a*c*f + 32*C*a*d*e))/(f^4*((e + f*x)^(1/2) - e
^(1/2))^4))/(((c + d*x)^(1/2) - c^(1/2))^8/((e + f*x)^(1/2) - e^(1/2))^8 +
 d^4/f^4 - (4*d*((c + d*x)^(1/2) - c^(1/2))^6)/(f*((e + f*x)^(1/2) - e^(1/
2))^6) - (4*d^3*((c + d*x)^(1/2) - c^(1/2))^2)/(f^3*((e + f*x)^(1/2) - e^(
1/2))^2) + (6*d^2*((c + d*x)^(1/2) - c^(1/2))^4)/(f^2*((e + f*x)^(1/2) - e
^(1/2))^4)) - ((((c + d*x)^(1/2) - c^(1/2))^3*((85*C*b*d^4*e^3)/12 + (85*C
*b*c^3*d*f^3)/12 + (17*C*b*c*d^3*e^2*f)/4 + (17*C*b*c^2*d^2*e*f^2)/4))/...
```

3.56 $\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$

3.56.1 Optimal result

Integrand size = 29, antiderivative size = 164

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = -\frac{(3Cde+5cCf-4Bdf)\sqrt{c+dx}\sqrt{e+fx}}{4d^2f^2}+\frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2d^2f} + \frac{(C(3d^2e^2+2cdef+3c^2f^2)+4df(2Adf-B(de+cf)))\,\text{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)}{4d^{5/2}f^{5/2}}$$

output
```
1/4*(C*(3*c^2*f^2+2*c*d*e*f+3*d^2*e^2)+4*d*f*(2*A*d*f-B*(c*f+d*e)))*arctan
h(f^(1/2)*(d*x+c)^(1/2)/d^(1/2)/(f*x+e)^(1/2))/d^(5/2)/f^(5/2)+1/2*C*(d*x+
c)^(3/2)*(f*x+e)^(1/2)/d^2/f-1/4*(-4*B*d*f+5*C*c*f+3*C*d*e)*(d*x+c)^(1/2)*
(f*x+e)^(1/2)/d^2/f^2
```

3.56.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \frac{\sqrt{c+dx}\sqrt{e+fx}(4Bdf+C(-3de-3cf+2dfx))}{4d^2f^2} + \frac{(C(3d^2e^2+2cdef+3c^2f^2)+4df(2Adf-B(de+cf)))\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{f}\sqrt{c+dx}}\right)}{4d^{5/2}f^{5/2}}$$

input
```
Integrate[(A + B*x + C*x^2)/(Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

output
```
(Sqrt[c + d*x]*Sqrt[e + f*x]*(4*B*d*f + C*(-3*d*e - 3*c*f + 2*d*f*x)))/(4*
d^2*f^2) + ((C*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d*f*(2*A*d*f - B*(d
*e + c*f)))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/(Sqrt[f]*Sqrt[c + d*x])])/(4*d
^(5/2)*f^(5/2))
```

3.56.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1194, 27, 90, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}}dx$$

$$\Big\downarrow \text{1194}$$

$$\frac{\int -\frac{Cfc^2+3Cdec-4Ad^2f+d(3Cde+5cCf-4Bdf)x}{2\sqrt{c+dx}\sqrt{e+fx}}dx}{2d^2f}+\frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2d^2f}$$

$$\Big\downarrow \text{27}$$

$$\frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2d^2f}-\frac{\int \frac{Cfc^2+3Cdec-4Ad^2f+d(3Cde+5cCf-4Bdf)x}{\sqrt{c+dx}\sqrt{e+fx}}dx}{4d^2f}$$

$$\Big\downarrow \text{90}$$

$$\frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2d^2f}-\frac{\frac{\sqrt{c+dx}\sqrt{e+fx}(-4Bdf+5cCf+3Cde)}{f}-\frac{(4df(2Adf-B(cf+de))+C(3c^2f^2+2cdef+3d^2e^2))\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}}dx}{2f}}{4d^2f}$$

$$\Big\downarrow \text{66}$$

$$\frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2d^2f}-\frac{\frac{\sqrt{c+dx}\sqrt{e+fx}(-4Bdf+5cCf+3Cde)}{f}-\frac{(4df(2Adf-B(cf+de))+C(3c^2f^2+2cdef+3d^2e^2))\int \frac{1}{d-\frac{f(c+dx)}{e+fx}}d\frac{\sqrt{c+dx}}{\sqrt{e+fx}}}{f}}{4d^2f}$$

$$\Big\downarrow \text{221}$$

$$\frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2d^2f}-\frac{\frac{\sqrt{c+dx}\sqrt{e+fx}(-4Bdf+5cCf+3Cde)}{f}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)\left(4df(2Adf-B(cf+de))+C\left(3c^2f^2+2cdef+3d^2e^2\right)\right)}{\sqrt{d}f^{3/2}}}{4d^2f}$$

input
```
Int[(A + B*x + C*x^2)/(Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

output
```
(C*(c + d*x)^(3/2)*Sqrt[e + f*x])/(2*d^2*f) - (((3*C*d*e + 5*c*C*f - 4*B*d
*f)*Sqrt[c + d*x]*Sqrt[e + f*x])/f - ((C*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^
2) + 4*d*f*(2*A*d*f - B*(d*e + c*f)))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqr
t[d]*Sqrt[e + f*x])])/(Sqrt[d]*f^(3/2)))/(4*d^2*f)
```

3.56.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1194
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Simp[1/(g*e^(2*p)*(m + n + 2
*p + 1))   Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(
2*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)
*(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ
[p, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && NeQ[m + n + 2*p + 1, 0]
```

3.56.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(138) = 276$.

Time = 5.70 (sec) , antiderivative size = 425, normalized size of antiderivative = 2.59

method	result
default	$\left(8A\ln\left(\frac{2dfx+2\sqrt{(dx+c)(fx+e)}\,\sqrt{df}+cf+de}{2\sqrt{df}}\right)d^2f^2-4B\ln\left(\frac{2dfx+2\sqrt{(dx+c)(fx+e)}\,\sqrt{df}+cf+de}{2\sqrt{df}}\right)cd\,f^2-4B\ln\left(\frac{2dfx+2\sqrt{(dx+c)(fx+e)}\,\sqrt{df}}{2\sqrt{df}}\right.\right.$

input
```
int((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(8*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d
*f)^(1/2))*d^2*f^2-4*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/
2)+c*f+d*e)/(d*f)^(1/2))*c*d*f^2-4*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(
1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*d^2*e*f+4*C*((d*x+c)*(f*x+e))^(1/2)
*(d*f)^(1/2)*d*f*x+3*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/
2)+c*f+d*e)/(d*f)^(1/2))*c^2*f^2+2*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(
1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c*d*e*f+3*C*ln(1/2*(2*d*f*x+2*((d*x
+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*d^2*e^2+8*B*(d*f)^(1/
2)*((d*x+c)*(f*x+e))^(1/2)*d*f-6*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*c*f
-6*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*d*e)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/
(d*f)^(1/2)/f^2/d^2/((d*x+c)*(f*x+e))^(1/2)
```

3.56.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.32

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$$
$$= \left[\frac{(3\,Cd^2e^2+2\,(Ccd-2\,Bd^2)ef+(3\,Cc^2-4\,Bcd+8\,Ad^2)f^2)\sqrt{df}\log\left(8\,d^2f^2x^2+d^2e^2+6\,cdef+c^2f^2-\right.}{}\right.$$
$$-\frac{(3\,Cd^2e^2+2\,(Ccd-2\,Bd^2)ef+(3\,Cc^2-4\,Bcd+8\,Ad^2)f^2)\sqrt{-df}\arctan\left(\frac{(2\,dfx+de+cf)\sqrt{-df}\sqrt{dx+c}\sqrt{fx+e}}{2\,(d^2f^2x^2+cdef+(d^2ef+cdf^2)x)}\right.}{8\,d^3f^3}$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*((3*C*d^2*e^2 + 2*(C*c*d - 2*B*d^2)*e*f + (3*C*c^2 - 4*B*c*d + 8*A*d
^2)*f^2)*sqrt(d*f)*log(8*d^2*f^2*x^2 + d^2*e^2 + 6*c*d*e*f + c^2*f^2 + 4*(
2*d*f*x + d*e + c*f)*sqrt(d*f)*sqrt(d*x + c)*sqrt(f*x + e) + 8*(d^2*e*f +
c*d*f^2)*x) + 4*(2*C*d^2*f^2*x - 3*C*d^2*e*f - (3*C*c*d - 4*B*d^2)*f^2)*sq
rt(d*x + c)*sqrt(f*x + e))/(d^3*f^3), -1/8*((3*C*d^2*e^2 + 2*(C*c*d - 2*B*
d^2)*e*f + (3*C*c^2 - 4*B*c*d + 8*A*d^2)*f^2)*sqrt(-d*f)*arctan(1/2*(2*d*f
*x + d*e + c*f)*sqrt(-d*f)*sqrt(d*x + c)*sqrt(f*x + e)/(d^2*f^2*x^2 + c*d*
e*f + (d^2*e*f + c*d*f^2)*x)) - 2*(2*C*d^2*f^2*x - 3*C*d^2*e*f - (3*C*c*d
- 4*B*d^2)*f^2)*sqrt(d*x + c)*sqrt(f*x + e))/(d^3*f^3)]
```

3.56.6 Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}}\,dx=\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(sqrt(c + d*x)*sqrt(e + f*x)), x)
```

3.56.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

3.56.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.16

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \frac{\left(\sqrt{d^2e+(dx+c)df-cdf}\sqrt{dx+c}\left(\frac{2\,(dx+c)C}{d^3f}-\frac{3\,Cd^6ef+5\,Ccd^5f^2-4\,Bd^6f^2}{d^8f^3}\right)-\frac{(3\,Cd^2e^2+2\,Ccdef-4\,Bd^2ef+3\,Cc^2f^2-4}{}\right.}{4\,|d|}$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
1/4*(sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*sqrt(d*x + c)*(2*(d*x + c)*C/(d^3
*f) - (3*C*d^6*e*f + 5*C*c*d^5*f^2 - 4*B*d^6*f^2)/(d^8*f^3)) - (3*C*d^2*e^
2 + 2*C*c*d*e*f - 4*B*d^2*e*f + 3*C*c^2*f^2 - 4*B*c*d*f^2 + 8*A*d^2*f^2)*l
og(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt(d^2*e + (d*x + c)*d*f - c*d*f)))/(s
qrt(d*f)*d^2*f^2))*d/abs(d)
```

3.56.9 Mupad [B] (verification not implemented)

Time = 50.86 (sec) , antiderivative size = 833, normalized size of antiderivative = 5.08

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$$
$$= \frac{\frac{(2\,B\,c\,f+2\,B\,d\,e)\,(\sqrt{c+d\,x}-\sqrt{c})}{f^3\,(\sqrt{e+f\,x}-\sqrt{e})}+\frac{(2\,B\,c\,f+2\,B\,d\,e)\,(\sqrt{c+d\,x}-\sqrt{c})^3}{d\,f^2\,(\sqrt{e+f\,x}-\sqrt{e})^3}-\frac{8\,B\,\sqrt{c}\,\sqrt{e}\,(\sqrt{c+d\,x}-\sqrt{c})^2}{f^2\,(\sqrt{e+f\,x}-\sqrt{e})^2}}{\frac{(\sqrt{c+d\,x}-\sqrt{c})^4}{(\sqrt{e+f\,x}-\sqrt{e})^4}+\frac{d^2}{f^2}-\frac{2\,d\,(\sqrt{c+d\,x}-\sqrt{c})^2}{f\,(\sqrt{e+f\,x}-\sqrt{e})^2}}$$
$$-\frac{\frac{(\sqrt{c+d\,x}-\sqrt{c})\left(\frac{3\,C\,c^2\,d\,f^2}{2}+C\,c\,d^2\,e\,f+\frac{3\,C\,d^3\,e^2}{2}\right)}{f^6\,(\sqrt{e+f\,x}-\sqrt{e})}-\frac{(\sqrt{c+d\,x}-\sqrt{c})^3\left(\frac{11\,C\,c^2\,f^2}{2}+25\,C\,c\,d\,e\,f+\frac{11\,C\,d^2\,e^2}{2}\right)}{f^5\,(\sqrt{e+f\,x}-\sqrt{e})^3}+\frac{(\sqrt{c+d\,x}-\sqrt{c})^7\left(\frac{3\,C\,c^2\,f^2}{2}+\cdots\right.}{d^2\,f^3\,(\sqrt{e+f\,x}\cdots}}{\frac{(\sqrt{c+d\,x}-\sqrt{c})^8}{(\sqrt{e+f\,x}-\sqrt{e})^8}+\frac{d^4}{f^4}-\frac{4\,d\,(\sqrt{c+d\,x}-\sqrt{c})^6}{f\,(\sqrt{e+f\,x}-\sqrt{e})^6}-\frac{4\,d^3\cdots}{f^3\,(\cdots}}$$
$$-\frac{4\,A\,\mathrm{atan}\left(\frac{d\,(\sqrt{e+f\,x}-\sqrt{e})}{\sqrt{-d\,f}\,(\sqrt{c+d\,x}-\sqrt{c})}\right)}{\sqrt{-d\,f}}-\frac{2\,B\,\mathrm{atanh}\left(\frac{\sqrt{f}\,(\sqrt{c+d\,x}-\sqrt{c})}{\sqrt{d}\,(\sqrt{e+f\,x}-\sqrt{e})}\right)\,(c\,f+d\,e)}{d^{3/2}\,f^{3/2}}$$
$$+\frac{C\,\mathrm{atanh}\left(\frac{\sqrt{f}\,(\sqrt{c+d\,x}-\sqrt{c})}{\sqrt{d}\,(\sqrt{e+f\,x}-\sqrt{e})}\right)\,(3\,c^2\,f^2+2\,c\,d\,e\,f+3\,d^2\,e^2)}{2\,d^{5/2}\,f^{5/2}}$$

input
```
int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(c + d*x)^(1/2)),x)
```

output
```
(((2*B*c*f + 2*B*d*e)*((c + d*x)^(1/2) - c^(1/2)))/(f^3*((e + f*x)^(1/2) -
 e^(1/2))) + ((2*B*c*f + 2*B*d*e)*((c + d*x)^(1/2) - c^(1/2))^3)/(d*f^2*((
e + f*x)^(1/2) - e^(1/2))^3) - (8*B*c^(1/2)*e^(1/2)*((c + d*x)^(1/2) - c^(
1/2))^2)/(f^2*((e + f*x)^(1/2) - e^(1/2))^2))/(((c + d*x)^(1/2) - c^(1/2))
^4/((e + f*x)^(1/2) - e^(1/2))^4 + d^2/f^2 - (2*d*((c + d*x)^(1/2) - c^(1/
2))^2)/(f*((e + f*x)^(1/2) - e^(1/2))^2)) - ((((c + d*x)^(1/2) - c^(1/2))*
((3*C*d^3*e^2)/2 + (3*C*c^2*d*f^2)/2 + C*c*d^2*e*f))/(f^6*((e + f*x)^(1/2)
 - e^(1/2))) - (((c + d*x)^(1/2) - c^(1/2))^3*((11*C*c^2*f^2)/2 + (11*C*d^
2*e^2)/2 + 25*C*c*d*e*f))/(f^5*((e + f*x)^(1/2) - e^(1/2))^3) + (((c + d*x
)^(1/2) - c^(1/2))^7*((3*C*c^2*f^2)/2 + (3*C*d^2*e^2)/2 + C*c*d*e*f))/(d^2
*f^3*((e + f*x)^(1/2) - e^(1/2))^7) - (((c + d*x)^(1/2) - c^(1/2))^5*((11*
C*c^2*f^2)/2 + (11*C*d^2*e^2)/2 + 25*C*c*d*e*f))/(d*f^4*((e + f*x)^(1/2) -
 e^(1/2))^5) + (c^(1/2)*e^(1/2)*(32*C*c*f + 32*C*d*e)*((c + d*x)^(1/2) - c
^(1/2))^4)/(f^4*((e + f*x)^(1/2) - e^(1/2))^4))/(((c + d*x)^(1/2) - c^(1/2
))^8/((e + f*x)^(1/2) - e^(1/2))^8 + d^4/f^4 - (4*d*((c + d*x)^(1/2) - c^(
1/2))^6)/(f*((e + f*x)^(1/2) - e^(1/2))^6) - (4*d^3*((c + d*x)^(1/2) - c^(
1/2))^2)/(f^3*((e + f*x)^(1/2) - e^(1/2))^2) + (6*d^2*((c + d*x)^(1/2) - c
^(1/2))^4)/(f^2*((e + f*x)^(1/2) - e^(1/2))^4)) - (4*A*atan((d*((e + f*x)^
(1/2) - e^(1/2)))/((-d*f)^(1/2)*((c + d*x)^(1/2) - c^(1/2)))))/(-d*f)^(1/2
) - (2*B*atanh((f^(1/2)*((c + d*x)^(1/2) - c^(1/2)))/(d^(1/2)*((e + f*x...
```

3.57 $\int \frac{A+Bx+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}\,dx$

3.57.1 Optimal result

Integrand size = 36, antiderivative size = 188

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}\,dx = {} & \frac{C\sqrt{c+dx}\sqrt{e+fx}}{bdf} \\ & - \frac{(2aCdf+b(Cde+cCf-2Bdf))\text{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)}{b^2d^{3/2}f^{3/2}} \\ & - \frac{2(Ab^2-a(bB-aC))\,\text{arctanh}\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{bc-ad}\sqrt{e+fx}}\right)}{b^2\sqrt{bc-ad}\sqrt{be-af}}\end{aligned}$$

output
```
-(2*a*C*d*f+b*(-2*B*d*f+C*c*f+C*d*e))*arctanh(f^(1/2)*(d*x+c)^(1/2)/d^(1/2
)/(f*x+e)^(1/2))/b^2/d^(3/2)/f^(3/2)-2*(A*b^2-a*(B*b-C*a))*arctanh((-a*f+b
*e)^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2)/(f*x+e)^(1/2))/b^2/(-a*d+b*c)^(1/
2)/(-a*f+b*e)^(1/2)+C*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/d/f
```

3.57.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}\,dx = \frac{\frac{bC\sqrt{c+dx}\sqrt{e+fx}}{df} + \frac{2(Ab^2+a(-bB+aC))\arctan\left(\frac{\sqrt{bc-ad}\sqrt{e+fx}}{\sqrt{-be+af}\sqrt{c+dx}}\right)}{\sqrt{bc-ad}\sqrt{-be+af}} - \frac{(2aCdf+b(Cde+cCf-2Bdf))\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{f}\sqrt{c+dx}}\right)}{d^{3/2}f^{3/2}}}{b^2}$$

input
```
Integrate[(A + B*x + C*x^2)/((a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

output
```
((b*C*Sqrt[c + d*x]*Sqrt[e + f*x])/(d*f) + (2*(A*b^2 + a*(-(b*B) + a*C))*A
rcTan[(Sqrt[b*c - a*d]*Sqrt[e + f*x])/(Sqrt[-(b*e) + a*f]*Sqrt[c + d*x])])
/(Sqrt[b*c - a*d]*Sqrt[-(b*e) + a*f]) - ((2*a*C*d*f + b*(C*d*e + c*C*f - 2
*B*d*f))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/(Sqrt[f]*Sqrt[c + d*x])])/(d^(3/2
)*f^(3/2)))/b^2
```

3.57.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2118, 27, 175, 66, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}dx$$

$$\Big\downarrow \text{2118}$$

$$\frac{\int \frac{b(2Abdf-aC(de+cf)-(2aCdf+b(Cde+cCf-2Bdf))x)}{2(a+bx)\sqrt{c+dx}\sqrt{e+fx}}dx}{b^2df}+\frac{C\sqrt{c+dx}\sqrt{e+fx}}{bdf}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{2Abdf-aC(de+cf)-(2aCdf+b(Cde+cCf-2Bdf))x}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}dx}{2bdf}+\frac{C\sqrt{c+dx}\sqrt{e+fx}}{bdf}$$

$$\Big\downarrow \text{175}$$

$$\frac{\frac{2df(Ab^2-a(bB-aC))\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}dx}{b}-\frac{(2aCdf+b(-2Bdf+cCf+Cde))\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}}dx}{b}}{2bdf}+\frac{C\sqrt{c+dx}\sqrt{e+fx}}{bdf}$$

$$\Big\downarrow \text{66}$$

$$\frac{\frac{2df(Ab^2-a(bB-aC))\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}dx}{b}-\frac{2(2aCdf+b(-2Bdf+cCf+Cde))\int \frac{1}{d-\frac{f(c+dx)}{e+fx}}d\frac{\sqrt{c+dx}}{\sqrt{e+fx}}}{b}}{2bdf}+\frac{C\sqrt{c+dx}\sqrt{e+fx}}{bdf}$$

$$\Big\downarrow \text{104}$$

$$\frac{\frac{4df(Ab^2-a(bB-aC))\int \frac{1}{-bc+ad+\frac{(be-af)(c+dx)}{e+fx}}d\frac{\sqrt{c+dx}}{\sqrt{e+fx}}}{b}-\frac{2(2aCdf+b(-2Bdf+cCf+Cde))\int \frac{1}{d-\frac{f(c+dx)}{e+fx}}d\frac{\sqrt{c+dx}}{\sqrt{e+fx}}}{b}}{2bdf}+\frac{C\sqrt{c+dx}\sqrt{e+fx}}{bdf}$$

$$\Big\downarrow \text{221}$$

$$\frac{-\frac{4df(Ab^2-a(bB-aC))\text{arctanh}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{bc-ad}}\right)}{b\sqrt{bc-ad}\sqrt{be-af}}-\frac{2\text{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)(2aCdf+b(-2Bdf+cCf+Cde))}{b\sqrt{d}\sqrt{f}}}{2bdf}+\frac{C\sqrt{c+dx}\sqrt{e+fx}}{bdf}$$

input
```
Int[(A + B*x + C*x^2)/((a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

output
```
(C*Sqrt[c + d*x]*Sqrt[e + f*x])/(b*d*f) + ((-2*(2*a*C*d*f + b*(C*d*e + c*C
*f - 2*B*d*f))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(
b*Sqrt[d]*Sqrt[f]) - (4*(A*b^2 - a*(b*B - a*C))*d*f*ArcTanh[(Sqrt[b*e - a*
f]*Sqrt[c + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x])])/(b*Sqrt[b*c - a*d]*Sqr
t[b*e - a*f]))/(2*b*d*f)
```

3.57.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

3.57.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 745 vs. $2(160) = 320$.

Time = 1.69 (sec) , antiderivative size = 746, normalized size of antiderivative = 3.97

method	result
default	$-\frac{\left(2A\sqrt{df}\,\ln\left(\frac{-2adfx+bcfx+bdex+2\sqrt{\frac{a^2df-acfb-abde+b^2ce}{b^2}}\,\sqrt{(dx+c)(fx+e)}\,b-acf-ade+2bce}{bx+a}\right)b^2df-2B\ln\left(\frac{2dfx+2\sqrt{(dx+c)(fx+e)}\,\sqrt{}}{2\sqrt{df}}\right.}{}$

input
```
int((C*x^2+B*x+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
-1/2*(2*A*(d*f)^(1/2)*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a
*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/
(b*x+a))*b^2*d*f-2*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)
+c*f+d*e)/(d*f)^(1/2))*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b^2*d
*f-2*B*(d*f)^(1/2)*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*
d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*
x+a))*a*b*d*f+2*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*
f+d*e)/(d*f)^(1/2))*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*a*b*d*f+
C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/
2))*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b^2*c*f+C*ln(1/2*(2*d*f*
x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*((a^2*d*f-a*
b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b^2*d*e+2*C*(d*f)^(1/2)*ln((-2*a*d*f*x+b
*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f
*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^2*d*f-2*C*((d*x+c)*(f*x+e))
^(1/2)*(d*f)^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b^2)*(d*x
+c)^(1/2)*(f*x+e)^(1/2)/((d*x+c)*(f*x+e))^(1/2)/d/(d*f)^(1/2)/b^3/((a^2*d*
f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)/f
```

3.57.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="
fricas")
```

output
```
Timed out
```

3.57.6 Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{A+Bx+Cx^2}{(a+bx)\,\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(b*x+a)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/((a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)), x)
```

3.57.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(((-(2*a*d*f)/b^2)>0)', see `assu
me?` for m
```

3.57.8 Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.57.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Hanged}$$

input
```
int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)*(c + d*x)^(1/2)),x)
```

output
```
\text{Hanged}
```

3.58 $\int \frac{A+Bx+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}\,dx$

3.58.1 Optimal result

Integrand size = 36, antiderivative size = 254

$$\int \frac{A+Bx+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}\,dx$$
$$= -\frac{(Ab^2-a(bB-aC))\sqrt{c+dx}\sqrt{e+fx}}{b(bc-ad)(be-af)(a+bx)} + \frac{2C\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)}{b^2\sqrt{d}\sqrt{f}}$$
$$+ \frac{(2a^3Cdf-3a^2bC(de+cf)-b^3(2Bce-Ade-Acf)+ab^2(4cCe+Bde+Bcf-2Adf))\operatorname{arctanh}\left(\frac{\sqrt{be}}{\sqrt{bc}}\right.}{b^2(bc-ad)^{3/2}(be-af)^{3/2}}$$

output
```
(2*a^3*C*d*f-3*a^2*b*C*(c*f+d*e)-b^3*(-A*c*f-A*d*e+2*B*c*e)+a*b^2*(-2*A*d*
f+B*c*f+B*d*e+4*C*c*e))*arctanh((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^
(1/2)/(f*x+e)^(1/2))/b^2/(-a*d+b*c)^(3/2)/(-a*f+b*e)^(3/2)+2*C*arctanh(f^(
1/2)*(d*x+c)^(1/2)/d^(1/2)/(f*x+e)^(1/2))/b^2/d^(1/2)/f^(1/2)-(A*b^2-a*(B*
b-C*a))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/(-a*d+b*c)/(-a*f+b*e)/(b*x+a)
```

3.58.2 Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}\,dx =$$
$$-\frac{\frac{b(Ab^2+a(-bB+aC))\sqrt{c+dx}\sqrt{e+fx}}{(bc-ad)(be-af)(a+bx)}+\frac{(-2a^3Cdf+3a^2bC(de+cf)-ab^2(4cCe+Bde+Bcf-2Adf)+b^3(2Bce-A(de+cf)))\arctan\left(\frac{\sqrt{bc-ad}\cdots}{\sqrt{-be+af}\cdots}\right)}{(bc-ad)^{3/2}(-be+af)^{3/2}}\cdots}{b^2}$$

input
```
Integrate[(A + B*x + C*x^2)/((a + b*x)^2*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

output
```
-(((b*(A*b^2 + a*(-(b*B) + a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/((b*c - a*d)
*(b*e - a*f)*(a + b*x)) + ((-2*a^3*C*d*f + 3*a^2*b*C*(d*e + c*f) - a*b^2*(
4*c*C*e + B*d*e + B*c*f - 2*A*d*f) + b^3*(2*B*c*e - A*(d*e + c*f)))*ArcTan
[(Sqrt[b*c - a*d]*Sqrt[e + f*x])/(Sqrt[-(b*e) + a*f]*Sqrt[c + d*x])])/((b*
c - a*d)^(3/2)*(-(b*e) + a*f)^(3/2)) - (2*C*ArcTanh[(Sqrt[d]*Sqrt[e + f*x]
)/(Sqrt[f]*Sqrt[c + d*x])])/(Sqrt[d]*Sqrt[f]))/b^2)
```

3.58.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.18, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2116, 27, 175, 66, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

$$\Big\downarrow \ 2116$$

$$-\frac{\int -\frac{C(de+cf)a^2-b(2cCe+Bde+Bcf-2Adf)a+b^2(2Bce-Ade-Acf)+2C(bc-ad)(be-af)x}{2b(a+bx)\sqrt{c+dx}\sqrt{e+fx}}dx}{(bc-ad)(be-af)}-\frac{\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b(a+bx)(bc-ad)(be-af)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{C(de+cf)a^2-b(2cCe+Bde+Bcf-2Adf)a+b^2(2Bce-A(de+cf))+2C(bc-ad)(be-af)x}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}dx}{2b(bc-ad)(be-af)} - \frac{\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b(a+bx)(bc-ad)(be-af)}$$

$\downarrow$ 175

$$\frac{\frac{2C(bc-ad)(be-af)\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}}dx}{b} - \frac{\left(2a^3Cdf-3a^2bC(cf+de)+ab^2(-2Adf+Bcf+Bde+4cCe)-b^3(-Acf-Ade+2Bce)\right)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+}}}{b}}{2b(bc-ad)(be-af)} - \frac{\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b(a+bx)(bc-ad)(be-af)}$$

$\downarrow$ 66

$$\frac{\frac{4C(bc-ad)(be-af)\int \frac{1}{d-\frac{f(c+dx)}{e+fx}}d\frac{\sqrt{c+dx}}{\sqrt{e+fx}}}{b} - \frac{\left(2a^3Cdf-3a^2bC(cf+de)+ab^2(-2Adf+Bcf+Bde+4cCe)-b^3(-Acf-Ade+2Bce)\right)\int \frac{1}{(a+bx)\sqrt{c+dx}}}{b}}{2b(bc-ad)(be-af)} - \frac{\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b(a+bx)(bc-ad)(be-af)}$$

$\downarrow$ 104

$$\frac{\frac{4C(bc-ad)(be-af)\int \frac{1}{d-\frac{f(c+dx)}{e+fx}}d\frac{\sqrt{c+dx}}{\sqrt{e+fx}}}{b} - \frac{2\left(2a^3Cdf-3a^2bC(cf+de)+ab^2(-2Adf+Bcf+Bde+4cCe)-b^3(-Acf-Ade+2Bce)\right)\int \frac{1}{-bc+ad+\frac{(be}{}}}{b}}{2b(bc-ad)(be-af)} - \frac{\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b(a+bx)(bc-ad)(be-af)}$$

$\downarrow$ 221

$$\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{bc-ad}}\right)\left(2a^3Cdf-3a^2bC(cf+de)+ab^2(-2Adf+Bcf+Bde+4cCe)-b^3(-Acf-Ade+2Bce)\right)}{b\sqrt{bc-ad}\sqrt{be-af}} + \frac{4C(bc-ad)(be-af)\text{arctanh}}{b\sqrt{d}\sqrt{f}}}{2b(bc-ad)(be-af)} - \frac{\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b(a+bx)(bc-ad)(be-af)}$$

input

```
Int[(A + B*x + C*x^2)/((a + b*x)^2*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

3.58. $\int \frac{A+Bx+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}\,dx$

output
```
-(((A*b^2 - a*(b*B - a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/(b*(b*c - a*d)*(b*
e - a*f)*(a + b*x))) + ((4*C*(b*c - a*d)*(b*e - a*f)*ArcTanh[(Sqrt[f]*Sqrt
[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(b*Sqrt[d]*Sqrt[f]) + (2*(2*a^3*C*d*f
 - 3*a^2*b*C*(d*e + c*f) - b^3*(2*B*c*e - A*d*e - A*c*f) + a*b^2*(4*c*C*e
+ B*d*e + B*c*f - 2*A*d*f))*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[
b*c - a*d]*Sqrt[e + f*x])])/(b*Sqrt[b*c - a*d]*Sqrt[b*e - a*f]))/(2*b*(b*c
 - a*d)*(b*e - a*f))
```

3.58.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2116
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Si
mp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*
(e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1
) - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x]
, x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && ILtQ[m,
-1]
```

3.58.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2972 vs. 2(226) = 452.

Time = 1.69 (sec) , antiderivative size = 2973, normalized size of antiderivative = 11.70

method	result	size
default	Expression too large to display	2973

input
```
int((C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
-1/2*(f*x+e)^(1/2)*(d*x+c)^(1/2)*(-2*B*a*b^3*((d*x+c)*(f*x+e))^(1/2)*(d*f)
^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+2*C*a^2*b^2*((d*x+c)*
(f*x+e))^(1/2)*(d*f)^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-A
*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^
(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*b^4*c*f*x*(d
*f)^(1/2)-A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2
*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*b
^4*d*e*x*(d*f)^(1/2)+2*B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*
f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*
e)/(b*x+a))*b^4*c*e*x*(d*f)^(1/2)-2*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^
(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^4*c*e*x*((a^2*d*f-a*b*c*f-a*b*d*
e+b^2*c*e)/b^2)^(1/2)+2*A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c
*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c
*e)/(b*x+a))*a^2*b^2*d*f*(d*f)^(1/2)-B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((
a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*
f-a*d*e+2*b*c*e)/(b*x+a))*a^2*b^2*d*e*(d*f)^(1/2)+2*B*ln((-2*a*d*f*x+b*c*f
*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e
))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a*b^3*c*e*(d*f)^(1/2)-2*C*ln(1/2*
(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^3*b
*d*f*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-2*C*ln((-2*a*d*f*x+b...
```

3.58.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm
="fricas")
```

output
```
Timed out
```

3.58.6 Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}\,dx=\int \frac{A+Bx+Cx^2}{(a+bx)^2\,\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(b*x+a)**2/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/((a + b*x)**2*sqrt(c + d*x)*sqrt(e + f*x)), x)
```

3.58.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(((-(2*a*d*f)/b^2)>0)', see `assu
me?` for m
```

3.58.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1319 vs. $2(225) = 450$.

Time = 1.10 (sec) , antiderivative size = 1319, normalized size of antiderivative = 5.19

$$\int \frac{A+Bx+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}\,dx=\text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm
="giac")
```

output
```
(4*sqrt(d*f)*C*a*b^2*c*d^2*e - 2*sqrt(d*f)*B*b^3*c*d^2*e - 3*sqrt(d*f)*C*a
^2*b*d^3*e + sqrt(d*f)*B*a*b^2*d^3*e + sqrt(d*f)*A*b^3*d^3*e - 3*sqrt(d*f)
*C*a^2*b*c*d^2*f + sqrt(d*f)*B*a*b^2*c*d^2*f + sqrt(d*f)*A*b^3*c*d^2*f + 2
*sqrt(d*f)*C*a^3*d^3*f - 2*sqrt(d*f)*A*a*b^2*d^3*f)*arctan(-1/2*(b*d^2*e +
 b*c*d*f - 2*a*d^2*f - (sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d
*f - c*d*f))^2*b)/(sqrt(-b^2*c*d*e*f + a*b*d^2*e*f + a*b*c*d*f^2 - a^2*d^2
*f^2)*d))/((b^4*c*e*abs(d) - a*b^3*d*e*abs(d) - a*b^3*c*f*abs(d) + a^2*b^2
*d*f*abs(d))*sqrt(-b^2*c*d*e*f + a*b*d^2*e*f + a*b*c*d*f^2 - a^2*d^2*f^2)*
d) - 2*(sqrt(d*f)*C*a^2*b*d^5*e^2 - sqrt(d*f)*B*a*b^2*d^5*e^2 + sqrt(d*f)*
A*b^3*d^5*e^2 - 2*sqrt(d*f)*C*a^2*b*c*d^4*e*f + 2*sqrt(d*f)*B*a*b^2*c*d^4*
e*f - 2*sqrt(d*f)*A*b^3*c*d^4*e*f + sqrt(d*f)*C*a^2*b*c^2*d^3*f^2 - sqrt(d
*f)*B*a*b^2*c^2*d^3*f^2 + sqrt(d*f)*A*b^3*c^2*d^3*f^2 - sqrt(d*f)*(sqrt(d*
f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^2*b*d^3*e +
sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^
2*B*a*b^2*d^3*e - sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x +
 c)*d*f - c*d*f))^2*A*b^3*d^3*e - sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqr
t(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^2*b*c*d^2*f + sqrt(d*f)*(sqrt(d*f)
*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a*b^2*c*d^2*f -
sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^
2*A*b^3*c*d^2*f + 2*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (...
```

3.58.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Hanged}$$

input
```
int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)^2*(c + d*x)^(1/2)),x)
```

output
```
\text{Hanged}
```

3.59 $\int \frac{A+Bx+Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}\,dx$

3.59.1 Optimal result

Integrand size = 36, antiderivative size = 424

$$\int \frac{A+Bx+Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}\,dx = -\frac{(Ab^2-a(bB-aC))\sqrt{c+dx}\sqrt{e+fx}}{2b(bc-ad)(be-af)(a+bx)^2}$$
$$+\frac{(2a^3Cdf+ab^2(8cCe+Bde+Bcf-6Adf)-b^3(4Bce-3A(de+cf))+a^2b(2Bdf-5C(de+cf)))\sqrt{}}{4b(bc-ad)^2(be-af)^2(a+bx)}$$
$$-\frac{(b^2(3Ad^2e^2-2cde(2Be-Af)+c^2(8Ce^2-4Bef+3Af^2))+ab(d^2e(Be-8Af)-c^2f(8Ce-Bf)-}{4(bc-}$$

output

```
-1/4*(b^2*(3*A*d^2*e^2-2*c*d*e*(-A*f+2*B*e)+c^2*(3*A*f^2-4*B*e*f+8*C*e^2))
+a*b*(d^2*e*(-8*A*f+B*e)-c^2*f*(-B*f+8*C*e)-2*c*d*(4*A*f^2-7*B*e*f+4*C*e^2
))+a^2*(C*(3*c^2*f^2+2*c*d*e*f+3*d^2*e^2)+4*d*f*(2*A*d*f-B*(c*f+d*e))))*ar
ctanh((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2)/(f*x+e)^(1/2))/(-a*d
+b*c)^(5/2)/(-a*f+b*e)^(5/2)-1/2*(A*b^2-a*(B*b-C*a))*(d*x+c)^(1/2)*(f*x+e)
^(1/2)/b/(-a*d+b*c)/(-a*f+b*e)/(b*x+a)^2+1/4*(2*a^3*C*d*f+a*b^2*(-6*A*d*f+
B*c*f+B*d*e+8*C*c*e)-b^3*(4*B*c*e-3*A*(c*f+d*e))+a^2*b*(2*B*d*f-5*C*(c*f+d
*e)))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/(-a*d+b*c)^2/(-a*f+b*e)^2/(b*x+a)
```

3.59.2 Mathematica [A] (verified)

Time = 2.40 (sec) , antiderivative size = 420, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx+Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}\,dx$$
$$=\frac{1}{4}\left(-\frac{\sqrt{c+dx}\sqrt{e+fx}(4b^3Bcex-ab^2(8cCex+B(-2ce+dex+cfx))+a^2b(5Cdex+Bd(e-2fx)+c}{}\right.$$
$$+\frac{(b^2(3Ad^2e^2+2cde(-2Be+Af)+c^2(8Ce^2-4Bef+3Af^2))+ab(d^2e(Be-8Af)+c^2f(-8Ce+Bf}{(bc-}$$

input
```
Integrate[(A + B*x + C*x^2)/((a + b*x)^3*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

output
```
(-((Sqrt[c + d*x]*Sqrt[e + f*x]*(4*b^3*B*c*e*x - a*b^2*(8*c*C*e*x + B*(-2*
c*e + d*e*x + c*f*x)) + a^2*b*(5*C*d*e*x + B*d*(e - 2*f*x) + c*(-6*C*e + B
*f + 5*C*f*x)) + a^3*(-4*B*d*f + C*(3*d*e + 3*c*f - 2*d*f*x)) + A*b*(8*a^2
*d*f + b^2*(2*c*e - 3*d*e*x - 3*c*f*x) + a*b*(-5*d*e - 5*c*f + 6*d*f*x))))
/((b*c - a*d)^2*(b*e - a*f)^2*(a + b*x)^2)) + ((b^2*(3*A*d^2*e^2 + 2*c*d*e
*(-2*B*e + A*f) + c^2*(8*C*e^2 - 4*B*e*f + 3*A*f^2)) + a*b*(d^2*e*(B*e - 8
*A*f) + c^2*f*(-8*C*e + B*f) - 2*c*d*(4*C*e^2 - 7*B*e*f + 4*A*f^2)) + a^2*
(C*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d*f*(2*A*d*f - B*(d*e + c*f))))
*ArcTan[(Sqrt[b*c - a*d]*Sqrt[e + f*x])/(Sqrt[-(b*e) + a*f]*Sqrt[c + d*x])
])/((b*c - a*d)^(5/2)*(-(b*e) + a*f)^(5/2)))/4
```

3.59.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2116, 27, 168, 27, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}\,dx$$
$$\Big\downarrow \; 2116$$

$$-\frac{\int -\frac{C(de+cf)a^2-b(4cCe+Bde+Bcf-4Adf)a+b^2(4Bce-3A(de+cf))+2b\left(\frac{Cdfa^2}{b}-2Cdea-2cCfa+Bdfa+2bcCe-Abdf\right)x}{2b(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}dx}{2(bc-ad)(be-af)}-\frac{\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{2b(a+bx)^2(bc-ad)(be-af)}$$

$\downarrow$ 27

$$\frac{\int \frac{C(de+cf)a^2-b(4cCe+Bde+Bcf-4Adf)a+b^2(4Bce-3A(de+cf))+2b\left(\frac{Cdfa^2}{b}+Bdfa-2C(de+cf)a+b(2cCe-Adf)\right)x}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}dx}{4b(bc-ad)(be-af)}-\frac{\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{2b(a+bx)^2(bc-ad)(be-af)}$$

$\downarrow$ 168

$$\frac{\frac{\sqrt{c+dx}\sqrt{e+fx}(2a^3Cdf+a^2b(2Bdf-5C(cf+de))+ab^2(-6Adf+Bcf+Bde+8cCe)-b^3(4Bce-3A(cf+de)))}{(a+bx)(bc-ad)(be-af)}-\frac{\int -\frac{b\left(\left(C\left(3d^2e^2+2cdfe+3c^2f^2\right)+4d\right.\right.}{}}{}}{4b(bc-ad)(be}$$

$$\frac{\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{2b(a+bx)^2(bc-ad)(be-af)}$$

$\downarrow$ 27

$$\frac{\frac{b\left(a^2\left(4df(2Adf-B(cf+de))+C\left(3c^2f^2+2cdef+3d^2e^2\right)\right)+ab\left(-2cd\left(4Af^2-7Bef+4Ce^2\right)+d^2e(Be-8Af)+c^2(-f)(8Ce-Bf)\right)+b^2\left(c^2\left(3Af^2-4Be\right.\right.}{2(bc-ad)(be-af)}}{}$$

$$\frac{\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{2b(a+bx)^2(bc-ad)(be-af)}$$

$\downarrow$ 104

$$\frac{\frac{b\left(a^2\left(4df(2Adf-B(cf+de))+C\left(3c^2f^2+2cdef+3d^2e^2\right)\right)+ab\left(-2cd\left(4Af^2-7Bef+4Ce^2\right)+d^2e(Be-8Af)+c^2(-f)(8Ce-Bf)\right)+b^2\left(c^2\left(3Af^2-4Be\right.\right.}{(bc-ad)(be-af)}}{}$$

$$\frac{\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{2b(a+bx)^2(bc-ad)(be-af)}$$

$\downarrow$ 221

3.59. $\int \frac{A+Bx+Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}\,dx$

$$\frac{\sqrt{c+dx}\sqrt{e+fx}(2a^3Cdf+a^2b(2Bdf-5C(cf+de))+ab^2(-6Adf+Bcf+Bde+8cCe)-b^3(4Bce-3A(cf+de)))}{(a+bx)(bc-ad)(be-af)} - \frac{b\,\text{arctanh}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{bc-ad}}\right)(a^2(4$$

$$\frac{\sqrt{c+dx}\sqrt{e+fx}(Ab^2-a(bB-aC))}{2b(a+bx)^2(bc-ad)(be-af)}$$

input
```
Int[(A + B*x + C*x^2)/((a + b*x)^3*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

output
```
-1/2*((A*b^2 - a*(b*B - a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/(b*(b*c - a*d)*
(b*e - a*f)*(a + b*x)^2) + (((2*a^3*C*d*f + a*b^2*(8*c*C*e + B*d*e + B*c*f
 - 6*A*d*f) - b^3*(4*B*c*e - 3*A*(d*e + c*f)) + a^2*b*(2*B*d*f - 5*C*(d*e
+ c*f)))*Sqrt[c + d*x]*Sqrt[e + f*x])/((b*c - a*d)*(b*e - a*f)*(a + b*x))
- (b*(b^2*(3*A*d^2*e^2 - 2*c*d*e*(2*B*e - A*f) + c^2*(8*C*e^2 - 4*B*e*f +
3*A*f^2)) + a*b*(d^2*e*(B*e - 8*A*f) - c^2*f*(8*C*e - B*f) - 2*c*d*(4*C*e^
2 - 7*B*e*f + 4*A*f^2)) + a^2*(C*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d
*f*(2*A*d*f - B*(d*e + c*f))))*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sq
rt[b*c - a*d]*Sqrt[e + f*x])])/((b*c - a*d)^(3/2)*(b*e - a*f)^(3/2)))/(4*b
*(b*c - a*d)*(b*e - a*f))
```

3.59.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2116
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Si
mp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*
(e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1
) - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x]
, x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && ILtQ[m,
-1]
```

3.59.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 7118 vs. 2(398) = 796.

Time = 1.69 (sec) , antiderivative size = 7119, normalized size of antiderivative = 16.79

method	result	size
default	Expression too large to display	7119

input
```
int((C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
result too large to display
```

3.59.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1998 vs. 2(397) = 794.

Time = 164.61 (sec) , antiderivative size = 4058, normalized size of antiderivative = 9.57

$$\int \frac{A+Bx+Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}\,dx=\text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm
="fricas")
```

output
```
[1/16*(((8*C*a^2*b^2*c^2 - 4*(2*C*a^3*b + B*a^2*b^2)*c*d + (3*C*a^4 + B*a^
3*b + 3*A*a^2*b^2)*d^2)*e^2 - 2*(2*(2*C*a^3*b + B*a^2*b^2)*c^2 - (C*a^4 +
7*B*a^3*b + A*a^2*b^2)*c*d + 2*(B*a^4 + 2*A*a^3*b)*d^2)*e*f + (8*A*a^4*d^2
 + (3*C*a^4 + B*a^3*b + 3*A*a^2*b^2)*c^2 - 4*(B*a^4 + 2*A*a^3*b)*c*d)*f^2
+ ((8*C*b^4*c^2 - 4*(2*C*a*b^3 + B*b^4)*c*d + (3*C*a^2*b^2 + B*a*b^3 + 3*A
*b^4)*d^2)*e^2 - 2*(2*(2*C*a*b^3 + B*b^4)*c^2 - (C*a^2*b^2 + 7*B*a*b^3 + A
*b^4)*c*d + 2*(B*a^2*b^2 + 2*A*a*b^3)*d^2)*e*f + (8*A*a^2*b^2*d^2 + (3*C*a
^2*b^2 + B*a*b^3 + 3*A*b^4)*c^2 - 4*(B*a^2*b^2 + 2*A*a*b^3)*c*d)*f^2)*x^2
+ 2*((8*C*a*b^3*c^2 - 4*(2*C*a^2*b^2 + B*a*b^3)*c*d + (3*C*a^3*b + B*a^2*b
^2 + 3*A*a*b^3)*d^2)*e^2 - 2*(2*(2*C*a^2*b^2 + B*a*b^3)*c^2 - (C*a^3*b + 7
*B*a^2*b^2 + A*a*b^3)*c*d + 2*(B*a^3*b + 2*A*a^2*b^2)*d^2)*e*f + (8*A*a^3*
b*d^2 + (3*C*a^3*b + B*a^2*b^2 + 3*A*a*b^3)*c^2 - 4*(B*a^3*b + 2*A*a^2*b^2
)*c*d)*f^2)*x)*sqrt((b^2*c - a*b*d)*e - (a*b*c - a^2*d)*f)*log((a^2*c^2*f^
2 + (8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*e^2 - 2*(4*a*b*c^2 - 3*a^2*c*d)*e*f
+ (b^2*d^2*e^2 + 2*(3*b^2*c*d - 4*a*b*d^2)*e*f + (b^2*c^2 - 8*a*b*c*d + 8*
a^2*d^2)*f^2)*x^2 + 4*(a*c*f - (2*b*c - a*d)*e - (b*d*e + (b*c - 2*a*d)*f)
*x)*sqrt((b^2*c - a*b*d)*e - (a*b*c - a^2*d)*f)*sqrt(d*x + c)*sqrt(f*x + e
) + 2*((4*b^2*c*d - 3*a*b*d^2)*e^2 + 2*(2*b^2*c^2 - 5*a*b*c*d + 2*a^2*d^2)
*e*f - (3*a*b*c^2 - 4*a^2*c*d)*f^2)*x)/(b^2*x^2 + 2*a*b*x + a^2)) + 4*((2*
(3*C*a^2*b^3 - B*a*b^4 - A*b^5)*c^2 - (9*C*a^3*b^2 - B*a^2*b^3 - 7*A*a*...
```

3.59.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/(b*x+a)**3/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

output
```
Timed out
```

3.59.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A + Bx + Cx^2}{(a + bx)^3 \sqrt{c + dx} \sqrt{e + fx}} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((a*d-b*c)>0)', see `assume?` for
 more deta
```

3.59.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7939 vs. 2(397) = 794.

Time = 38.67 (sec) , antiderivative size = 7939, normalized size of antiderivative = 18.72

$$\int \frac{A + Bx + Cx^2}{(a + bx)^3 \sqrt{c + dx} \sqrt{e + fx}} \, dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm
="giac")
```

output
```
-1/4*(8*sqrt(d*f)*C*b^2*c^2*d^2*e^2 - 8*sqrt(d*f)*C*a*b*c*d^3*e^2 - 4*sqrt
(d*f)*B*b^2*c*d^3*e^2 + 3*sqrt(d*f)*C*a^2*d^4*e^2 + sqrt(d*f)*B*a*b*d^4*e^
2 + 3*sqrt(d*f)*A*b^2*d^4*e^2 - 8*sqrt(d*f)*C*a*b*c^2*d^2*e*f - 4*sqrt(d*f
)*B*b^2*c^2*d^2*e*f + 2*sqrt(d*f)*C*a^2*c*d^3*e*f + 14*sqrt(d*f)*B*a*b*c*d
^3*e*f + 2*sqrt(d*f)*A*b^2*c*d^3*e*f - 4*sqrt(d*f)*B*a^2*d^4*e*f - 8*sqrt(
d*f)*A*a*b*d^4*e*f + 3*sqrt(d*f)*C*a^2*c^2*d^2*f^2 + sqrt(d*f)*B*a*b*c^2*d
^2*f^2 + 3*sqrt(d*f)*A*b^2*c^2*d^2*f^2 - 4*sqrt(d*f)*B*a^2*c*d^3*f^2 - 8*s
qrt(d*f)*A*a*b*c*d^3*f^2 + 8*sqrt(d*f)*A*a^2*d^4*f^2)*arctan(-1/2*(b*d^2*e
 + b*c*d*f - 2*a*d^2*f - (sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)
*d*f - c*d*f))^2*b)/(sqrt(-b^2*c*d*e*f + a*b*d^2*e*f + a*b*c*d*f^2 - a^2*d
^2*f^2)*d))/((b^4*c^2*e^2*abs(d) - 2*a*b^3*c*d*e^2*abs(d) + a^2*b^2*d^2*e^
2*abs(d) - 2*a*b^3*c^2*e*f*abs(d) + 4*a^2*b^2*c*d*e*f*abs(d) - 2*a^3*b*d^2
*e*f*abs(d) + a^2*b^2*c^2*f^2*abs(d) - 2*a^3*b*c*d*f^2*abs(d) + a^4*d^2*f^
2*abs(d))*sqrt(-b^2*c*d*e*f + a*b*d^2*e*f + a*b*c*d*f^2 - a^2*d^2*f^2)*d)
+ 1/2*(8*sqrt(d*f)*C*a*b^4*c*d^9*e^5 - 4*sqrt(d*f)*B*b^5*c*d^9*e^5 - 5*sqr
t(d*f)*C*a^2*b^3*d^10*e^5 + sqrt(d*f)*B*a*b^4*d^10*e^5 + 3*sqrt(d*f)*A*b^5
*d^10*e^5 - 32*sqrt(d*f)*C*a*b^4*c^2*d^8*e^4*f + 16*sqrt(d*f)*B*b^5*c^2*d^
8*e^4*f + 15*sqrt(d*f)*C*a^2*b^3*c*d^9*e^4*f - 3*sqrt(d*f)*B*a*b^4*c*d^9*e
^4*f - 9*sqrt(d*f)*A*b^5*c*d^9*e^4*f + 2*sqrt(d*f)*C*a^3*b^2*d^10*e^4*f +
2*sqrt(d*f)*B*a^2*b^3*d^10*e^4*f - 6*sqrt(d*f)*A*a*b^4*d^10*e^4*f + 48*...
```

3.59.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Hanged}$$

input
```
int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)^3*(c + d*x)^(1/2)),x)
```

output
```
\text{Hanged}
```

3.60 $\int \frac{A+Bx+Cx^2}{(a+bx)^4\sqrt{c+dx}\sqrt{e+fx}}\,dx$

3.60.1 Optimal result

Integrand size = 36, antiderivative size = 826

$$\int \frac{A+Bx+Cx^2}{(a+bx)^4\sqrt{c+dx}\sqrt{e+fx}}\,dx = -\frac{(Ab^2-a(bB-aC))\sqrt{c+dx}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^3}$$
$$+\frac{(2a^3Cdf+ab^2(12cCe+Bde+Bcf-10Adf)-b^3(6Bce-5A(de+cf))+a^2b(4Bdf-7C(de+cf))}{12b(bc-ad)^2(be-af)^2(a+bx)^2}$$
$$+\frac{(4a^4Cd^2f^2+8a^3bdf(Bdf-2C(de+cf))-b^4(15Ad^2e^2-2cde(9Be-7Af)+3c^2(8Ce^2-6Bef+5A}{}$$
$$+\frac{(b^3(5Ad^3e^3-3cd^2e^2(2Be-Af)+c^2de(8Ce^2-4Bef+3Af^2)+c^3f(8Ce^2-6Bef+5Af^2))+ab^2(d}{}$$

output
```
1/8*(b^3*(5*A*d^3*e^3-3*c*d^2*e^2*(-A*f+2*B*e)+c^2*d*e*(3*A*f^2-4*B*e*f+8*
C*e^2)+c^3*f*(5*A*f^2-6*B*e*f+8*C*e^2))+a*b^2*(d^3*e^2*(-18*A*f+B*e)-c^3*f
^2*(-B*f+4*C*e)-c*d^2*e*(12*A*f^2-23*B*e*f+4*C*e^2)-c^2*d*f*(18*A*f^2-23*B
*e*f+40*C*e^2))-2*a^3*d*f*(C*(3*c^2*f^2+2*c*d*e*f+3*d^2*e^2)+4*d*f*(2*A*d*
f-B*(c*f+d*e)))+a^2*b*(C*(c^3*f^3+23*c^2*d*e*f^2+23*c*d^2*e^2*f+d^3*e^3)+4
*d*f*(6*A*d*f*(c*f+d*e)-B*(c^2*f^2+10*c*d*e*f+d^2*e^2))))*arctanh((-a*f+b*
e)^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2)/(f*x+e)^(1/2))/(-a*d+b*c)^(7/2)/(-
a*f+b*e)^(7/2)-1/3*(A*b^2-a*(B*b-C*a))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/(-a*d
+b*c)/(-a*f+b*e)/(b*x+a)^3+1/12*(2*a^3*C*d*f+a*b^2*(-10*A*d*f+B*c*f+B*d*e+
12*C*c*e)-b^3*(6*B*c*e-5*A*(c*f+d*e))+a^2*b*(4*B*d*f-7*C*(c*f+d*e)))*(d*x+
c)^(1/2)*(f*x+e)^(1/2)/b/(-a*d+b*c)^2/(-a*f+b*e)^2/(b*x+a)^2+1/24*(4*a^4*C
*d^2*f^2+8*a^3*b*d*f*(B*d*f-2*C*(c*f+d*e))-b^4*(15*A*d^2*e^2-2*c*d*e*(-7*A
*f+9*B*e)+3*c^2*(5*A*f^2-6*B*e*f+8*C*e^2))-a*b^3*(d^2*e*(-44*A*f+3*B*e)-3*
c^2*f*(-B*f+4*C*e)-2*c*d*(22*A*f^2-29*B*e*f+6*C*e^2))-a^2*b^2*(C*(3*c^2*f^
2-34*c*d*e*f+3*d^2*e^2)+2*d*f*(22*A*d*f-5*B*(c*f+d*e))))*(d*x+c)^(1/2)*(f*
x+e)^(1/2)/b/(-a*d+b*c)^3/(-a*f+b*e)^3/(b*x+a)
```

3.60.2 Mathematica [A] (verified)

Time = 6.96 (sec) , antiderivative size = 1036, normalized size of antiderivative = 1.25

$$\int \frac{A+Bx+Cx^2}{(a+bx)^4\sqrt{c+dx}\sqrt{e+fx}}\,dx =$$

$$-\frac{\sqrt{c+dx}\sqrt{e+fx}(6b^5cex(4cCex+B(2ce-3dex-3cfx))+6a^5df(-4Bdf+C(3de+3cf-2dfx))+}{}$$

$$+\frac{(ab^2(d^3e^2(Be-18Af)+c^3f^2(-4Ce+Bf)+c^2df(-40Ce^2+23Bef-18Af^2)+cd^2e(-4Ce^2+23B}{}$$

input
```
Integrate[(A + B*x + C*x^2)/((a + b*x)^4*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

output
```
-1/24*(Sqrt[c + d*x]*Sqrt[e + f*x]*(6*b^5*c*e*x*(4*c*C*e*x + B*(2*c*e - 3*
d*e*x - 3*c*f*x)) + 6*a^5*d*f*(-4*B*d*f + C*(3*d*e + 3*c*f - 2*d*f*x)) + a
*b^4*(-12*c*C*e*x*(-2*c*e + d*e*x + c*f*x) + B*(3*d^2*e^2*x^2 + 2*c*d*e*x*
(-25*e + 29*f*x) + c^2*(4*e^2 - 50*e*f*x + 3*f^2*x^2))) + a^4*b*(12*B*d*f*
(c*f + d*(e - 2*f*x)) - C*(3*c^2*f^2 + 2*c*d*f*(29*e - 25*f*x) + d^2*(3*e^
2 - 50*e*f*x + 4*f^2*x^2))) + a^2*b^3*(d^2*e*x*(8*B*e + 3*C*e*x - 10*B*f*x
) + c^2*(8*B*f*(-2*e + f*x) + C*(8*e^2 + 14*e*f*x + 3*f^2*x^2)) - 2*c*d*(C
*e*x*(-7*e + 17*f*x) + B*(8*e^2 - 62*e*f*x + 5*f^2*x^2))) + a^3*b^2*(c^2*f
*(10*C*e - 3*B*f - 8*C*f*x) + 2*c*d*(B*f*(17*e - 7*f*x) + C*(5*e^2 - 62*e*
f*x + 8*f^2*x^2)) - d^2*(8*C*e*x*(e - 2*f*x) + B*(3*e^2 + 14*e*f*x + 8*f^2
*x^2))) + A*b*(72*a^4*d^2*f^2 + 18*a^3*b*d*f*(-5*d*e - 5*c*f + 6*d*f*x) +
b^4*(15*d^2*e^2*x^2 + 2*c*d*e*x*(-5*e + 7*f*x) + c^2*(8*e^2 - 10*e*f*x + 1
5*f^2*x^2)) - 2*a*b^3*(c^2*f*(13*e - 20*f*x) + 2*d^2*e*x*(-10*e + 11*f*x)
+ c*d*(13*e^2 - 34*e*f*x + 22*f^2*x^2)) + a^2*b^2*(33*c^2*f^2 + 2*c*d*f*(4
3*e - 59*f*x) + d^2*(33*e^2 - 118*e*f*x + 44*f^2*x^2)))))/((b*c - a*d)^3*(
b*e - a*f)^3*(a + b*x)^3) + ((a*b^2*(d^3*e^2*(B*e - 18*A*f) + c^3*f^2*(-4*
C*e + B*f) + c^2*d*f*(-40*C*e^2 + 23*B*e*f - 18*A*f^2) + c*d^2*e*(-4*C*e^2
 + 23*B*e*f - 12*A*f^2)) + b^3*(5*A*d^3*e^3 + 3*c*d^2*e^2*(-2*B*e + A*f) +
 c^2*d*e*(8*C*e^2 - 4*B*e*f + 3*A*f^2) + c^3*f*(8*C*e^2 - 6*B*e*f + 5*A*f^
2)) - 2*a^3*d*f*(C*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d*f*(2*A*d*f...
```

3.60.3 Rubi [A] (verified)

Time = 1.65 (sec) , antiderivative size = 869, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2116, 27, 168, 27, 168, 27, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^4\sqrt{c+dx}\sqrt{e+fx}}dx$$

$$\Big\downarrow \text{2116}$$

$$-\frac{\int -\frac{C(de+cf)a^2-b(6cCe+Bde+Bcf-6Adf)a+b^2(6Bce-5A(de+cf))+2b\left(\frac{Cdfa^2}{b}-3Cdea-3cCfa+2Bdfa+3bcCe-2Abdf\right)x}{2b(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}dx}{3(bc-ad)(be-af)}-\frac{\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^3(bc-ad)(be-af)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{C(de+cf)a^2-b(6cCe+Bde+Bcf-6Adf)a+b^2(6Bce-5A(de+cf))+2b\left(\frac{Cdfa^2}{b}+2Bdfa-3C(de+cf)a+b(3cCe-2Adf)\right)x}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}dx}{6b(bc-ad)(be-af)}-\frac{\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^3(bc-ad)(be-af)}$$

$\downarrow$ 168

$$\frac{\frac{\sqrt{c+dx}\sqrt{e+fx}(2a^3Cdf+a^2b(4Bdf-7C(cf+de))+ab^2(-10Adf+Bcf+Bde+12cCe)-b^3(6Bce-5A(cf+de)))}{2(a+bx)^2(bc-ad)(be-af)}-\frac{\int -\frac{2Cdf(de+cf)a^3-b\left(8df(Bde+\cdots\right.}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^3(bc-ad)(be-af)}$$

$\downarrow$ 27

$$\frac{\int \frac{2Cdf(de+cf)a^3+b\left(C\left(3d^2e^2-10cdfe+3c^2f^2\right)+8df(3Adf-B(de+cf))\right)a^2+b^2\left(-3f(4Ce-Bf)c^2-2d\left(6Ce^2-23Bfe+17Af^2\right)c+d^2e(3Be-34Af)\right)a+b^3\left(3\left(8Ce^2\cdots\right.\right.}{(a+bx)^2\sqrt{c+dx}\sqrt{e\cdots}}}{4(bc-ad)(be-a\cdots}$$

$$\frac{\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^3(bc-ad)(be-af)}$$

$\downarrow$ 168

$$\frac{\frac{\sqrt{c+dx}\sqrt{e+fx}(2Cdfa^3+b(4Bdf-7C(de+cf))a^2+b^2(12cCe+Bde+Bcf-10Adf)a-b^3(6Bce-5A(de+cf)))}{2(bc-ad)(be-af)(a+bx)^2}+\frac{\left(4Cd^2f^2a^4+8bdf(Bdf-2C(de+cf))\cdots\right.}{\cdots}}{\cdots}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^3}$$

$\downarrow$ 27

$$\frac{\frac{\sqrt{c+dx}\sqrt{e+fx}(2Cdfa^3+b(4Bdf-7C(de+cf))a^2+b^2(12cCe+Bde+Bcf-10Adf)a-b^3(6Bce-5A(de+cf)))}{2(bc-ad)(be-af)(a+bx)^2}+\frac{\left(4Cd^2f^2a^4+8bdf(Bdf-2C(de+cf))\cdots\right.}{\cdots}}{\cdots}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^3}$$

$\downarrow$ 104

3.60. $\int \frac{A+Bx+Cx^2}{(a+bx)^4\sqrt{c+dx}\sqrt{e+fx}}\,dx$

$$\frac{\frac{\sqrt{c+dx}\sqrt{e+fx}(2Cdfa^3+b(4Bdf-7C(de+cf))a^2+b^2(12cCe+Bde+Bcf-10Adf)a-b^3(6Bce-5A(de+cf)))}{2(bc-ad)(be-af)(a+bx)^2}+\frac{\left(4Cd^2f^2a^4+8bdf(Bdf-2C(de+cf))\right.}{}}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^3}$$

$\Big\downarrow$ 221

$$\frac{\frac{\sqrt{c+dx}\sqrt{e+fx}(2Cdfa^3+b(4Bdf-7C(de+cf))a^2+b^2(12cCe+Bde+Bcf-10Adf)a-b^3(6Bce-5A(de+cf)))}{2(bc-ad)(be-af)(a+bx)^2}+\frac{\sqrt{c+dx}\sqrt{e+fx}\left(4Cd^2f^2a^4+8bdf(Bdf\right.}{}}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^3}$$

input
```
Int[(A + B*x + C*x^2)/((a + b*x)^4*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

output
```
-1/3*((A*b^2 - a*(b*B - a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/(b*(b*c - a*d)*
(b*e - a*f)*(a + b*x)^3) + (((2*a^3*C*d*f + a*b^2*(12*c*C*e + B*d*e + B*c*
f - 10*A*d*f) - b^3*(6*B*c*e - 5*A*(d*e + c*f)) + a^2*b*(4*B*d*f - 7*C*(d*
e + c*f)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(2*(b*c - a*d)*(b*e - a*f)*(a + b*
x)^2) + (((4*a^4*C*d^2*f^2 + 8*a^3*b*d*f*(B*d*f - 2*C*(d*e + c*f)) - b^4*(
15*A*d^2*e^2 - 2*c*d*e*(9*B*e - 7*A*f) + 3*c^2*(8*C*e^2 - 6*B*e*f + 5*A*f^
2)) - a*b^3*(d^2*e*(3*B*e - 44*A*f) - 3*c^2*f*(4*C*e - B*f) - 2*c*d*(6*C*e
^2 - 29*B*e*f + 22*A*f^2)) - a^2*b^2*(C*(3*d^2*e^2 - 34*c*d*e*f + 3*c^2*f^
2) + 2*d*f*(22*A*d*f - 5*B*(d*e + c*f))))*Sqrt[c + d*x]*Sqrt[e + f*x])/((b
*c - a*d)*(b*e - a*f)*(a + b*x)) + (3*b*(b^3*(5*A*d^3*e^3 - 3*c*d^2*e^2*(2
*B*e - A*f) + c^2*d*e*(8*C*e^2 - 4*B*e*f + 3*A*f^2) + c^3*f*(8*C*e^2 - 6*B
*e*f + 5*A*f^2)) + a*b^2*(d^3*e^2*(B*e - 18*A*f) - c^3*f^2*(4*C*e - B*f) -
 c*d^2*e*(4*C*e^2 - 23*B*e*f + 12*A*f^2) - c^2*d*f*(40*C*e^2 - 23*B*e*f +
18*A*f^2)) - 2*a^3*d*f*(C*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d*f*(2*A
*d*f - B*(d*e + c*f))) + a^2*b*(C*(d^3*e^3 + 23*c*d^2*e^2*f + 23*c^2*d*e*f
^2 + c^3*f^3) + 4*d*f*(6*A*d*f*(d*e + c*f) - B*(d^2*e^2 + 10*c*d*e*f + c^2
*f^2))))*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[b*c - a*d]*Sqrt[e +
 f*x])])/((b*c - a*d)^(3/2)*(b*e - a*f)^(3/2)))/(4*(b*c - a*d)*(b*e - a*f)
))/(6*b*(b*c - a*d)*(b*e - a*f))
```

3.60. $\int \frac{A+Bx+Cx^2}{(a+bx)^4\sqrt{c+dx}\sqrt{e+fx}}\,dx$

3.60.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2116
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Si
mp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*
(e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1
) - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x]
, x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && ILtQ[m,
-1]
```

3.60.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 18801 vs. 2(794) = 1588.

Time = 1.70 (sec) , antiderivative size = 18802, normalized size of antiderivative = 22.76

method	result	size
default	Expression too large to display	18802

input
```
int((C*x^2+B*x+A)/(b*x+a)^4/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
result too large to display
```

3.60.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^4\sqrt{c+dx}\sqrt{e+fx}}\,dx=\text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^4/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm
="fricas")
```

output
```
Timed out
```

3.60.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^4\sqrt{c+dx}\sqrt{e+fx}}\,dx=\text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/(b*x+a)**4/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

output
```
Timed out
```

3.60.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^4\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^4/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((a*d-b*c)>0)', see `assume?` for
 more deta
```

3.60.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25632 vs. 2(796) = 1592.

Time = 58.08 (sec) , antiderivative size = 25632, normalized size of antiderivative = 31.03

$$\int \frac{A+Bx+Cx^2}{(a+bx)^4\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^4/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm
="giac")
```

output
```
1/8*(8*sqrt(d*f)*C*b^3*c^2*d^3*e^3 - 4*sqrt(d*f)*C*a*b^2*c*d^4*e^3 - 6*sqr
t(d*f)*B*b^3*c*d^4*e^3 + sqrt(d*f)*C*a^2*b*d^5*e^3 + sqrt(d*f)*B*a*b^2*d^5
*e^3 + 5*sqrt(d*f)*A*b^3*d^5*e^3 + 8*sqrt(d*f)*C*b^3*c^3*d^2*e^2*f - 40*sq
rt(d*f)*C*a*b^2*c^2*d^3*e^2*f - 4*sqrt(d*f)*B*b^3*c^2*d^3*e^2*f + 23*sqrt(
d*f)*C*a^2*b*c*d^4*e^2*f + 23*sqrt(d*f)*B*a*b^2*c*d^4*e^2*f + 3*sqrt(d*f)*
A*b^3*c*d^4*e^2*f - 6*sqrt(d*f)*C*a^3*d^5*e^2*f - 4*sqrt(d*f)*B*a^2*b*d^5*
e^2*f - 18*sqrt(d*f)*A*a*b^2*d^5*e^2*f - 4*sqrt(d*f)*C*a*b^2*c^3*d^2*e*f^2
 - 6*sqrt(d*f)*B*b^3*c^3*d^2*e*f^2 + 23*sqrt(d*f)*C*a^2*b*c^2*d^3*e*f^2 +
23*sqrt(d*f)*B*a*b^2*c^2*d^3*e*f^2 + 3*sqrt(d*f)*A*b^3*c^2*d^3*e*f^2 - 4*s
qrt(d*f)*C*a^3*c*d^4*e*f^2 - 40*sqrt(d*f)*B*a^2*b*c*d^4*e*f^2 - 12*sqrt(d*
f)*A*a*b^2*c*d^4*e*f^2 + 8*sqrt(d*f)*B*a^3*d^5*e*f^2 + 24*sqrt(d*f)*A*a^2*
b*d^5*e*f^2 + sqrt(d*f)*C*a^2*b*c^3*d^2*f^3 + sqrt(d*f)*B*a*b^2*c^3*d^2*f^
3 + 5*sqrt(d*f)*A*b^3*c^3*d^2*f^3 - 6*sqrt(d*f)*C*a^3*c^2*d^3*f^3 - 4*sqrt
(d*f)*B*a^2*b*c^2*d^3*f^3 - 18*sqrt(d*f)*A*a*b^2*c^2*d^3*f^3 + 8*sqrt(d*f)
*B*a^3*c*d^4*f^3 + 24*sqrt(d*f)*A*a^2*b*c*d^4*f^3 - 16*sqrt(d*f)*A*a^3*d^5
*f^3)*arctan(-1/2*(b*d^2*e + b*c*d*f - 2*a*d^2*f - (sqrt(d*f)*sqrt(d*x + c
) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*b)/(sqrt(-b^2*c*d*e*f + a*b*d^2
*e*f + a*b*c*d*f^2 - a^2*d^2*f^2)*d))/((b^6*c^3*e^3*abs(d) - 3*a*b^5*c^2*d
*e^3*abs(d) + 3*a^2*b^4*c*d^2*e^3*abs(d) - a^3*b^3*d^3*e^3*abs(d) - 3*a*b^
5*c^3*e^2*f*abs(d) + 9*a^2*b^4*c^2*d*e^2*f*abs(d) - 9*a^3*b^3*c*d^2*e^2...
```

3.60.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^4\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Hanged}$$

input
```
int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)^4*(c + d*x)^(1/2)),x)
```

output
```
\text{Hanged}
```

3.61 $\int \sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)\,dx$

3.61.1 Optimal result

Integrand size = 38, antiderivative size = 1182

$$\int \sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)\,dx$$

$$= \frac{2(8a^3Cd^3f^3+3a^2bd^2f^2(Cde-cCf-4Bdf)-3ab^2df^2((c^2C-7Ad^2)f+Bd(de-2cf))-b^3(C(16d^3e^3}{315b^3d^3}$$

$$-\frac{2(7bdf(bcCe+aCde+acCf-3Abdf)+(adf-4b(de+cf))(2aCdf-b(3Bdf-2C(de+cf))))\sqrt{a-}}{105b^2d^2f^3}$$

$$-\frac{2(2aCdf-b(3Bdf-2C(de+cf)))\sqrt{a+bx}(c+dx)^{3/2}(e+fx)^{3/2}}{21bd^2f^2}$$

$$+\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}(e+fx)^{3/2}}{9bdf}$$

$$-\frac{2\sqrt{-bc+ad}(16a^4Cd^4f^4-8a^3bd^3f^3(Cde+cCf+3Bdf)+3a^2b^2d^2f^2(df(5Bde+5Bcf+14Adf)-2}{}$$

$$-\frac{2\sqrt{-bc+ad}(be-af)(de-cf)\left(8a^3Cd^3f^3+3a^2bd^2f^2(Cde-cCf-4Bdf)-3ab^2df^2((c^2C-7Ad^2)f\right.}{}$$

output
```
2/9*C*(b*x+a)^(3/2)*(d*x+c)^(3/2)*(f*x+e)^(3/2)/b/d/f-2/21*(2*a*C*d*f-b*(3
*B*d*f-2*C*(c*f+d*e)))*(d*x+c)^(3/2)*(f*x+e)^(3/2)*(b*x+a)^(1/2)/b/d^2/f^2
-2/105*(7*b*d*f*(-3*A*b*d*f+C*a*c*f+C*a*d*e+C*b*c*e)+(a*d*f-4*b*(c*f+d*e))
*(2*a*C*d*f-b*(3*B*d*f-2*C*(c*f+d*e))))*(f*x+e)^(3/2)*(b*x+a)^(1/2)*(d*x+c
)^(1/2)/b^2/d^2/f^3+2/315*(8*a^3*C*d^3*f^3+3*a^2*b*d^2*f^2*(-4*B*d*f-C*c*f
+C*d*e)-3*a*b^2*d*f^2*((-7*A*d^2+C*c^2)*f+B*d*(-2*c*f+d*e))-b^3*(C*(-8*c^3
*f^3-3*c^2*d*e*f^2+16*d^3*e^3)+3*d*f*(7*A*d*f*(-c*f+2*d*e)-B*(-4*c^2*f^2-c
*d*e*f+8*d^2*e^2))))*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^3/d^3/f^3
-2/315*(16*a^4*C*d^4*f^4-8*a^3*b*d^3*f^3*(3*B*d*f+C*c*f+C*d*e)+3*a^2*b^2*d
^2*f^2*(d*f*(14*A*d*f+5*B*c*f+5*B*d*e)-2*C*(c^2*f^2-c*d*e*f+d^2*e^2))-a*b^
3*d*f*(C*(8*c^3*f^3-6*c^2*d*e*f^2-6*c*d^2*e^2*f+8*d^3*e^3)+3*d*f*(14*A*d*f
*(c*f+d*e)-B*(5*c^2*f^2-6*c*d*e*f+5*d^2*e^2)))+b^4*(2*C*(8*c^4*f^4-4*c^3*d
*e*f^3-3*c^2*d^2*e^2*f^2-4*c*d^3*e^3*f+8*d^4*e^4)+3*d*f*(14*A*d*f*(c^2*f^2
-c*d*e*f+d^2*e^2)-B*(8*c^3*f^3-5*c^2*d*e*f^2-5*c*d^2*e^2*f+8*d^3*e^3))))*E
llipticE(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))
^(1/2))*(a*d-b*c)^(1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(f*x+e)^(1/2)/b^4/d^(
7/2)/f^4/(d*x+c)^(1/2)/(b*(f*x+e)/(-a*f+b*e))^(1/2)-2/315*(-a*f+b*e)*(-c*f
+d*e)*(8*a^3*C*d^3*f^3+3*a^2*b*d^2*f^2*(-4*B*d*f-C*c*f+C*d*e)-3*a*b^2*d*f^
2*((-7*A*d^2+C*c^2)*f+B*d*(-2*c*f+d*e))-b^3*(C*(-8*c^3*f^3-3*c^2*d*e*f^2+1
6*d^3*e^3)+3*d*f*(7*A*d*f*(-c*f+2*d*e)-B*(-4*c^2*f^2-c*d*e*f+8*d^2*e^2)...
```

3.61.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 33.21 (sec) , antiderivative size = 1422, normalized size of antiderivative = 1.20

$$\int \sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx$$

$$= \frac{2\left(-b^2\sqrt{-a+\frac{bc}{d}}(16a^4Cd^4f^4-8a^3bd^3f^3(Cde+cCf+3Bdf)+3a^2b^2d^2f^2(df(5Bde+5Bcf+14Adf)-\right.}{}$$

input
```
Integrate[Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2),x]
```

output
```
(2*(-(b^2*Sqrt[-a + (b*c)/d]*(16*a^4*C*d^4*f^4 - 8*a^3*b*d^3*f^3*(C*d*e +
c*C*f + 3*B*d*f) + 3*a^2*b^2*d^2*f^2*(d*f*(5*B*d*e + 5*B*c*f + 14*A*d*f) -
 2*C*(d^2*e^2 - c*d*e*f + c^2*f^2)) + a*b^3*d*f*(C*(-8*d^3*e^3 + 6*c*d^2*e
^2*f + 6*c^2*d*e*f^2 - 8*c^3*f^3) - 3*d*f*(14*A*d*f*(d*e + c*f) + B*(-5*d^
2*e^2 + 6*c*d*e*f - 5*c^2*f^2))) + b^4*(2*C*(8*d^4*e^4 - 4*c*d^3*e^3*f - 3
*c^2*d^2*e^2*f^2 - 4*c^3*d*e*f^3 + 8*c^4*f^4) + 3*d*f*(14*A*d*f*(d^2*e^2 -
 c*d*e*f + c^2*f^2) + B*(-8*d^3*e^3 + 5*c*d^2*e^2*f + 5*c^2*d*e*f^2 - 8*c^
3*f^3))))*(c + d*x)*(e + f*x)) + b^2*Sqrt[-a + (b*c)/d]*d*f*(a + b*x)*(c +
 d*x)*(e + f*x)*(8*a^3*C*d^3*f^3 - 3*a^2*b*d^2*f^2*(c*C*f + 4*B*d*f + C*d*
(e + 2*f*x)) + a*b^2*d*f*(3*d*f*(7*A*d*f + B*(2*d*e + 2*c*f + 3*d*f*x)) +
C*(-3*c^2*f^2 + 2*c*d*f*(e + f*x) + d^2*(-3*e^2 + 2*e*f*x + 5*f^2*x^2))) +
 b^3*(C*(8*c^3*f^3 - 3*c^2*d*f^2*(e + 2*f*x) + c*d^2*f*(-3*e^2 + 2*e*f*x +
 5*f^2*x^2) + d^3*(8*e^3 - 6*e^2*f*x + 5*e*f^2*x^2 + 35*f^3*x^3)) + 3*d*f*
(7*A*d*f*(c*f + d*(e + 3*f*x)) + B*(-4*c^2*f^2 + c*d*f*(2*e + 3*f*x) + d^2
*(-4*e^2 + 3*e*f*x + 15*f^2*x^2))))) - I*(b*c - a*d)*f*(16*a^4*C*d^4*f^4 -
 8*a^3*b*d^3*f^3*(C*d*e + c*C*f + 3*B*d*f) + 3*a^2*b^2*d^2*f^2*(d*f*(5*B*d
*e + 5*B*c*f + 14*A*d*f) - 2*C*(d^2*e^2 - c*d*e*f + c^2*f^2)) + a*b^3*d*f*
(C*(-8*d^3*e^3 + 6*c*d^2*e^2*f + 6*c^2*d*e*f^2 - 8*c^3*f^3) - 3*d*f*(14*A*
d*f*(d*e + c*f) + B*(-5*d^2*e^2 + 6*c*d*e*f - 5*c^2*f^2))) + b^4*(2*C*(8*d
^4*e^4 - 4*c*d^3*e^3*f - 3*c^2*d^2*e^2*f^2 - 4*c^3*d*e*f^3 + 8*c^4*f^4)...
```

3.61.3 Rubi [A] (verified)

Time = 2.51 (sec) , antiderivative size = 1213, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {2118, 27, 171, 27, 171, 27, 171, 27, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{2118}$$

$$\frac{2\int -\frac{3}{2}b\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}(bcCe+aCde+acCf-3Abdf-(3bBdf-2aCdf-2bC(de+cf))x)dx}{9b^2df}+\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}(e+fx)^{3/2}}{9bdf}$$

$$\Big\downarrow \text{27}$$

$$\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}(e+fx)^{3/2}}{9bdf}-\frac{\int \sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}(bcCe+aCde+acCf-3Abdf-(3bBdf-2aCdf-2bC(de+cf))x)dx}{3bdf}$$

$\downarrow$ 171

$$\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}(e+fx)^{3/2}}{9bdf}-\frac{\frac{2\int \frac{\sqrt{c+dx}\sqrt{e+fx}(7adf(bcCe+aCde+acCf-3Abdf)+(bce+3a(de+cf))(3bBdf-2aCdf-2bC(de+cf))+(7bdf(bcCe+aCde+acCf-3Abdf)-(adf-4b(de+cf))(3bBdj}{2\sqrt{a+bx}}}{7df}}{3bdf}$$

$\downarrow$ 27

$$\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}(e+fx)^{3/2}}{9bdf}-\frac{\frac{\int \frac{\sqrt{c+dx}\sqrt{e+fx}(7adf(bcCe+aCde+acCf-3Abdf)+(bce+3a(de+cf))(3bBdf-2aCdf-2bC(de+cf))+(7bdf(bcCe+aCde+acCf-3Abdf)-(adf-4b(de+cf))(3bBdf-}{\sqrt{a+bx}}}{7df}}{3bdf}$$

$\downarrow$ 171

$$\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}(e+fx)^{3/2}}{9bdf}-$$

$$2\int \frac{\sqrt{e+fx}\Big(5bcf(7adf(bcCe+aCde+acCf-3Abdf)+(bce+3a(de+cf))(3bBdf-2aCdf-2bC(de+cf)))-(bce+ade+3acf)(7bdf(bcCe+aCde+acCf-3Abdf)-(adf-4b}{}$$

$\downarrow$ 27

$$\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}(e+fx)^{3/2}}{9bdf}-$$

$$\int \frac{\sqrt{e+fx}\Big(5bcf(7adf(bcCe+aCde+acCf-3Abdf)+(bce+3a(de+cf))(3bBdf-2aCdf-2bC(de+cf)))-(bce+ade+3acf)(7bdf(bcCe+aCde+acCf-3Abdf)-(adf-4b(}{}$$

$\downarrow$ 171

$$\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}(e+fx)^{3/2}}{9bdf}-$$

$$\frac{2(7bdf(bcCe+aCde+acCf-3Abdf)-(adf-4b(de+cf))(3bBdf-2aCdf-2bC(de+cf)))\sqrt{a+bx}\sqrt{c+dx}(e+fx)^{3/2}}{5bf}+\frac{2\int \frac{3bde(5bcf(7adf(bcCe+aCde+acCf-3Abdf)+}{}}{}$$

$\downarrow$ 27

3.61. $\int \sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)\,dx$

$$\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}(e+fx)^{3/2}}{9bdf}-$$

$$\frac{2(7bdf(bcCe+aCde+acCf-3Abdf)-(adf-4b(de+cf))(3bBdf-2aCdf-2bC(de+cf)))\sqrt{a+bx}\sqrt{c+dx}(e+fx)^{3/2}}{5bf}+\frac{\int\frac{3bde(5bcf(7adf(bcCe+aCde+acCf-3Abdf)+(b\ldots}{\ldots}}{\ldots}$$

$\Big\downarrow$ 176

$$\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}(e+fx)^{3/2}}{9bdf}-$$

$$\frac{2(7bdf(bcCe+aCde+acCf-3Abdf)-(adf-4b(de+cf))(3bBdf-2aCdf-2bC(de+cf)))\sqrt{a+bx}\sqrt{c+dx}(e+fx)^{3/2}}{5bf}+\frac{\frac{(be-af)(de-cf)\left(-\left(\left(C\left(16d^3e^3-3c^2df^2e-8c^3f\ldots\right.\right.\right.\right.}{\ldots}}{\ldots}$$

$\Big\downarrow$ 124

$$\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}(e+fx)^{3/2}}{9bdf}-$$

$$\frac{2(7bdf(bcCe+aCde+acCf-3Abdf)-(adf-4b(de+cf))(3bBdf-2aCdf-2bC(de+cf)))\sqrt{a+bx}\sqrt{c+dx}(e+fx)^{3/2}}{5bf}+\frac{\frac{(be-af)(de-cf)\left(-\left(\left(C\left(16d^3e^3-3c^2df^2e-8c^3f\ldots\right.\right.\right.\right.}{\ldots}}{\ldots}$$

$\Big\downarrow$ 123

$$\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}(e+fx)^{3/2}}{9bdf}-$$

$$\frac{2(7bdf(bcCe+aCde+acCf-3Abdf)-(adf-4b(de+cf))(3bBdf-2aCdf-2bC(de+cf)))\sqrt{a+bx}\sqrt{c+dx}(e+fx)^{3/2}}{5bf}+\frac{\frac{2\sqrt{ad-bc}\left(\left(\left(2C\left(8d^4e^4-4cd^3fe^3-3c^2d^2f^2e^2-4c\ldots\right.\right.\right.\right.}{\ldots}}{\ldots}$$

$\Big\downarrow$ 131

$$\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}(e+fx)^{3/2}}{9bdf}-$$

$$\frac{2(7bdf(bcCe+aCde+acCf-3Abdf)-(adf-4b(de+cf))(3bBdf-2aCdf-2bC(de+cf)))\sqrt{a+bx}\sqrt{c+dx}(e+fx)^{3/2}}{5bf}+\frac{\frac{2\sqrt{ad-bc}\left(\left(\left(2C\left(8d^4e^4-4cd^3fe^3-3c^2d^2f^2e^2-4c\ldots\right.\right.\right.\right.}{\ldots}}{\ldots}$$

$\Big\downarrow$ 131

3.61. $\int\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)\,dx$

$$\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}(e+fx)^{3/2}}{9bdf} - \frac{\frac{2(7bdf(bcCe+aCde+acCf-3Abdf)-(adf-4b(de+cf))(3bBdf-2aCdf-2bC(de+cf)))\sqrt{a+bx}\sqrt{c+dx}(e+fx)^{3/2}}{5bf} + \frac{2\sqrt{ad-bc}\left(\left(2C\left(8d^4e^4-4cd^3fe^3-3c^2d^2f^2e^2-4c\cdots\right.\right.\right.}{\cdots}}{\cdots}$$

$\downarrow$ 130

$$\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}(e+fx)^{3/2}}{9bdf} - \frac{\frac{2(7bdf(bcCe+aCde+acCf-3Abdf)-(adf-4b(de+cf))(3bBdf-2aCdf-2bC(de+cf)))\sqrt{a+bx}\sqrt{c+dx}(e+fx)^{3/2}}{5bf} + \frac{2\sqrt{ad-bc}\left(\left(2C\left(8d^4e^4-4cd^3fe^3-3c^2d^2f^2e^2-4c\cdots\right.\right.\right.}{\cdots}}{\cdots}$$

input
```
Int[Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2),x]
```

output
```
(2*C*(a + b*x)^(3/2)*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(9*b*d*f) - ((-2*(3*
b*B*d*f - 2*a*C*d*f - 2*b*C*(d*e + c*f))*Sqrt[a + b*x]*(c + d*x)^(3/2)*(e
+ f*x)^(3/2))/(7*d*f) + ((2*(7*b*d*f*(b*c*C*e + a*C*d*e + a*c*C*f - 3*A*b*
d*f) - (a*d*f - 4*b*(d*e + c*f))*(3*b*B*d*f - 2*a*C*d*f - 2*b*C*(d*e + c*f
)))*Sqrt[a + b*x]*Sqrt[c + d*x]*(e + f*x)^(3/2))/(5*b*f) + ((-2*(8*a^3*C*d
^3*f^3 + 3*a^2*b*d^2*f^2*(C*d*e - c*C*f - 4*B*d*f) - 3*a*b^2*d*f^2*((c^2*C
 - 7*A*d^2)*f + B*d*(d*e - 2*c*f)) - b^3*(C*(16*d^3*e^3 - 3*c^2*d*e*f^2 -
8*c^3*f^3) + 3*d*f*(7*A*d*f*(2*d*e - c*f) - B*(8*d^2*e^2 - c*d*e*f - 4*c^2
*f^2))))*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])/(3*b*d) + ((2*Sqrt[-(b
*c) + a*d]*(16*a^4*C*d^4*f^4 - 8*a^3*b*d^3*f^3*(C*d*e + c*C*f + 3*B*d*f) +
 3*a^2*b^2*d^2*f^2*(d*f*(5*B*d*e + 5*B*c*f + 14*A*d*f) - 2*C*(d^2*e^2 - c*
d*e*f + c^2*f^2)) - a*b^3*d*f*(C*(8*d^3*e^3 - 6*c*d^2*e^2*f - 6*c^2*d*e*f^
2 + 8*c^3*f^3) + 3*d*f*(14*A*d*f*(d*e + c*f) - B*(5*d^2*e^2 - 6*c*d*e*f +
5*c^2*f^2))) + b^4*(2*C*(8*d^4*e^4 - 4*c*d^3*e^3*f - 3*c^2*d^2*e^2*f^2 - 4
*c^3*d*e*f^3 + 8*c^4*f^4) + 3*d*f*(14*A*d*f*(d^2*e^2 - c*d*e*f + c^2*f^2)
- B*(8*d^3*e^3 - 5*c*d^2*e^2*f - 5*c^2*d*e*f^2 + 8*c^3*f^3))))*Sqrt[(b*(c
+ d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x]
)/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(b*Sqrt[d]*f*Sqrt
[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]) + (2*Sqrt[-(b*c) + a*d]*(b*e -
a*f)*(d*e - c*f)*(8*a^3*C*d^3*f^3 + 3*a^2*b*d^2*f^2*(C*d*e - c*C*f - 4*...
```

3.61. $\int \sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)\,dx$

3.61.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

3.61.4 Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 2077, normalized size of antiderivative = 1.76

method	result	size
elliptic	Expression too large to display	2077
default	Expression too large to display	14766

input
```
int((b*x+a)^(1/2)*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
((b*x+a)*(d*x+c)*(f*x+e))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*
(2/9*C*x^3*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*
x+a*c*e)^(1/2)+2/7*(b*d*f*B+a*C*d*f+C*b*c*f+C*b*d*e-2/9*(4*a*d*f+4*b*c*f+4
*b*d*e)*C)/b/d/f*x^2*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*
e*x+b*c*e*x+a*c*e)^(1/2)+2/5*(A*b*d*f+B*a*d*f+B*b*c*f+b*B*d*e+C*a*c*f+C*a*
d*e+C*b*c*e-2/9*C*(7/2*a*c*f+7/2*a*d*e+7/2*b*c*e)-2/7*(b*d*f*B+a*C*d*f+C*b
*c*f+C*b*d*e-2/9*(4*a*d*f+4*b*c*f+4*b*d*e)*C)/b/d/f*(3*a*d*f+3*b*c*f+3*b*d
*e))/b/d/f*x*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*
e*x+a*c*e)^(1/2)+2/3*(A*a*d*f+A*b*c*f+A*b*d*e+B*a*c*f+B*a*d*e+B*b*c*e+1/3*
C*a*c*e-2/7*(b*d*f*B+a*C*d*f+C*b*c*f+C*b*d*e-2/9*(4*a*d*f+4*b*c*f+4*b*d*e)
*C)/b/d/f*(5/2*a*c*f+5/2*a*d*e+5/2*b*c*e)-2/5*(A*b*d*f+B*a*d*f+B*b*c*f+b*B
*d*e+C*a*c*f+C*a*d*e+C*b*c*e-2/9*C*(7/2*a*c*f+7/2*a*d*e+7/2*b*c*e)-2/7*(b*
d*f*B+a*C*d*f+C*b*c*f+C*b*d*e-2/9*(4*a*d*f+4*b*c*f+4*b*d*e)*C)/b/d/f*(3*a*
d*f+3*b*c*f+3*b*d*e))/b/d/f*(2*a*d*f+2*b*c*f+2*b*d*e))/b/d/f*(b*d*f*x^3+a*
d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)+2*(A*a*c*
e-2/5*(A*b*d*f+B*a*d*f+B*b*c*f+b*B*d*e+C*a*c*f+C*a*d*e+C*b*c*e-2/9*C*(7/2*
a*c*f+7/2*a*d*e+7/2*b*c*e)-2/7*(b*d*f*B+a*C*d*f+C*b*c*f+C*b*d*e-2/9*(4*a*d
*f+4*b*c*f+4*b*d*e)*C)/b/d/f*(3*a*d*f+3*b*c*f+3*b*d*e))/b/d/f*a*c*e-2/3*(A
*a*d*f+A*b*c*f+A*b*d*e+B*a*c*f+B*a*d*e+B*b*c*e+1/3*C*a*c*e-2/7*(b*d*f*B+a*
C*d*f+C*b*c*f+C*b*d*e-2/9*(4*a*d*f+4*b*c*f+4*b*d*e)*C)/b/d/f*(5/2*a*c*f...
```

3.61.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.17 (sec) , antiderivative size = 1916, normalized size of antiderivative = 1.62

$$\int \sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)\ dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(1/2)*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algor
ithm="fricas")
```

output
```
2/945*(3*(35*C*b^5*d^5*f^5*x^3 + 8*C*b^5*d^5*e^3*f^2 - 3*(C*b^5*c*d^4 + (C
*a*b^4 + 4*B*b^5)*d^5)*e^2*f^3 - (3*C*b^5*c^2*d^3 - 2*(C*a*b^4 + 3*B*b^5)*
c*d^4 + 3*(C*a^2*b^3 - 2*B*a*b^4 - 7*A*b^5)*d^5)*e*f^4 + (8*C*b^5*c^3*d^2
- 3*(C*a*b^4 + 4*B*b^5)*c^2*d^3 - 3*(C*a^2*b^3 - 2*B*a*b^4 - 7*A*b^5)*c*d^
4 + (8*C*a^3*b^2 - 12*B*a^2*b^3 + 21*A*a*b^4)*d^5)*f^5 + 5*(C*b^5*d^5*e*f^
4 + (C*b^5*c*d^4 + (C*a*b^4 + 9*B*b^5)*d^5)*f^5)*x^2 - (6*C*b^5*d^5*e^2*f^
3 - (2*C*b^5*c*d^4 + (2*C*a*b^4 + 9*B*b^5)*d^5)*e*f^4 + (6*C*b^5*c^2*d^3 -
 (2*C*a*b^4 + 9*B*b^5)*c*d^4 + 3*(2*C*a^2*b^3 - 3*B*a*b^4 - 21*A*b^5)*d^5)
*f^5)*x)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e) + (16*C*b^5*d^5*e^5 - 8
*(2*C*b^5*c*d^4 + (2*C*a*b^4 + 3*B*b^5)*d^5)*e^4*f - (5*C*b^5*c^2*d^3 - (2
0*C*a*b^4 + 27*B*b^5)*c*d^4 + (5*C*a^2*b^3 - 27*B*a*b^4 - 42*A*b^5)*d^5)*e
^3*f^2 - (5*C*b^5*c^3*d^2 - 6*(C*a*b^4 + 2*B*b^5)*c^2*d^3 - 3*(2*C*a^2*b^3
 - 14*B*a*b^4 - 21*A*b^5)*c*d^4 + (5*C*a^3*b^2 - 12*B*a^2*b^3 + 63*A*a*b^4
)*d^5)*e^2*f^3 - (16*C*b^5*c^4*d - (20*C*a*b^4 + 27*B*b^5)*c^3*d^2 - 3*(2*
C*a^2*b^3 - 14*B*a*b^4 - 21*A*b^5)*c^2*d^3 - 2*(10*C*a^3*b^2 - 21*B*a^2*b^
3 + 126*A*a*b^4)*c*d^4 + (16*C*a^4*b - 27*B*a^3*b^2 + 63*A*a^2*b^3)*d^5)*e
*f^4 + (16*C*b^5*c^5 - 8*(2*C*a*b^4 + 3*B*b^5)*c^4*d - (5*C*a^2*b^3 - 27*B
*a*b^4 - 42*A*b^5)*c^3*d^2 - (5*C*a^3*b^2 - 12*B*a^2*b^3 + 63*A*a*b^4)*c^2
*d^3 - (16*C*a^4*b - 27*B*a^3*b^2 + 63*A*a^2*b^3)*c*d^4 + 2*(8*C*a^5 - 12*
B*a^4*b + 21*A*a^3*b^2)*d^5)*f^5)*sqrt(b*d*f)*weierstrassPInverse(4/3*(...
```

3.61.6 Sympy [F]

$$\int \sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\,(A+Bx+Cx^2)\;dx = \int \sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\,(A+Bx+Cx^2)\;dx$$

input
```
integrate((b*x+a)**(1/2)*(C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2), x)
```

3.61.7 Maxima [F]

$$\int \sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)\ dx = \int (Cx^2+Bx+A)\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}\,dx$$

input
```
integrate((b*x+a)^(1/2)*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algor
ithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e), x)
```

3.61.8 Giac [F]

$$\int \sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)\ dx = \int (Cx^2+Bx+A)\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}\,dx$$

input
```
integrate((b*x+a)^(1/2)*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algor
ithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e), x)
```

3.61.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)\ dx = \int \sqrt{e+f\,x}\,\sqrt{a+b\,x}\,\sqrt{c+d\,x}\,\left(C\,x^2+B\,x+A\right)\,dx$$

input
```
int((e + f*x)^(1/2)*(a + b*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2),x)
```

output
```
int((e + f*x)^(1/2)*(a + b*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2), x)
```

3.62 $\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{\sqrt{a+bx}}\,dx$

3.62.1 Optimal result

Integrand size = 38, antiderivative size = 774

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{\sqrt{a+bx}}\,dx =$$

$$-\frac{2(5bdf(3aC(de+cf)+b(cCe-7Adf))-(2bde-bcf+4adf)(6aCdf-b(7Bdf-4C(de+cf))))\sqrt{a}}{105b^3d^2f^2}$$

$$-\frac{2(6aCdf-b(7Bdf-4C(de+cf)))\sqrt{a+bx}\sqrt{c+dx}(e+fx)^{3/2}}{35b^2df^2}$$

$$+\frac{2C\sqrt{a+bx}(c+dx)^{3/2}(e+fx)^{3/2}}{7bdf}$$

$$-\frac{2\sqrt{-bc+ad}(3bdf(5bcf(3aC(de+cf)+b(cCe-7Adf))-(bce+ade+3acf)(6aCdf-b(7Bdf-4C}{}$$

$$-\frac{2\sqrt{-bc+ad}(be-af)(de-cf)\left(24a^2Cd^2f^2+abdf(13Cde-5cCf-28Bdf)-b^2(7df(2Bde-Bcf-\right.}{105b^4d^{5/2}f^3\sqrt{c+d}}$$

output
```
2/7*C*(d*x+c)^(3/2)*(f*x+e)^(3/2)*(b*x+a)^(1/2)/b/d/f-2/35*(6*a*C*d*f-b*(7
*B*d*f-4*C*(c*f+d*e)))*(f*x+e)^(3/2)*(b*x+a)^(1/2)*(d*x+c)^(1/2)/b^2/d/f^2
-2/105*(5*b*d*f*(3*a*C*(c*f+d*e)+b*(-7*A*d*f+C*c*e))-(4*a*d*f-b*c*f+2*b*d*
e)*(6*a*C*d*f-b*(7*B*d*f-4*C*(c*f+d*e))))*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x
+e)^(1/2)/b^3/d^2/f^2-2/105*(3*b*d*f*(5*b*c*f*(3*a*C*(c*f+d*e)+b*(-7*A*d*f
+C*c*e))-(3*a*c*f+a*d*e+b*c*e)*(6*a*C*d*f-b*(7*B*d*f-4*C*(c*f+d*e))))+2*(1
/2*b*d*e-(a*d+b*c)*f)*(5*b*d*f*(3*a*C*(c*f+d*e)+b*(-7*A*d*f+C*c*e))-(4*a*d
*f-b*c*f+2*b*d*e)*(6*a*C*d*f-b*(7*B*d*f-4*C*(c*f+d*e)))))*EllipticE(d^(1/2
)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(a*d-b*
c)^(1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(f*x+e)^(1/2)/b^4/d^(5/2)/f^3/(d*x+c
)^(1/2)/(b*(f*x+e)/(-a*f+b*e))^(1/2)-2/105*(-a*f+b*e)*(-c*f+d*e)*(24*a^2*C
*d^2*f^2+a*b*d*f*(-28*B*d*f-5*C*c*f+13*C*d*e)-b^2*(7*d*f*(-5*A*d*f-B*c*f+2
*B*d*e)-C*(-4*c^2*f^2-c*d*e*f+8*d^2*e^2)))*EllipticF(d^(1/2)*(b*x+a)^(1/2)
/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(a*d-b*c)^(1/2)*(b*(d*
x+c)/(-a*d+b*c))^(1/2)*(b*(f*x+e)/(-a*f+b*e))^(1/2)/b^4/d^(5/2)/f^3/(d*x+c
)^(1/2)/(f*x+e)^(1/2)
```

3.62.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 28.73 (sec) , antiderivative size = 917, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{\sqrt{a+bx}}\,dx =$$

$$-\frac{2\left(b^2\sqrt{-a+\frac{bc}{d}}(48a^3Cd^3f^3-8a^2bd^2f^2(7Bdf+2C(de+cf))+ab^2df(7df(3Bde+3Bcf+10Adf)+\right.}{}$$

input
```
Integrate[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/Sqrt[a + b*x],x]
```

output
```
(-2*(b^2*Sqrt[-a + (b*c)/d]*(48*a^3*C*d^3*f^3 - 8*a^2*b*d^2*f^2*(7*B*d*f +
 2*C*(d*e + c*f)) + a*b^2*d*f*(7*d*f*(3*B*d*e + 3*B*c*f + 10*A*d*f) + C*(-
9*d^2*e^2 + 8*c*d*e*f - 9*c^2*f^2)) + b^3*(C*(-8*d^3*e^3 + 5*c*d^2*e^2*f +
 5*c^2*d*e*f^2 - 8*c^3*f^3) - 7*d*f*(5*A*d*f*(d*e + c*f) - 2*B*(d^2*e^2 -
c*d*e*f + c^2*f^2))))*(c + d*x)*(e + f*x) + b^2*Sqrt[-a + (b*c)/d]*d*f*(a
+ b*x)*(c + d*x)*(e + f*x)*(-24*a^2*C*d^2*f^2 + a*b*d*f*(28*B*d*f + C*(5*d
*e + 5*c*f + 18*d*f*x)) + b^2*(-7*d*f*(B*c*f + 5*A*d*f + B*d*(e + 3*f*x))
+ C*(4*c^2*f^2 - c*d*f*(2*e + 3*f*x) + d^2*(4*e^2 - 3*e*f*x - 15*f^2*x^2))
)) + I*(b*c - a*d)*f*(48*a^3*C*d^3*f^3 - 8*a^2*b*d^2*f^2*(7*B*d*f + 2*C*(d
*e + c*f)) + a*b^2*d*f*(7*d*f*(3*B*d*e + 3*B*c*f + 10*A*d*f) + C*(-9*d^2*e
^2 + 8*c*d*e*f - 9*c^2*f^2)) + b^3*(C*(-8*d^3*e^3 + 5*c*d^2*e^2*f + 5*c^2*
d*e*f^2 - 8*c^3*f^3) - 7*d*f*(5*A*d*f*(d*e + c*f) - 2*B*(d^2*e^2 - c*d*e*f
 + c^2*f^2))))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(
e + f*x))/(f*(a + b*x))]*EllipticE[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b
*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)] - I*b*(b*c - a*d)*f*(d*e - c*f)*(24
*a^2*C*d^2*f^2 + a*b*d*f*(-5*C*d*e + 13*c*C*f - 28*B*d*f) + b^2*(7*d*f*(B*
d*e - 2*B*c*f + 5*A*d*f) - C*(4*d^2*e^2 + c*d*e*f - 8*c^2*f^2)))*(a + b*x)
^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]
*EllipticF[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b
*c*f - a*d*f)]))/(105*b^5*Sqrt[-a + (b*c)/d]*d^3*f^3*Sqrt[a + b*x]*Sqrt...
```

3.62.3 Rubi [A] (verified)

Time = 1.71 (sec) , antiderivative size = 789, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2118, 27, 171, 27, 171, 27, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{\sqrt{a+bx}}dx$$

$$\Big\downarrow \text{2118}$$

$$\frac{2\int -\frac{b\sqrt{c+dx}\sqrt{e+fx}(3aC(de+cf)+b(cCe-7Adf)-(7bBdf-6aCdf-4bC(de+cf))x)}{2\sqrt{a+bx}}dx}{7b^2df}+\frac{2C\sqrt{a+bx}(c+dx)^{3/2}(e+fx)^{3/2}}{7bdf}$$

$$\Big\downarrow \text{27}$$

$$\frac{2C\sqrt{a+bx}(c+dx)^{3/2}(e+fx)^{3/2}}{7bdf}-\frac{\int \frac{\sqrt{c+dx}\sqrt{e+fx}(3aC(de+cf)+b(cCe-7Adf)-(7bBdf-6aCdf-4bC(de+cf))x)}{\sqrt{a+bx}}dx}{7bdf}$$

$\downarrow$ 171

$$\frac{2C\sqrt{a+bx}(c+dx)^{3/2}(e+fx)^{3/2}}{7bdf}-\frac{\frac{2\int \frac{\sqrt{e+fx}((bce+ade+3acf)(7bBdf-6aCdf-4bC(de+cf))+5bcf(3aC(de+cf)+b(cCe-7Adf))+((2bde-bcf+4adf)(7bBdf-6aCdf-4bC(de+cf))+5bdf(3aC(de+cf}{2\sqrt{a+bx}\sqrt{c+dx}}}{5bf}}{7bdf}$$

$\downarrow$ 27

$$\frac{2C\sqrt{a+bx}(c+dx)^{3/2}(e+fx)^{3/2}}{7bdf}-\frac{\frac{\int \frac{\sqrt{e+fx}((bce+ade+3acf)(7bBdf-6aCdf-4bC(de+cf))+5bcf(3aC(de+cf)+b(cCe-7Adf))+((2bde-bcf+4adf)(7bBdf-6aCdf-4bC(de+cf))+5bdf(3aC(de+cf)}{\sqrt{a+bx}\sqrt{c+dx}}}{5bf}}{7bdf}$$

$\downarrow$ 171

$$\frac{2C\sqrt{a+bx}(c+dx)^{3/2}(e+fx)^{3/2}}{7bdf}-$$

$$2\int \frac{3bde((bce+ade+3acf)(7bBdf-6aCdf-4bC(de+cf))+5bcf(3aC(de+cf)+b(cCe-7Adf)))-(bce+ade+acf)((2bde-bcf+4adf)(7bBdf-6aCdf-4bC(de+cf))+5bd}{}$$

$\downarrow$ 27

$$\frac{2C\sqrt{a+bx}(c+dx)^{3/2}(e+fx)^{3/2}}{7bdf}-$$

$$\int \frac{3bde((bce+ade+3acf)(7bBdf-6aCdf-4bC(de+cf))+5bcf(3aC(de+cf)+b(cCe-7Adf)))-(bce+ade+acf)((2bde-bcf+4adf)(7bBdf-6aCdf-4bC(de+cf))+5bdf}{}$$

$\downarrow$ 176

$$\frac{2C\sqrt{a+bx}(c+dx)^{3/2}(e+fx)^{3/2}}{7bdf}-$$

$$\frac{(be-af)(de-cf)\left(24a^2Cd^2f^2+abdf(-28Bdf-5cCf+13Cde)-\left(b^2\left(7df(-5Adf-Bcf+2Bde)-C\left(-4c^2f^2-cdef+8d^2e^2\right)\right)\right)\right)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx}{f}+\frac{\left(3bdf(5b\right.}{}$$

$\downarrow$ 124

3.62. $\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}}\,dx$

$$\frac{2C\sqrt{a+bx}(c+dx)^{3/2}(e+fx)^{3/2}}{7bdf}-$$

$$\frac{(be-af)(de-cf)\left(24a^2Cd^2f^2+abdf(-28Bdf-5cCf+13Cde)-\left(b^2\left(7df(-5Adf-Bcf+2Bde)-C\left(-4c^2f^2-cdef+8d^2e^2\right)\right)\right)\right)\int\frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx}{f}+\frac{\sqrt{e+fx}\sqrt{}}{}$$

$\downarrow$ 123

$$\frac{2C\sqrt{a+bx}(c+dx)^{3/2}(e+fx)^{3/2}}{7bdf}-$$

$$\frac{(be-af)(de-cf)\left(24a^2Cd^2f^2+abdf(-28Bdf-5cCf+13Cde)-\left(b^2\left(7df(-5Adf-Bcf+2Bde)-C\left(-4c^2f^2-cdef+8d^2e^2\right)\right)\right)\right)\int\frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx}{f}+\frac{2\sqrt{e+fx}}{}$$

$\downarrow$ 131

$$\frac{2C\sqrt{a+bx}(c+dx)^{3/2}(e+fx)^{3/2}}{7bdf}-$$

$$\frac{(be-af)(de-cf)\sqrt{\frac{b(c+dx)}{bc-ad}}\left(24a^2Cd^2f^2+abdf(-28Bdf-5cCf+13Cde)-\left(b^2\left(7df(-5Adf-Bcf+2Bde)-C\left(-4c^2f^2-cdef+8d^2e^2\right)\right)\right)\right)\int\frac{1}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}}}{f\sqrt{c+dx}}$$

$\downarrow$ 131

$$\frac{2C\sqrt{a+bx}(c+dx)^{3/2}(e+fx)^{3/2}}{7bdf}-$$

$$\frac{(be-af)(de-cf)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\left(24a^2Cd^2f^2+abdf(-28Bdf-5cCf+13Cde)-\left(b^2\left(7df(-5Adf-Bcf+2Bde)-C\left(-4c^2f^2-cdef+8d^2e^2\right)\right)\right)\right)\int\frac{}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-}}}}{f\sqrt{c+dx}\sqrt{e+fx}}$$

$\downarrow$ 130

$$\frac{2C\sqrt{a+bx}(c+dx)^{3/2}(e+fx)^{3/2}}{7bdf}-$$

$$\frac{2\sqrt{ad-bc}(be-af)(de-cf)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\left(24a^2Cd^2f^2+abdf(-28Bdf-5cCf+13Cde)-\left(b^2\left(7df(-5Adf-Bcf+2Bde)-C\left(-4c^2f^2-cdef+8d^2e^2\right)\right)\right)\right)\text{Ellipti}}{b\sqrt{df}\sqrt{c+dx}\sqrt{e+fx}}$$

3.62. $\int\frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{\sqrt{a+bx}}dx$

input
```
Int[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/Sqrt[a + b*x],x]
```

output
```
(2*C*Sqrt[a + b*x]*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(7*b*d*f) - ((-2*(7*b*
B*d*f - 6*a*C*d*f - 4*b*C*(d*e + c*f))*Sqrt[a + b*x]*Sqrt[c + d*x]*(e + f*
x)^(3/2))/(5*b*f) + ((2*((2*b*d*e - b*c*f + 4*a*d*f)*(7*b*B*d*f - 6*a*C*d*
f - 4*b*C*(d*e + c*f)) + 5*b*d*f*(3*a*C*(d*e + c*f) + b*(c*C*e - 7*A*d*f))
)*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])/(3*b*d) + ((2*Sqrt[-(b*c) + a
*d]*(3*b*d*f*((b*c*e + a*d*e + 3*a*c*f)*(7*b*B*d*f - 6*a*C*d*f - 4*b*C*(d*
e + c*f)) + 5*b*c*f*(3*a*C*(d*e + c*f) + b*(c*C*e - 7*A*d*f))) + 2*((b*d*e
)/2 - (b*c + a*d)*f)*((2*b*d*e - b*c*f + 4*a*d*f)*(7*b*B*d*f - 6*a*C*d*f -
 4*b*C*(d*e + c*f)) + 5*b*d*f*(3*a*C*(d*e + c*f) + b*(c*C*e - 7*A*d*f))))*
Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE[ArcSin[(Sqrt[d]*Sq
rt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(b*Sqr
t[d]*f*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]) + (2*Sqrt[-(b*c) + a
*d]*(b*e - a*f)*(d*e - c*f)*(24*a^2*C*d^2*f^2 + a*b*d*f*(13*C*d*e - 5*c*C*
f - 28*B*d*f) - b^2*(7*d*f*(2*B*d*e - B*c*f - 5*A*d*f) - C*(8*d^2*e^2 - c*
d*e*f - 4*c^2*f^2)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b
*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], (
(b*c - a*d)*f)/(d*(b*e - a*f))])/(b*Sqrt[d]*f*Sqrt[c + d*x]*Sqrt[e + f*x])
)/(3*b*d))/(5*b*f))/(7*b*d*f)
```

3.62.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

3.62. $\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{\sqrt{a+bx}}\,dx$

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

3.62.4 Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 1205, normalized size of antiderivative = 1.56

method	result	size
elliptic	Expression too large to display	1205
default	Expression too large to display	9543

input
```
int((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
((b*x+a)*(d*x+c)*(f*x+e))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*
(2/7*C/b*x^2*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*
e*x+a*c*e)^(1/2)+2/5*(B*d*f+C*c*f+C*d*e-2/7*C/b*(3*a*d*f+3*b*c*f+3*b*d*e))
/b/d/f*x*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+
a*c*e)^(1/2)+2/3*(A*d*f+B*c*f+d*B*e+C*c*e-2/7*C/b*(5/2*a*c*f+5/2*a*d*e+5/2
*b*c*e)-2/5*(B*d*f+C*c*f+C*d*e-2/7*C/b*(3*a*d*f+3*b*c*f+3*b*d*e))/b/d/f*(2
*a*d*f+2*b*c*f+2*b*d*e))/b/d/f*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*
c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)+2*(A*c*e-2/5*(B*d*f+C*c*f+C*d*e-2/7*C/b
*(3*a*d*f+3*b*c*f+3*b*d*e))/b/d/f*a*c*e-2/3*(A*d*f+B*c*f+d*B*e+C*c*e-2/7*C
/b*(5/2*a*c*f+5/2*a*d*e+5/2*b*c*e)-2/5*(B*d*f+C*c*f+C*d*e-2/7*C/b*(3*a*d*f
+3*b*c*f+3*b*d*e))/b/d/f*(2*a*d*f+2*b*c*f+2*b*d*e))/b/d/f*(1/2*a*c*f+1/2*a
*d*e+1/2*b*c*e))*(e/f-c/d)*((x+e/f)/(e/f-c/d))^(1/2)*((x+a/b)/(-e/f+a/b))^
(1/2)*((x+c/d)/(-e/f+c/d))^(1/2)/(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+
a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*EllipticF(((x+e/f)/(e/f-c/d))^(1/2),(
(-e/f+c/d)/(-e/f+a/b))^(1/2))+2*(A*c*f+A*d*e+B*c*e-4/7*C/b*a*c*e-2/5*(B*d*
f+C*c*f+C*d*e-2/7*C/b*(3*a*d*f+3*b*c*f+3*b*d*e))/b/d/f*(3/2*a*c*f+3/2*a*d*
e+3/2*b*c*e)-2/3*(A*d*f+B*c*f+d*B*e+C*c*e-2/7*C/b*(5/2*a*c*f+5/2*a*d*e+5/2
*b*c*e)-2/5*(B*d*f+C*c*f+C*d*e-2/7*C/b*(3*a*d*f+3*b*c*f+3*b*d*e))/b/d/f*(2
*a*d*f+2*b*c*f+2*b*d*e))/b/d/f*(a*d*f+b*c*f+b*d*e))*(e/f-c/d)*((x+e/f)/(e/
f-c/d))^(1/2)*((x+a/b)/(-e/f+a/b))^(1/2)*((x+c/d)/(-e/f+c/d))^(1/2)/(b*...
```

3.62.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 1393, normalized size of antiderivative = 1.80

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{\sqrt{a+bx}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(1/2),x, algor
ithm="fricas")
```

output
```
2/315*(3*(15*C*b^4*d^4*f^4*x^2 - 4*C*b^4*d^4*e^2*f^2 + (2*C*b^4*c*d^3 - (5
*C*a*b^3 - 7*B*b^4)*d^4)*e*f^3 - (4*C*b^4*c^2*d^2 + (5*C*a*b^3 - 7*B*b^4)*
c*d^3 - (24*C*a^2*b^2 - 28*B*a*b^3 + 35*A*b^4)*d^4)*f^4 + 3*(C*b^4*d^4*e*f
^3 + (C*b^4*c*d^3 - (6*C*a*b^3 - 7*B*b^4)*d^4)*f^4)*x)*sqrt(b*x + a)*sqrt(
d*x + c)*sqrt(f*x + e) - (8*C*b^4*d^4*e^4 - (9*C*b^4*c*d^3 - (5*C*a*b^3 -
14*B*b^4)*d^4)*e^3*f - (4*C*b^4*c^2*d^2 + 7*(C*a*b^3 - 3*B*b^4)*c*d^3 - (1
0*C*a^2*b^2 - 14*B*a*b^3 + 35*A*b^4)*d^4)*e^2*f^2 - (9*C*b^4*c^3*d + 7*(C*
a*b^3 - 3*B*b^4)*c^2*d^2 + 14*(3*C*a^2*b^2 - 4*B*a*b^3 + 10*A*b^4)*c*d^3 -
 (40*C*a^3*b - 49*B*a^2*b^2 + 70*A*a*b^3)*d^4)*e*f^3 + (8*C*b^4*c^4 + (5*C
*a*b^3 - 14*B*b^4)*c^3*d + (10*C*a^2*b^2 - 14*B*a*b^3 + 35*A*b^4)*c^2*d^2
+ (40*C*a^3*b - 49*B*a^2*b^2 + 70*A*a*b^3)*c*d^3 - 2*(24*C*a^4 - 28*B*a^3*
b + 35*A*a^2*b^2)*d^4)*f^4)*sqrt(b*d*f)*weierstrassPInverse(4/3*(b^2*d^2*e
^2 - (b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2
*f^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2
*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2
*b*c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3), 1/3*(3*b*d*f*x + b*d*e + (b*c +
a*d)*f)/(b*d*f)) - 3*(8*C*b^4*d^4*e^3*f - (5*C*b^4*c*d^3 - (9*C*a*b^3 - 14
*B*b^4)*d^4)*e^2*f^2 - (5*C*b^4*c^2*d^2 + 2*(4*C*a*b^3 - 7*B*b^4)*c*d^3 -
(16*C*a^2*b^2 - 21*B*a*b^3 + 35*A*b^4)*d^4)*e*f^3 + (8*C*b^4*c^3*d + (9*C*
a*b^3 - 14*B*b^4)*c^2*d^2 + (16*C*a^2*b^2 - 21*B*a*b^3 + 35*A*b^4)*c*d^...
```

3.62.6 Sympy [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{\sqrt{a+bx}}\,dx = \int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{\sqrt{a+bx}}\,dx$$

input
```
integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2)/(b*x+a)**(1/2),x)
```

output
```
Integral(sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2)/sqrt(a + b*x), x)
```

3.62.7 Maxima [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{\sqrt{a+bx}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{dx+c}\sqrt{fx+e}}{\sqrt{bx+a}}\,dx$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(1/2),x, algor
ithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(d*x + c)*sqrt(f*x + e)/sqrt(b*x + a), x)
```

3.62.8 Giac [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{\sqrt{a+bx}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{dx+c}\sqrt{fx+e}}{\sqrt{bx+a}}\,dx$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(1/2),x, algor
ithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(d*x + c)*sqrt(f*x + e)/sqrt(b*x + a), x)
```

3.62.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{\sqrt{a+bx}}\,dx = \int \frac{\sqrt{e+f\,x}\,\sqrt{c+d\,x}\,(C\,x^2+B\,x+A)}{\sqrt{a+b\,x}}\,dx$$

input
```
int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x)^(1/2),x)
```

output
```
int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x)^(1/2), x
)
```

3.63 $\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{3/2}}\,dx$

3.63.1 Optimal result

Integrand size = 38, antiderivative size = 706

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{3/2}}\,dx = \frac{2(24a^2Cdf^2 - abf(7Cde + cCf + 20Bdf) + b^2(5df(Be + 3Af)}{15b^3df(be - af)}$$

$$+ \frac{2(6a^2Cdf + b^2(cCe + 5Adf) - ab(Cde + cCf + 5Bdf))\sqrt{a+bx}\sqrt{c+dx}(e+fx)^{3/2}}{5b^2(bc-ad)f(be-af)}$$

$$- \frac{2(Ab^2 - a(bB - aC))(c+dx)^{3/2}(e+fx)^{3/2}}{b(bc-ad)(be-af)\sqrt{a+bx}}$$

$$+ \frac{2\sqrt{-bc+ad}(48a^2Cd^2f^2 - 8abdf(Cde + cCf + 5Bdf) + b^2(5df(Bde + Bcf + 6Adf) - 2C(d^2e^2 - cdef +}{15b^4d^{3/2}f^2\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}$$

$$- \frac{2\sqrt{-bc+ad}(de-cf)(24a^2Cdf^2 - abf(7Cde + cCf + 20Bdf) + b^2(5df(Be + 3Af) - Ce(2de - cf)))\sqrt{}}{15b^4d^{3/2}f^2\sqrt{c+dx}\sqrt{e+fx}}$$

output
```
-2*(A*b^2-a*(B*b-C*a))*(d*x+c)^(3/2)*(f*x+e)^(3/2)/b/(-a*d+b*c)/(-a*f+b*e)
/(b*x+a)^(1/2)+2/5*(6*a^2*C*d*f+b^2*(5*A*d*f+C*c*e)-a*b*(5*B*d*f+C*c*f+C*d
*e))*(f*x+e)^(3/2)*(b*x+a)^(1/2)*(d*x+c)^(1/2)/b^2/(-a*d+b*c)/f/(-a*f+b*e)
+2/15*(24*a^2*C*d*f^2-a*b*f*(20*B*d*f+C*c*f+7*C*d*e)+b^2*(5*d*f*(3*A*f+B*e
)-C*e*(-c*f+2*d*e)))*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^3/d/f/(-a
*f+b*e)+2/15*(48*a^2*C*d^2*f^2-8*a*b*d*f*(5*B*d*f+C*c*f+C*d*e)+b^2*(5*d*f*
(6*A*d*f+B*c*f+B*d*e)-2*C*(c^2*f^2-c*d*e*f+d^2*e^2)))*EllipticE(d^(1/2)*(b
*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(a*d-b*c)^(
1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(f*x+e)^(1/2)/b^4/d^(3/2)/f^2/(d*x+c)^(1
/2)/(b*(f*x+e)/(-a*f+b*e))^(1/2)-2/15*(-c*f+d*e)*(24*a^2*C*d*f^2-a*b*f*(20
*B*d*f+C*c*f+7*C*d*e)+b^2*(5*d*f*(3*A*f+B*e)-C*e*(-c*f+2*d*e)))*EllipticF(
d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(
a*d-b*c)^(1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(b*(f*x+e)/(-a*f+b*e))^(1/2)/b
^4/d^(3/2)/f^2/(d*x+c)^(1/2)/(f*x+e)^(1/2)
```

3.63.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.90 (sec) , antiderivative size = 633, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{3/2}}\,dx =$$

$$-\frac{2\left(-b^2\sqrt{-a+\frac{bc}{d}}(48a^2Cd^2f^2-8abdf(Cde+cCf+5Bdf)+b^2(5df(Bde+Bcf+6Adf)-2C(d^2e^2-c\right.}{}$$

input
```
Integrate[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^(3/2),
x]
```

output
```
(-2*(-(b^2*Sqrt[-a + (b*c)/d]*(48*a^2*C*d^2*f^2 - 8*a*b*d*f*(C*d*e + c*C*f
 + 5*B*d*f) + b^2*(5*d*f*(B*d*e + B*c*f + 6*A*d*f) - 2*C*(d^2*e^2 - c*d*e*
f + c^2*f^2)))*(c + d*x)*(e + f*x)) + b^2*Sqrt[-a + (b*c)/d]*d*f*(c + d*x)
*(e + f*x)*(15*(A*b^2 + a*(-(b*B) + a*C))*d*f - (-9*a*C*d*f + b*(C*d*e + c
*C*f + 5*B*d*f))*(a + b*x) - 3*b*C*d*f*x*(a + b*x)) - I*(b*c - a*d)*f*(48*
a^2*C*d^2*f^2 - 8*a*b*d*f*(C*d*e + c*C*f + 5*B*d*f) + b^2*(5*d*f*(B*d*e +
B*c*f + 6*A*d*f) - 2*C*(d^2*e^2 - c*d*e*f + c^2*f^2)))*(a + b*x)^(3/2)*Sqr
t[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticE
[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d
*f)] - I*b*f*(d*e - c*f)*(24*a^2*C*d^2*f - a*b*d*(C*d*e + 7*c*C*f + 20*B*d
*f) + b^2*(-2*c^2*C*f + 15*A*d^2*f + c*d*(C*e + 5*B*f)))*(a + b*x)^(3/2)*S
qrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*Ellipti
cF[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a
*d*f)]))/(15*b^5*Sqrt[-a + (b*c)/d]*d^2*f^2*Sqrt[a + b*x]*Sqrt[c + d*x]*Sq
rt[e + f*x])
```

3.63.3 Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 729, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2117, 27, 171, 27, 171, 27, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{3/2}}dx$$

$$\Big\downarrow \text{2117}$$

$$-\frac{2\int -\frac{\sqrt{c+dx}\sqrt{e+fx}\left(3C(de+cf)a^2-b(cCe+3Bde+3Bcf-Adf)a+b^2(Bce+2A(de+cf))+b\left(\frac{6Cdfa^2}{b}-(Cde+cCf+5Bdf)a+b(cCe+5Adf)\right)x}{2b\sqrt{a+bx}}}{(bc-ad)(be-af)}$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(3C(de+cf)a^2-b(cCe+3Bde+3Bcf-Adf)a+b^2(Bce+2A(de+cf))+b\left(\frac{6Cdfa^2}{b}-(Cde+cCf+5Bdf)a+b(cCe+5Adf)\right)x\right)}{\sqrt{a+bx}}dx}{b(bc-ad)(be-af)} -$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$\downarrow$ 171

$$\frac{\frac{2\int \frac{(bc-ad)\sqrt{e+fx}\left(6Cf(de+3cf)a^2-b(5Bf(de+3cf)+Ce(de+7cf))a-b^2\left(cCe^2-5Adfe-5cf(Be+2Af)\right)+\left((5df(Be+3Af)-Ce(2de-cf))b^2-af(7Cde+cCf+20B\right.}{2\sqrt{a+bx}\sqrt{c+dx}}}{5bf}}{b(bc-ad)(be-af)}$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$\downarrow$ 27

$$\frac{\frac{(bc-ad)\int \frac{\sqrt{e+fx}\left(6Cf(de+3cf)a^2-b(5Bf(de+3cf)+Ce(de+7cf))a-b^2\left(cCe^2-5Adfe-5cf(Be+2Af)\right)+\left((5df(Be+3Af)-Ce(2de-cf))b^2-af(7Cde+cCf+20B\right.}{\sqrt{a+bx}\sqrt{c+dx}}}{5bf}}{b(bc-ad)(be-af)}$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$\downarrow$ 171

$$(bc-ad)\left(\frac{2\int \frac{(be-af)\left(24Cdf(de+cf)a^2-b\left(20Bdf(de+cf)+C\left(d^2e^2+14cdfe+c^2f^2\right)\right)a-b^2\left(Cefc^2+d\left(Ce^2-5f(2Be+3Af)\right)c-15Ad^2ef\right)+\left(\left(5df(Bde+Bcf+6A\right.\right.}{2\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}}{3bd}\right.$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$\downarrow$ 27

$$(bc-ad)\left(\frac{(be-af)\int \frac{24Cdf(de+cf)a^2-b\left(20Bdf(de+cf)+C\left(d^2e^2+14cdfe+c^2f^2\right)\right)a-b^2\left(Cefc^2+d\left(Ce^2-5f(2Be+3Af)\right)c-15Ad^2ef\right)+\left(\left(5df(Bde+Bcf+6Adf\right)\right.}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}}{3bd}\right.$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$\downarrow$ 176

3.63. $\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{3/2}}\,dx$

$$(bc-ad)\left((be-af)\left(\frac{\left(48a^2Cd^2f^2-8abdf(5Bdf+cCf+Cde)+b^2\left(5df(6Adf+Bcf+Bde)-2C\left(c^2f^2-cdef+d^2e^2\right)\right)\right)\int \frac{\sqrt{e+fx}}{\sqrt{a+bx}\sqrt{c+dx}}dx}{f}-\frac{(de-cf)\left(24a^2Cdf^2-abf\right.}{}\right.\right.$$

$$\overline{\qquad 3bd \qquad}$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 124$$

$$(bc-ad)\left((be-af)\left(\frac{\sqrt{e+fx}\sqrt{\frac{b(c+dx)}{bc-ad}}\left(48a^2Cd^2f^2-8abdf(5Bdf+cCf+Cde)+b^2\left(5df(6Adf+Bcf+Bde)-2C\left(c^2f^2-cdef+d^2e^2\right)\right)\right)\int \frac{\sqrt{\frac{be}{be-af}+\frac{bfx}{be-af}}}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}}d}{f\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}\right.\right.$$

$$\overline{\qquad 3bd \qquad}$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 123$$

$$(bc-ad)\left((be-af)\left(\frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}\left(48a^2Cd^2f^2-8abdf(5Bdf+cCf+Cde)+b^2\left(5df(6Adf+Bcf+Bde)-2C\left(c^2f^2-cdef+d^2e^2\right)\right)\right)E\left(\arcsin\left(\frac{\sqrt{d}\sqrt{a+}}{\sqrt{ad-b}}\right.\right.}{b\sqrt{d}f\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}\right.\right.$$

$$\overline{\qquad 3bd \qquad}$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 131$$

3.63. $\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)}{(a+bx)^{3/2}}\,dx$

$$(bc-ad)\left(\frac{(be-af)\left(\frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}\left(48a^2Cd^2f^2-8abdf(5Bdf+cCf+Cde)+b^2\left(5df(6Adf+Bcf+Bde)-2C\left(c^2f^2-cdef+d^2e^2\right)\right)\right)E\left(\arcsin\left(\frac{\sqrt{d}\sqrt{a+\ldots}}{\sqrt{ad-b\ldots}}\right.\right.}{b\sqrt{d}f\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}\right.}{3b\ldots}\right.$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$$\Big\downarrow \ 131$$

$$(bc-ad)\left(\frac{(be-af)\left(\frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}\left(48a^2Cd^2f^2-8abdf(5Bdf+cCf+Cde)+b^2\left(5df(6Adf+Bcf+Bde)-2C\left(c^2f^2-cdef+d^2e^2\right)\right)\right)E\left(\arcsin\left(\frac{\sqrt{d}\sqrt{a+\ldots}}{\sqrt{ad-b\ldots}}\right.\right.}{b\sqrt{d}f\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}\right.}{\ldots}\right.$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$$\Big\downarrow \ 130$$

$$(bc-ad)\left(\frac{(be-af)\left(\frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}\left(48a^2Cd^2f^2-8abdf(5Bdf+cCf+Cde)+b^2\left(5df(6Adf+Bcf+Bde)-2C\left(c^2f^2-cdef+d^2e^2\right)\right)\right)E\left(\arcsin\left(\frac{\sqrt{d}\sqrt{a+\ldots}}{\sqrt{ad-b\ldots}}\right.\right.}{b\sqrt{d}f\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}\right.}{\ldots}\right.$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

input
```
Int[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^(3/2),x]
```

3.63. $\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)}{(a+bx)^{3/2}}\,dx$

output
```
(-2*(A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(b*(b*c - a*d
)*(b*e - a*f)*Sqrt[a + b*x]) + ((2*((6*a^2*C*d*f)/b + b*(c*C*e + 5*A*d*f)
- a*(C*d*e + c*C*f + 5*B*d*f))*Sqrt[a + b*x]*Sqrt[c + d*x]*(e + f*x)^(3/2)
)/(5*f) + ((b*c - a*d)*((2*(24*a^2*C*d*f^2 - a*b*f*(7*C*d*e + c*C*f + 20*B
*d*f) + b^2*(5*d*f*(B*e + 3*A*f) - C*e*(2*d*e - c*f)))*Sqrt[a + b*x]*Sqrt[
c + d*x]*Sqrt[e + f*x])/(3*b*d) + ((b*e - a*f)*((2*Sqrt[-(b*c) + a*d]*(48*
a^2*C*d^2*f^2 - 8*a*b*d*f*(C*d*e + c*C*f + 5*B*d*f) + b^2*(5*d*f*(B*d*e +
B*c*f + 6*A*d*f) - 2*C*(d^2*e^2 - c*d*e*f + c^2*f^2)))*Sqrt[(b*(c + d*x))/
(b*c - a*d)]*Sqrt[e + f*x]*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-
(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(b*Sqrt[d]*f*Sqrt[c + d*x
]*Sqrt[(b*(e + f*x))/(b*e - a*f)]) - (2*Sqrt[-(b*c) + a*d]*(d*e - c*f)*(24
*a^2*C*d*f^2 - a*b*f*(7*C*d*e + c*C*f + 20*B*d*f) + b^2*(5*d*f*(B*e + 3*A*
f) - C*e*(2*d*e - c*f)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x)
)/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]
], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(b*Sqrt[d]*f*Sqrt[c + d*x]*Sqrt[e + f
*x])))/(3*b*d)))/(5*b*f))/(b*(b*c - a*d)*(b*e - a*f))
```

3.63.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 2117
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Si
mp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*
(e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1
) - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x]
, x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n, 2*p]
```

3.63. $\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{3/2}}\,dx$

3.63.4 Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 1163, normalized size of antiderivative = 1.65

method	result	size
elliptic	Expression too large to display	1163
default	Expression too large to display	5787

input
```
int((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(3/2),x,method=_RETU
RNVERBOSE)
```

output
```
((b*x+a)*(d*x+c)*(f*x+e))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*
(-2*(b*d*f*x^2+b*c*f*x+b*d*e*x+b*c*e)*(A*b^2-B*a*b+C*a^2)/b^4/((x+a/b)*(b*
d*f*x^2+b*c*f*x+b*d*e*x+b*c*e))^(1/2)+2/5*C/b^2*x*(b*d*f*x^3+a*d*f*x^2+b*c
*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)+2/3*(1/b^2*(B*b*d*f-
C*a*d*f+C*b*c*f+C*b*d*e)-2/5*C/b^2*(2*a*d*f+2*b*c*f+2*b*d*e))/b/d/f*(b*d*f
*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)+2*
(-(A*a*b^2*d*f-A*b^3*c*f-A*b^3*d*e-B*a^2*b*d*f+B*a*b^2*c*f+B*a*b^2*d*e-B*b
^3*c*e+C*a^3*d*f-C*a^2*b*c*f-C*a^2*b*d*e+C*a*b^2*c*e)/b^4+(A*b^2-B*a*b+C*a
^2)/b^4*(a*d*f-b*c*f-b*d*e)+(b*c*f+b*d*e)*(A*b^2-B*a*b+C*a^2)/b^4-2/5*C/b^
2*a*c*e-2/3*(1/b^2*(B*b*d*f-C*a*d*f+C*b*c*f+C*b*d*e)-2/5*C/b^2*(2*a*d*f+2*
b*c*f+2*b*d*e))/b/d/f*(1/2*a*c*f+1/2*a*d*e+1/2*b*c*e))*(e/f-c/d)*((x+e/f)/
(e/f-c/d))^(1/2)*((x+a/b)/(-e/f+a/b))^(1/2)*((x+c/d)/(-e/f+c/d))^(1/2)/(b*
d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)
*EllipticF(((x+e/f)/(e/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1/2))+2*(1/b
^3*(A*b^2*d*f-B*a*b*d*f+B*b^2*c*f+B*b^2*d*e+C*a^2*d*f-C*a*b*c*f-C*a*b*d*e+
C*b^2*c*e)+(A*b^2-B*a*b+C*a^2)/b^3*d*f-2/5*C/b^2*(3/2*a*c*f+3/2*a*d*e+3/2*
b*c*e)-2/3*(1/b^2*(B*b*d*f-C*a*d*f+C*b*c*f+C*b*d*e)-2/5*C/b^2*(2*a*d*f+2*b
*c*f+2*b*d*e))/b/d/f*(a*d*f+b*c*f+b*d*e))*(e/f-c/d)*((x+e/f)/(e/f-c/d))^(1
/2)*((x+a/b)/(-e/f+a/b))^(1/2)*((x+c/d)/(-e/f+c/d))^(1/2)/(b*d*f*x^3+a*d*f
*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*((-e/f+a/...
```

3.63.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.17 (sec) , antiderivative size = 1463, normalized size of antiderivative = 2.07

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(3/2),x, algor
ithm="fricas")
```

output
```
2/45*(3*(3*C*b^4*d^3*f^3*x^2 + C*a*b^3*d^3*e*f^2 + (C*a*b^3*c*d^2 - (24*C*
a^2*b^2 - 20*B*a*b^3 + 15*A*b^4)*d^3)*f^3 + (C*b^4*d^3*e*f^2 + (C*b^4*c*d^
2 - (6*C*a*b^3 - 5*B*b^4)*d^3)*f^3)*x)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*
x + e) + (2*C*a*b^3*d^3*e^3 - (3*C*a*b^3*c*d^2 - (7*C*a^2*b^2 - 5*B*a*b^3)
*d^3)*e^2*f - (3*C*a*b^3*c^2*d + 4*(7*C*a^2*b^2 - 5*B*a*b^3)*c*d^2 - (32*C
*a^3*b - 25*B*a^2*b^2 + 15*A*a*b^3)*d^3)*e*f^2 + (2*C*a*b^3*c^3 + (7*C*a^2
*b^2 - 5*B*a*b^3)*c^2*d + (32*C*a^3*b - 25*B*a^2*b^2 + 15*A*a*b^3)*c*d^2 -
 2*(24*C*a^4 - 20*B*a^3*b + 15*A*a^2*b^2)*d^3)*f^3 + (2*C*b^4*d^3*e^3 - (3
*C*b^4*c*d^2 - (7*C*a*b^3 - 5*B*b^4)*d^3)*e^2*f - (3*C*b^4*c^2*d + 4*(7*C*
a*b^3 - 5*B*b^4)*c*d^2 - (32*C*a^2*b^2 - 25*B*a*b^3 + 15*A*b^4)*d^3)*e*f^2
 + (2*C*b^4*c^3 + (7*C*a*b^3 - 5*B*b^4)*c^2*d + (32*C*a^2*b^2 - 25*B*a*b^3
 + 15*A*b^4)*c*d^2 - 2*(24*C*a^3*b - 20*B*a^2*b^2 + 15*A*a*b^3)*d^3)*f^3)*
x)*sqrt(b*d*f)*weierstrassPInverse(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*
e*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e
^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*
b*d^3)*e*f^2 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3
)/(b^3*d^3*f^3), 1/3*(3*b*d*f*x + b*d*e + (b*c + a*d)*f)/(b*d*f)) + 3*(2*C
*a*b^3*d^3*e^2*f - (2*C*a*b^3*c*d^2 - (8*C*a^2*b^2 - 5*B*a*b^3)*d^3)*e*f^2
 + (2*C*a*b^3*c^2*d + (8*C*a^2*b^2 - 5*B*a*b^3)*c*d^2 - 2*(24*C*a^3*b - 20
*B*a^2*b^2 + 15*A*a*b^3)*d^3)*f^3 + (2*C*b^4*d^3*e^2*f - (2*C*b^4*c*d^2...
```

3.63.6 Sympy [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{3/2}}\,dx = \int \frac{\sqrt{c+dx}\sqrt{e+fx}\,(A+Bx+Cx^2)}{(a+bx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2)/(b*x+a)**(3/2),x)
```

output
```
Integral(sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2)/(a + b*x)**(3/2),
x)
```

3.63.7 Maxima [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{3/2}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{dx+c}\sqrt{fx+e}}{(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(3/2),x, algor
ithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(d*x + c)*sqrt(f*x + e)/(b*x + a)^(3/2), x
)
```

3.63.8 Giac [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{3/2}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{dx+c}\sqrt{fx+e}}{(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(3/2),x, algor
ithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(d*x + c)*sqrt(f*x + e)/(b*x + a)^(3/2), x
)
```

3.63.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{3/2}}\,dx = \int \frac{\sqrt{e+f\,x}\,\sqrt{c+d\,x}\,\left(C\,x^2+B\,x+A\right)}{{\left(a+b\,x\right)}^{3/2}}\,dx$$

input
```
int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x)^(3/2),x)
```

output
```
int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x)^(3/2), x
)
```

3.64 $\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{5/2}}\,dx$

3.64.1 Optimal result

Integrand size = 38, antiderivative size = 687

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{5/2}}\,dx = \frac{2(8a^2Cdf+b^2(cCe+3Bcf+Adf)-ab(Cde+7cCf+4Bdf))}{3b^3(bc-ad)(be-af)}$$

$$-\frac{2(bB-2aC)\sqrt{c+dx}(e+fx)^{3/2}}{b^2(be-af)\sqrt{a+bx}}-\frac{2(Ab^2-a(bB-aC))\,(c+dx)^{3/2}(e+fx)^{3/2}}{3b(bc-ad)(be-af)(a+bx)^{3/2}}$$

$$+\frac{2(16a^3Cd^2f^2-8a^2bdf(Bdf+2C(de+cf))-b^3(c^2Cef+Ad^2ef+cd(Ce^2+6Bef+Af^2))+ab^2(df(7B}{3b^4\sqrt{d}\sqrt{-bc+ad}f(be-a}$$

$$+\frac{2(de-cf)\,(8a^2Cdf+b^2(cCe+3Bcf+Adf)-ab(Cde+7cCf+4Bdf))\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\,\text{EllipticF}\left(\text{ar}\right.}{3b^4\sqrt{d}\sqrt{-bc+ad}f\sqrt{c+dx}\sqrt{e+fx}}$$

output
```
-2/3*(A*b^2-a*(B*b-C*a))*(d*x+c)^(3/2)*(f*x+e)^(3/2)/b/(-a*d+b*c)/(-a*f+b*
e)/(b*x+a)^(3/2)-2*(B*b-2*C*a)*(f*x+e)^(3/2)*(d*x+c)^(1/2)/b^2/(-a*f+b*e)/
(b*x+a)^(1/2)+2/3*(8*a^2*C*d*f+b^2*(A*d*f+3*B*c*f+C*c*e)-a*b*(4*B*d*f+7*C*
c*f+C*d*e))*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^3/(-a*d+b*c)/(-a*f
+b*e)+2/3*(16*a^3*C*d^2*f^2-8*a^2*b*d*f*(B*d*f+2*C*(c*f+d*e))-b^3*(c^2*C*e
*f+A*d^2*e*f+c*d*(A*f^2+6*B*e*f+C*e^2))+a*b^2*(d*f*(2*A*d*f+7*B*c*f+7*B*d*
e)+C*(c^2*f^2+16*c*d*e*f+d^2*e^2)))*EllipticE(d^(1/2)*(b*x+a)^(1/2)/(a*d-b
*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(b*(d*x+c)/(-a*d+b*c))^(1/2)*
(f*x+e)^(1/2)/b^4/f/(-a*f+b*e)/d^(1/2)/(a*d-b*c)^(1/2)/(d*x+c)^(1/2)/(b*(f
*x+e)/(-a*f+b*e))^(1/2)+2/3*(-c*f+d*e)*(8*a^2*C*d*f+b^2*(A*d*f+3*B*c*f+C*c
*e)-a*b*(4*B*d*f+7*C*c*f+C*d*e))*EllipticF(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)
^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(b*
(f*x+e)/(-a*f+b*e))^(1/2)/b^4/f/d^(1/2)/(a*d-b*c)^(1/2)/(d*x+c)^(1/2)/(f*x
+e)^(1/2)
```

3.64.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 28.99 (sec) , antiderivative size = 815, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{5/2}}\,dx =$$

$$-\frac{2\left(b^2\sqrt{-a+\frac{bc}{d}}df(c+dx)(e+fx)\left(\left(Ab^2+a(-bB+aC)\right)(bc-ad)(be-af)+\left(-8a^3Cdf+b^3(3Bce+A\right.\right.\right.}{}$$

input
```
Integrate[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^(5/2),
x]
```

output
```
(-2*(b^2*Sqrt[-a + (b*c)/d]*d*f*(c + d*x)*(e + f*x)*((A*b^2 + a*(-(b*B) +
a*C))*(b*c - a*d)*(b*e - a*f) + (-8*a^3*C*d*f + b^3*(3*B*c*e + A*d*e + A*c
*f) - 2*a*b^2*(3*c*C*e + 2*B*d*e + 2*B*c*f + A*d*f) + a^2*b*(5*B*d*f + 7*C
*(d*e + c*f)))*(a + b*x) - C*(b*c - a*d)*(b*e - a*f)*(a + b*x)^2) + (a + b
*x)*(b^2*Sqrt[-a + (b*c)/d]*(16*a^3*C*d^2*f^2 - 8*a^2*b*d*f*(B*d*f + 2*C*(
d*e + c*f)) - b^3*(c^2*C*e*f + A*d^2*e*f + c*d*(C*e^2 + 6*B*e*f + A*f^2))
+ a*b^2*(d*f*(7*B*d*e + 7*B*c*f + 2*A*d*f) + C*(d^2*e^2 + 16*c*d*e*f + c^2
*f^2)))*(c + d*x)*(e + f*x) + I*(b*c - a*d)*f*(16*a^3*C*d^2*f^2 - 8*a^2*b*
d*f*(B*d*f + 2*C*(d*e + c*f)) - b^3*(c^2*C*e*f + A*d^2*e*f + c*d*(C*e^2 +
6*B*e*f + A*f^2)) + a*b^2*(d*f*(7*B*d*e + 7*B*c*f + 2*A*d*f) + C*(d^2*e^2
+ 16*c*d*e*f + c^2*f^2)))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))
]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticE[I*ArcSinh[Sqrt[-a + (b*c)/d]
/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)] - I*b*(b*c - a*d)*f*(d*e
 - c*f)*(8*a^2*C*d*f + b^2*(c*C*e + 3*B*d*e + A*d*f) - a*b*(7*C*d*e + c*C*
f + 4*B*d*f))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e
 + f*x))/(f*(a + b*x))]*EllipticF[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*
x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)])))/(3*b^5*Sqrt[-a + (b*c)/d]*d*(b*c
- a*d)*f*(b*e - a*f)*(a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x])
```

3.64.3 Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 697, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2117, 27, 167, 27, 171, 27, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)}{(a+bx)^{5/2}}dx$$

$$\Big\downarrow \text{ 2117}$$

$$-\frac{2\int -\frac{3\sqrt{c+dx}\sqrt{e+fx}\left(C(de+cf)a^2-b(cCe+Bde+Bcf-Adf)a+b^2Bce+b\left(\frac{2Cdfa^2}{b}-(Cde+cCf+Bdf)a+b(cCe+Adf)\right)x\right)}{2b(a+bx)^{3/2}}dx}{3(bc-ad)(be-af)}-\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(C(de+cf)a^2-b(cCe+Bde+Bcf-Adf)a+b^2Bce+b\left(\frac{2Cdfa^2}{b}-(Cde+cCf+Bdf)a+b(cCe+Adf)\right)x\right)}{(a+bx)^{3/2}}dx}{b(bc-ad)(be-af)}-\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$\Big\downarrow$ 167

$$\frac{\frac{2\int \frac{(be-af)\sqrt{e+fx}\left(2Cd(de+3cf)a^2-b\left(5Cfc^2+3d(Ce+Bf)c+Bd^2e\right)a+b^2c(cCe+Bde+2Bcf+Adf)+d\left(8Cdfa^2-b(Cde+7cCf+4Bdf)a+b^2(cCe+3Bcf+Adf)\right)x\right)}{2\sqrt{a+bx}\sqrt{c+dx}}}{b(be-af)}}{b(bc-ad)(be-af)}$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{\sqrt{e+fx}\left(2Cd(de+3cf)a^2-b\left(5Cfc^2+3d(Ce+Bf)c+Bd^2e\right)a+b^2c(cCe+Bde+2Bcf+Adf)+d\left(8Cdfa^2-b(Cde+7cCf+4Bdf)a+b^2(cCe+3Bcf+Adf)\right)x\right)}{\sqrt{a+bx}\sqrt{c+dx}}dx}{b}-2\ldots}{b(bc-ad)(be-af)}$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$\Big\downarrow$ 171

$$2\int -\frac{d\left((bce+ade+acf)\left(8Cdfa^2-b(Cde+7cCf+4Bdf)a+b^2(cCe+3Bcf+Adf)\right)-3be\left(2Cd(de+3cf)a^2-b\left(5Cfc^2+3d(Ce+Bf)c+Bd^2e\right)a+b^2c(cCe+Bde+2Bcf+A\ldots\right.\right.}{2\sqrt{a+bx}\sqrt{c+\ldots}} \ldots \frac{}{3bd}$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$\Big\downarrow$ 27

$$\frac{2\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\left(8a^2Cdf-ab(4Bdf+7cCf+Cde)+b^2(Adf+3Bcf+cCe)\right)}{3b}-\int \frac{(bce+ade+acf)\left(8Cdfa^2-b(Cde+7cCf+4Bdf)a+b^2(cCe+3Bcf+Adf)\right)-3be\left(\ldots\right.}{\ldots}$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$\Big\downarrow$ 176

3.64. $\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{5/2}}\,dx$

$$\frac{\frac{2\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\left(8a^2Cdf-ab(4Bdf+7cCf+Cde)+b^2(Adf+3Bcf+cCe)\right)}{3b}-\frac{\frac{(be-af)(de-cf)\left(8a^2Cdf-ab(4Bdf+7cCf+Cde)+b^2(Adf+3Bcf+cCe)\right)\int \frac{}{\sqrt{a+}}}{f}}{}}{}$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$\downarrow$ 124

$$\frac{\frac{2\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\left(8a^2Cdf-ab(4Bdf+7cCf+Cde)+b^2(Adf+3Bcf+cCe)\right)}{3b}-\frac{\frac{(be-af)(de-cf)\left(8a^2Cdf-ab(4Bdf+7cCf+Cde)+b^2(Adf+3Bcf+cCe)\right)\int \frac{}{\sqrt{a+}}}{f}}{}}{}$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$\downarrow$ 123

$$\frac{\frac{2\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\left(8a^2Cdf-ab(4Bdf+7cCf+Cde)+b^2(Adf+3Bcf+cCe)\right)}{3b}-\frac{\frac{(be-af)(de-cf)\left(8a^2Cdf-ab(4Bdf+7cCf+Cde)+b^2(Adf+3Bcf+cCe)\right)\int \frac{}{\sqrt{a+}}}{f}}{}}{}$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$\downarrow$ 131

$$\frac{\frac{2\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\left(8a^2Cdf-ab(4Bdf+7cCf+Cde)+b^2(Adf+3Bcf+cCe)\right)}{3b}-\frac{\frac{(be-af)(de-cf)\sqrt{\frac{b(c+dx)}{bc-ad}}\left(8a^2Cdf-ab(4Bdf+7cCf+Cde)+b^2(Adf+3Bcf+cC}{f\sqrt{c+dx}}}{}}{}$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$\downarrow$ 131

3.64. $\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)}{(a+bx)^{5/2}}\,dx$

$$\frac{2\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\left(8a^2Cdf-ab(4Bdf+7cCf+Cde)+b^2(Adf+3Bcf+cCe)\right)}{3b} - \frac{(be-af)(de-cf)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\left(8a^2Cdf-ab(4Bdf+7cCf+Cde)+b^2(Adf\cdots}{f\sqrt{c+dx}\sqrt{e+fx}\cdots}$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$\big\downarrow$ 130

$$\frac{2\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\left(8a^2Cdf-ab(4Bdf+7cCf+Cde)+b^2(Adf+3Bcf+cCe)\right)}{3b} - \frac{2\sqrt{ad-bc}(be-af)(de-cf)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\left(8a^2Cdf-ab(4Bdf+7cCf+Cde)\cdots}{b\sqrt{d}f\sqrt{c+dx}\sqrt{e+\cdots}}$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

input
```
Int[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^(5/2),x]
```

output
```
(-2*(A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(3*b*(b*c - a
*d)*(b*e - a*f)*(a + b*x)^(3/2)) + ((-2*(b*B - 2*a*C)*(b*c - a*d)*Sqrt[c +
 d*x]*(e + f*x)^(3/2))/(b*Sqrt[a + b*x]) + ((2*(8*a^2*C*d*f + b^2*(c*C*e +
 3*B*c*f + A*d*f) - a*b*(C*d*e + 7*c*C*f + 4*B*d*f))*Sqrt[a + b*x]*Sqrt[c
+ d*x]*Sqrt[e + f*x])/(3*b) - ((2*Sqrt[-(b*c) + a*d]*(16*a^3*C*d^2*f^2 - 8
*a^2*b*d*f*(B*d*f + 2*C*(d*e + c*f)) - b^3*(c^2*C*e*f + A*d^2*e*f + c*d*(C
*e^2 + 6*B*e*f + A*f^2)) + a*b^2*(d*f*(7*B*d*e + 7*B*c*f + 2*A*d*f) + C*(d
^2*e^2 + 16*c*d*e*f + c^2*f^2)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e +
f*x]*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c -
 a*d)*f)/(d*(b*e - a*f))])/(b*Sqrt[d]*f*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(
b*e - a*f)]) + (2*Sqrt[-(b*c) + a*d]*(b*e - a*f)*(d*e - c*f)*(8*a^2*C*d*f
+ b^2*(c*C*e + 3*B*c*f + A*d*f) - a*b*(C*d*e + 7*c*C*f + 4*B*d*f))*Sqrt[(b
*(c + d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[
(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f
))])/(b*Sqrt[d]*f*Sqrt[c + d*x]*Sqrt[e + f*x]))/(3*b))/b)/(b*(b*c - a*d)*(
b*e - a*f))
```

3.64. $\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{5/2}}\,dx$

3.64.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 167
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2*p]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 2117
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Si
mp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*
(e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1
) - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x]
, x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n, 2*p]
```

3.64. $\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{5/2}}\,dx$

3.64.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1377 vs. $2(627) = 1254$.

Time = 5.45 (sec) , antiderivative size = 1378, normalized size of antiderivative = 2.01

method	result	size
elliptic	Expression too large to display	1378
default	Expression too large to display	15769

input
```
int((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(5/2),x,method=_RETU
RNVERBOSE)
```

output
```
((b*x+a)*(d*x+c)*(f*x+e))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*
(-2/3*(A*b^2-B*a*b+C*a^2)/b^5*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c
*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)/(x+a/b)^2+2/3*(b*d*f*x^2+b*c*f*x+b*d*e*x
+b*c*e)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^4*(2*A*a*b^2*d*f-A*b^3*c*f-A*b
^3*d*e-5*B*a^2*b*d*f+4*B*a*b^2*c*f+4*B*a*b^2*d*e-3*B*b^3*c*e+8*C*a^3*d*f-7
*C*a^2*b*c*f-7*C*a^2*b*d*e+6*C*a*b^2*c*e)/((x+a/b)*(b*d*f*x^2+b*c*f*x+b*d*
e*x+b*c*e))^(1/2)+2/3*C/b^3*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f
*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)+2*((A*b^2*d*f-2*B*a*b*d*f+B*b^2*c*f+B*b^2*
d*e+3*C*a^2*d*f-2*C*a*b*c*f-2*C*a*b*d*e+C*b^2*c*e)/b^4-1/3*(A*b^2-B*a*b+C*
a^2)/b^4*d*f-1/3/b^4*(a*d*f-b*c*f-b*d*e)*(2*A*a*b^2*d*f-A*b^3*c*f-A*b^3*d*
e-5*B*a^2*b*d*f+4*B*a*b^2*c*f+4*B*a*b^2*d*e-3*B*b^3*c*e+8*C*a^3*d*f-7*C*a^
2*b*c*f-7*C*a^2*b*d*e+6*C*a*b^2*c*e)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)-1/3
*(b*c*f+b*d*e)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^4*(2*A*a*b^2*d*f-A*b^3*
c*f-A*b^3*d*e-5*B*a^2*b*d*f+4*B*a*b^2*c*f+4*B*a*b^2*d*e-3*B*b^3*c*e+8*C*a^
3*d*f-7*C*a^2*b*c*f-7*C*a^2*b*d*e+6*C*a*b^2*c*e)-2/3*C/b^3*(1/2*a*c*f+1/2*
a*d*e+1/2*b*c*e))*(e/f-c/d)*((x+e/f)/(e/f-c/d))^(1/2)*((x+a/b)/(-e/f+a/b))
^(1/2)*((x+c/d)/(-e/f+c/d))^(1/2)/(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2
+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*EllipticF(((x+e/f)/(e/f-c/d))^(1/2),
((-e/f+c/d)/(-e/f+a/b))^(1/2))+2*(1/b^3*(B*b*d*f-2*C*a*d*f+C*b*c*f+C*b*d*e
)-1/3/b^3*d*f*(2*A*a*b^2*d*f-A*b^3*c*f-A*b^3*d*e-5*B*a^2*b*d*f+4*B*a*b^...
```

3.64.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.27 (sec) , antiderivative size = 2588, normalized size of antiderivative = 3.77

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(5/2),x, algor
ithm="fricas")
```

output
```
2/9*(3*(((6*C*a^2*b^4 - 2*B*a*b^5 - A*b^6)*c*d^2 - (7*C*a^3*b^3 - 3*B*a^2*
b^4)*d^3)*e*f^2 - ((7*C*a^3*b^3 - 3*B*a^2*b^4)*c*d^2 - (8*C*a^4*b^2 - 4*B*
a^3*b^3 + A*a^2*b^4)*d^3)*f^3 + ((C*b^6*c*d^2 - C*a*b^5*d^3)*e*f^2 - (C*a*
b^5*c*d^2 - C*a^2*b^4*d^3)*f^3)*x^2 + (((8*C*a*b^5 - 3*B*b^6)*c*d^2 - (9*C
*a^2*b^4 - 4*B*a*b^5 + A*b^6)*d^3)*e*f^2 - ((9*C*a^2*b^4 - 4*B*a*b^5 + A*b
^6)*c*d^2 - (10*C*a^3*b^3 - 5*B*a^2*b^4 + 2*A*a*b^5)*d^3)*f^3)*x)*sqrt(b*x
 + a)*sqrt(d*x + c)*sqrt(f*x + e) - ((C*a^2*b^4*c*d^2 - C*a^3*b^3*d^3)*e^3
 - (4*C*a^2*b^4*c^2*d - (11*C*a^3*b^3 - 3*B*a^2*b^4)*c*d^2 + (6*C*a^4*b^2
- 2*B*a^3*b^3 - A*a^2*b^4)*d^3)*e^2*f + (C*a^2*b^4*c^3 + (11*C*a^3*b^3 - 3
*B*a^2*b^4)*c^2*d - 2*(19*C*a^4*b^2 - 8*B*a^3*b^3 + 2*A*a^2*b^4)*c*d^2 + (
24*C*a^5*b - 11*B*a^4*b^2 + 2*A*a^3*b^3)*d^3)*e*f^2 - (C*a^3*b^3*c^3 + (6*
C*a^4*b^2 - 2*B*a^3*b^3 - A*a^2*b^4)*c^2*d - (24*C*a^5*b - 11*B*a^4*b^2 +
2*A*a^3*b^3)*c*d^2 + 2*(8*C*a^6 - 4*B*a^5*b + A*a^4*b^2)*d^3)*f^3 + ((C*b^
6*c*d^2 - C*a*b^5*d^3)*e^3 - (4*C*b^6*c^2*d - (11*C*a*b^5 - 3*B*b^6)*c*d^2
 + (6*C*a^2*b^4 - 2*B*a*b^5 - A*b^6)*d^3)*e^2*f + (C*b^6*c^3 + (11*C*a*b^5
 - 3*B*b^6)*c^2*d - 2*(19*C*a^2*b^4 - 8*B*a*b^5 + 2*A*b^6)*c*d^2 + (24*C*a
^3*b^3 - 11*B*a^2*b^4 + 2*A*a*b^5)*d^3)*e*f^2 - (C*a*b^5*c^3 + (6*C*a^2*b^
4 - 2*B*a*b^5 - A*b^6)*c^2*d - (24*C*a^3*b^3 - 11*B*a^2*b^4 + 2*A*a*b^5)*c
*d^2 + 2*(8*C*a^4*b^2 - 4*B*a^3*b^3 + A*a^2*b^4)*d^3)*f^3)*x^2 + 2*((C*a*b
^5*c*d^2 - C*a^2*b^4*d^3)*e^3 - (4*C*a*b^5*c^2*d - (11*C*a^2*b^4 - 3*B*...
```

3.64.6 Sympy [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{5/2}}\,dx = \int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2)/(b*x+a)**(5/2),x)
```

output
```
Integral(sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2)/(a + b*x)**(5/2),
x)
```

3.64.7 Maxima [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{5/2}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{dx+c}\sqrt{fx+e}}{(bx+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(5/2),x, algor
ithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(d*x + c)*sqrt(f*x + e)/(b*x + a)^(5/2), x
)
```

3.64.8 Giac [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{5/2}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{dx+c}\sqrt{fx+e}}{(bx+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(5/2),x, algor
ithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(d*x + c)*sqrt(f*x + e)/(b*x + a)^(5/2), x
)
```

3.64.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{5/2}}\,dx = \int \frac{\sqrt{e+f\,x}\,\sqrt{c+d\,x}\,\left(C\,x^2+B\,x+A\right)}{{\left(a+b\,x\right)}^{5/2}}\,dx$$

input
```
int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x)^(5/2),x)
```

output
```
int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x)^(5/2), x
)
```

3.64. $\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{5/2}}\,dx$

3.65 $\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{7/2}}\,dx$

3.65.1 Optimal result

Integrand size = 38, antiderivative size = 964

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{7/2}}\,dx = \frac{2(24a^3Cd^2f-a^2bd(23Cde+41cCf+4Bdf)-b^3(15c^2Ce-2A\cdots}{15\cdots}$$

$$+\frac{2(6a^3Cdf+ab^2(10cCe+3Bde+3Bcf-4Adf)-b^3(5Bce-2A(de+cf))-a^2b(Bdf+8C(de+cf)))\,\sqrt{\cdots}}{15b^2(bc-ad)(be-af)^2(a+bx)^{3/2}}$$

$$-\frac{2(Ab^2-a(bB-aC))\,(c+dx)^{3/2}(e+fx)^{3/2}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$$+\frac{2\sqrt{d}(48a^4Cd^2f^2-8a^3bdf(Bdf+11C(de+cf))-b^4(2Ad^2e^2-cde(5Be+2Af)-c^2(30Ce^2+5Bef-2\cdots}{\cdots}$$

$$+\frac{2(de-cf)\,(24a^3Cd^2f-a^2bd(23Cde+41cCf+4Bdf)-b^3(15c^2Ce-2Ad^2e+cd(5Be+Af))+ab^2(15c\cdots}{15b^4\sqrt{d}(-bc+ad)^{3/2}(be-\cdots}$$

output
```
-2/5*(A*b^2-a*(B*b-C*a))*(d*x+c)^(3/2)*(f*x+e)^(3/2)/b/(-a*d+b*c)/(-a*f+b*
e)/(b*x+a)^(5/2)+2/15*(6*a^3*C*d*f+a*b^2*(-4*A*d*f+3*B*c*f+3*B*d*e+10*C*c*
e)-b^3*(5*B*c*e-2*A*(c*f+d*e))-a^2*b*(B*d*f+8*C*(c*f+d*e)))*(f*x+e)^(3/2)*
(d*x+c)^(1/2)/b^2/(-a*d+b*c)/(-a*f+b*e)^2/(b*x+a)^(3/2)+2/15*(24*a^3*C*d^2
*f-a^2*b*d*(4*B*d*f+41*C*c*f+23*C*d*e)-b^3*(15*c^2*C*e-2*A*d^2*e+c*d*(A*f+
5*B*e))+a*b^2*(15*c^2*C*f+d^2*(-A*f+3*B*e)+c*(6*B*d*f+40*C*d*e)))*(d*x+c)^
(1/2)*(f*x+e)^(1/2)/b^3/(-a*d+b*c)^2/(-a*f+b*e)/(b*x+a)^(1/2)+2/15*(48*a^4
*C*d^2*f^2-8*a^3*b*d*f*(B*d*f+11*C*(c*f+d*e))-b^4*(2*A*d^2*e^2-c*d*e*(2*A*
f+5*B*e)-c^2*(-2*A*f^2+5*B*e*f+30*C*e^2))-a*b^3*(d^2*e*(-2*A*f+3*B*e)+c^2*
f*(3*B*f+70*C*e)+2*c*d*(-A*f^2+11*B*e*f+35*C*e^2))+a^2*b^2*(2*C*(19*c^2*f^
2+81*c*d*e*f+19*d^2*e^2)-d*f*(2*A*d*f-13*B*(c*f+d*e))))*EllipticE(d^(1/2)*
(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*d^(1/2)*(
b*(d*x+c)/(-a*d+b*c))^(1/2)*(f*x+e)^(1/2)/b^4/(a*d-b*c)^(3/2)/(-a*f+b*e)^2
/(d*x+c)^(1/2)/(b*(f*x+e)/(-a*f+b*e))^(1/2)+2/15*(-c*f+d*e)*(24*a^3*C*d^2*
f-a^2*b*d*(4*B*d*f+41*C*c*f+23*C*d*e)-b^3*(15*c^2*C*e-2*A*d^2*e+c*d*(A*f+5
*B*e))+a*b^2*(15*c^2*C*f+d^2*(-A*f+3*B*e)+c*(6*B*d*f+40*C*d*e)))*EllipticF
(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*
(b*(d*x+c)/(-a*d+b*c))^(1/2)*(b*(f*x+e)/(-a*f+b*e))^(1/2)/b^4/(a*d-b*c)^(3
/2)/(-a*f+b*e)/d^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)
```

3.65.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 33.80 (sec) , antiderivative size = 1444, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{7/2}}\,dx = \sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\Bigg(-\frac{2(Ab^2-abB+a^2C)}{5b^3(a+bx)^3}$$

$$-\frac{2(5b^3Bce-10ab^2cCe+Ab^3de-6ab^2Bde+11a^2bCde+Ab^3cf-6ab^2Bcf+11a^2bcCf-2aAb^2df+7a}{15b^3(bc-ad)(be-af)(a+bx)^2}$$

$$-\frac{2(15b^4c^2Ce^2+5b^4Bcde^2-40ab^3cCde^2-2Ab^4d^2e^2-3ab^3Bd^2e^2+23a^2b^2Cd^2e^2+5b^4Bc^2ef-40ab^3c^2C}{}$$

$$+\frac{2(a+bx)^{3/2}\left(\sqrt{-a+\frac{bc}{d}}(48a^4Cd^2f^2-8a^3bdf(Bdf+11C(de+cf))+b^4(-2Ad^2e^2+cde(5Be+2Af)+\right.}{}$$

input
```
Integrate[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^(7/2),
x]
```

output
```
Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*((-2*(A*b^2 - a*b*B + a^2*C))/(5
*b^3*(a + b*x)^3) - (2*(5*b^3*B*c*e - 10*a*b^2*c*C*e + A*b^3*d*e - 6*a*b^2
*B*d*e + 11*a^2*b*C*d*e + A*b^3*c*f - 6*a*b^2*B*c*f + 11*a^2*b*c*C*f - 2*a
*A*b^2*d*f + 7*a^2*b*B*d*f - 12*a^3*C*d*f))/(15*b^3*(b*c - a*d)*(b*e - a*f
)*(a + b*x)^2) - (2*(15*b^4*c^2*C*e^2 + 5*b^4*B*c*d*e^2 - 40*a*b^3*c*C*d*e
^2 - 2*A*b^4*d^2*e^2 - 3*a*b^3*B*d^2*e^2 + 23*a^2*b^2*C*d^2*e^2 + 5*b^4*B*
c^2*e*f - 40*a*b^3*c^2*C*e*f + 2*A*b^4*c*d*e*f - 22*a*b^3*B*c*d*e*f + 102*
a^2*b^2*c*C*d*e*f + 2*a*A*b^3*d^2*e*f + 13*a^2*b^2*B*d^2*e*f - 58*a^3*b*C*
d^2*e*f - 2*A*b^4*c^2*f^2 - 3*a*b^3*B*c^2*f^2 + 23*a^2*b^2*c^2*C*f^2 + 2*a
*A*b^3*c*d*f^2 + 13*a^2*b^2*B*c*d*f^2 - 58*a^3*b*c*C*d*f^2 - 2*a^2*A*b^2*d
^2*f^2 - 8*a^3*b*B*d^2*f^2 + 33*a^4*C*d^2*f^2))/(15*b^3*(b*c - a*d)^2*(b*e
 - a*f)^2*(a + b*x))) + (2*(a + b*x)^(3/2)*(Sqrt[-a + (b*c)/d]*(48*a^4*C*d
^2*f^2 - 8*a^3*b*d*f*(B*d*f + 11*C*(d*e + c*f)) + b^4*(-2*A*d^2*e^2 + c*d*
e*(5*B*e + 2*A*f) + c^2*(30*C*e^2 + 5*B*e*f - 2*A*f^2)) - a*b^3*(d^2*e*(3*
B*e - 2*A*f) + c^2*f*(70*C*e + 3*B*f) + 2*c*d*(35*C*e^2 + 11*B*e*f - A*f^2
)) + a^2*b^2*(2*C*(19*d^2*e^2 + 81*c*d*e*f + 19*c^2*f^2) + d*f*(-2*A*d*f +
 13*B*(d*e + c*f))))*(d + (b*c)/(a + b*x) - (a*d)/(a + b*x))*(f + (b*e)/(a
 + b*x) - (a*f)/(a + b*x)) + (I*(-(b*c) + a*d)*f*(-48*a^4*C*d^2*f^2 + 8*a^
3*b*d*f*(B*d*f + 11*C*(d*e + c*f)) - b^4*(-2*A*d^2*e^2 + c*d*e*(5*B*e + 2*
A*f) + c^2*(30*C*e^2 + 5*B*e*f - 2*A*f^2)) + a*b^3*(d^2*e*(3*B*e - 2*A*...
```

3.65.3 Rubi [A] (verified)

Time = 2.17 (sec) , antiderivative size = 996, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2117, 27, 167, 27, 167, 27, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)}{(a+bx)^{7/2}}\,dx$$

$$\Big\downarrow \text{2117}$$

$$-\frac{2\int -\frac{\sqrt{c+dx}\sqrt{e+fx}\left(3C(de+cf)a^2-b(5cCe+3Bde+3Bcf-5Adf)a+b^2(5Bce-2A(de+cf))-b\left(-\frac{6Cdfa^2}{b}+Bdfa+5C(de+cf)a-b(5cCe+Ad\right.\right.}{2b(a+bx)^{5/2}}}{5(bc-ad)(be-af)}$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{5b(a+bx)^{5/2}(bc-ad)(be-af)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(3C(de+cf)a^2-b(5cCe+3Bde+3Bcf-5Adf)a+b^2(5Bce-2A(de+cf))-b\left(-\frac{6Cdfa^2}{b}+Bdfa+5C(de+cf)a-b(5cCe+Adf)\right)x\right)}{(a+bx)^{5/2}}}{5b(bc-ad)(be-af)}$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{5b(a+bx)^{5/2}(bc-ad)(be-af)}$$

$\downarrow$ 167

$$\frac{2\int -\frac{\sqrt{e+fx}\left(6Cdf(de+3cf)a^3-b\left(Bdf(de+3cf)+C\left(8d^2e^2+41cdfe+15c^2f^2\right)\right)a^2+b^2\left(30Cefc^2+d\left(25Ce^2+6Bfe+3Af^2\right)c+d^2e(3Be-4Af)\right)a-b^3e\left(15Cec^2+d(5\right.}{2(a+bx)^{3/2}\sqrt{}}}{3b(be-af)}$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{5b(a+bx)^{5/2}(bc-ad)(be-af)}$$

$\downarrow$ 27

$$\frac{2\sqrt{c+dx}(e+fx)^{3/2}(6a^3Cdf-a^2b(Bdf+8C(cf+de))+ab^2(-4Adf+3Bcf+3Bde+10cCe)-b^3(5Bce-2A(cf+de)))}{3b(a+bx)^{3/2}(be-af)}-\frac{\int\frac{\sqrt{e+fx}\left(6Cdf(de+3cf)a^3-b\right.}{}}{}$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{5b(a+bx)^{5/2}(bc-ad)(be-af)}$$

$\downarrow$ 167

$$\frac{2(6Cdfa^3-b(Bdf+8C(de+cf))a^2+b^2(10cCe+3Bde+3Bcf-4Adf)a-b^3(5Bce-2A(de+cf)))\sqrt{c+dx}(e+fx)^{3/2}}{3b(be-af)(a+bx)^{3/2}}-\frac{2\int-\frac{24Cd^2f^2(de+cf)a^4-bdf\left(4\right.}{}}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}(e+fx)^{3/2}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$\downarrow$ 27

$$\frac{2(6Cdfa^3-b(Bdf+8C(de+cf))a^2+b^2(10cCe+3Bde+3Bcf-4Adf)a-b^3(5Bce-2A(de+cf)))\sqrt{c+dx}(e+fx)^{3/2}}{3b(be-af)(a+bx)^{3/2}}-\frac{-\frac{2(be-af)\sqrt{c+dx}\sqrt{e+fx}\left(24Cd\right.}{}}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}(e+fx)^{3/2}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$\downarrow$ 176

3.65. $\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{7/2}}\,dx$

$$\frac{\frac{2\sqrt{c+dx}(e+fx)^{3/2}\left(6a^3Cdf-a^2b(Bdf+8C(cf+de))+ab^2(-4Adf+3Bcf+3Bde+10cCe)-b^3(5Bce-2A(cf+de))\right)}{3b(a+bx)^{3/2}(be-af)} - \frac{-\frac{(be-af)(de-cf)\left(24a^3Cd^2f\right.}{}}{}}{}$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{5b(a+bx)^{5/2}(bc-ad)(be-af)}$$

$\Big\downarrow$ 124

$$\frac{\frac{2\left(6Cdfa^3-b(Bdf+8C(de+cf))a^2+b^2(10cCe+3Bde+3Bcf-4Adf)a-b^3(5Bce-2A(de+cf))\right)\sqrt{c+dx}(e+fx)^{3/2}}{3b(be-af)(a+bx)^{3/2}} - \frac{-\frac{2(be-af)\sqrt{c+dx}\sqrt{e+fx}\left(24Cd\right.}{}}{}}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}(e+fx)^{3/2}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$\Big\downarrow$ 123

$$\frac{\frac{2\left(6Cdfa^3-b(Bdf+8C(de+cf))a^2+b^2(10cCe+3Bde+3Bcf-4Adf)a-b^3(5Bce-2A(de+cf))\right)\sqrt{c+dx}(e+fx)^{3/2}}{3b(be-af)(a+bx)^{3/2}} - \frac{-\frac{2(be-af)\sqrt{c+dx}\sqrt{e+fx}\left(24Cd\right.}{}}{}}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}(e+fx)^{3/2}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$\Big\downarrow$ 131

$$\frac{\frac{2\left(6Cdfa^3-b(Bdf+8C(de+cf))a^2+b^2(10cCe+3Bde+3Bcf-4Adf)a-b^3(5Bce-2A(de+cf))\right)\sqrt{c+dx}(e+fx)^{3/2}}{3b(be-af)(a+bx)^{3/2}} - \frac{-\frac{2(be-af)\sqrt{c+dx}\sqrt{e+fx}\left(24Cd\right.}{}}{}}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}(e+fx)^{3/2}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$\Big\downarrow$ 131

3.65. $\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)}{(a+bx)^{7/2}}\,dx$

$$\frac{\frac{2\left(6Cdfa^3-b(Bdf+8C(de+cf))a^2+b^2(10cCe+3Bde+3Bcf-4Adf)a-b^3(5Bce-2A(de+cf))\right)\sqrt{c+dx}(e+fx)^{3/2}}{3b(be-af)(a+bx)^{3/2}}-\frac{-\frac{2(be-af)\sqrt{c+dx}\sqrt{e+fx}\left(24Cd\right.}{}}{}}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}(e+fx)^{3/2}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$$\Big\downarrow \; 130$$

$$\frac{\frac{2\left(6Cdfa^3-b(Bdf+8C(de+cf))a^2+b^2(10cCe+3Bde+3Bcf-4Adf)a-b^3(5Bce-2A(de+cf))\right)\sqrt{c+dx}(e+fx)^{3/2}}{3b(be-af)(a+bx)^{3/2}}-\frac{-\frac{2(be-af)\sqrt{c+dx}\sqrt{e+fx}\left(24Cd\right.}{}}{}}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}(e+fx)^{3/2}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

input
```
Int[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^(7/2),x]
```

output
```
(-2*(A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(5*b*(b*c - a
*d)*(b*e - a*f)*(a + b*x)^(5/2)) + ((2*(6*a^3*C*d*f + a*b^2*(10*c*C*e + 3*
B*d*e + 3*B*c*f - 4*A*d*f) - b^3*(5*B*c*e - 2*A*(d*e + c*f)) - a^2*b*(B*d*
f + 8*C*(d*e + c*f)))*Sqrt[c + d*x]*(e + f*x)^(3/2))/(3*b*(b*e - a*f)*(a +
 b*x)^(3/2)) - ((-2*(b*e - a*f)*(24*a^3*C*d^2*f - a^2*b*d*(23*C*d*e + 41*c
*C*f + 4*B*d*f) - b^3*(15*c^2*C*e - 2*A*d^2*e + c*d*(5*B*e + A*f)) + a*b^2
*(15*c^2*C*f + d^2*(3*B*e - A*f) + c*(40*C*d*e + 6*B*d*f)))*Sqrt[c + d*x]*
Sqrt[e + f*x])/(b*(b*c - a*d)*Sqrt[a + b*x]) - ((2*Sqrt[d]*Sqrt[-(b*c) + a
*d]*(48*a^4*C*d^2*f^2 - 8*a^3*b*d*f*(B*d*f + 11*C*(d*e + c*f)) - b^4*(2*A*
d^2*e^2 - c*d*e*(5*B*e + 2*A*f) - c^2*(30*C*e^2 + 5*B*e*f - 2*A*f^2)) - a*
b^3*(d^2*e*(3*B*e - 2*A*f) + c^2*f*(70*C*e + 3*B*f) + 2*c*d*(35*C*e^2 + 11
*B*e*f - A*f^2)) + a^2*b^2*(2*C*(19*d^2*e^2 + 81*c*d*e*f + 19*c^2*f^2) - d
*f*(2*A*d*f - 13*B*(d*e + c*f))))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e +
 f*x]*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c
- a*d)*f)/(d*(b*e - a*f))])/(b*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f
)]) + (2*Sqrt[-(b*c) + a*d]*(b*e - a*f)*(d*e - c*f)*(24*a^3*C*d^2*f - a^2*
b*d*(23*C*d*e + 41*c*C*f + 4*B*d*f) - b^3*(15*c^2*C*e - 2*A*d^2*e + c*d*(5
*B*e + A*f)) + a*b^2*(15*c^2*C*f + d^2*(3*B*e - A*f) + c*(40*C*d*e + 6*B*d
*f)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*Elli
pticF[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*...
```

3.65. $\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)}{(a+bx)^{7/2}}\,dx$

3.65.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

3.65. $\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{7/2}}\,dx$

rule 167
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2*p]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 2117
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Si
mp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*
(e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1
) - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x]
, x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n, 2*p]
```

3.65.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2291 vs. $2(902) = 1804$.

Time = 4.64 (sec) , antiderivative size = 2292, normalized size of antiderivative = 2.38

method	result	size
elliptic	Expression too large to display	2292
default	Expression too large to display	34614

input
```
int((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(7/2),x,method=_RETU
RNVERBOSE)
```

output
```
((b*x+a)*(d*x+c)*(f*x+e))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*
(-2/5*(A*b^2-B*a*b+C*a^2)/b^6*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c
*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)/(x+a/b)^3+2/15*(2*A*a*b^2*d*f-A*b^3*c*f-
A*b^3*d*e-7*B*a^2*b*d*f+6*B*a*b^2*c*f+6*B*a*b^2*d*e-5*B*b^3*c*e+12*C*a^3*d
*f-11*C*a^2*b*c*f-11*C*a^2*b*d*e+10*C*a*b^2*c*e)/b^5/(a^2*d*f-a*b*c*f-a*b*
d*e+b^2*c*e)*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*
e*x+a*c*e)^(1/2)/(x+a/b)^2+2/15*(b*d*f*x^2+b*c*f*x+b*d*e*x+b*c*e)/(a^2*d*f
-a*b*c*f-a*b*d*e+b^2*c*e)^2/b^4*(2*A*a^2*b^2*d^2*f^2-2*A*a*b^3*c*d*f^2-2*A
*a*b^3*d^2*e*f+2*A*b^4*c^2*f^2-2*A*b^4*c*d*e*f+2*A*b^4*d^2*e^2+8*B*a^3*b*d
^2*f^2-13*B*a^2*b^2*c*d*f^2-13*B*a^2*b^2*d^2*e*f+3*B*a*b^3*c^2*f^2+22*B*a*
b^3*c*d*e*f+3*B*a*b^3*d^2*e^2-5*B*b^4*c^2*e*f-5*B*b^4*c*d*e^2-33*C*a^4*d^2
*f^2+58*C*a^3*b*c*d*f^2+58*C*a^3*b*d^2*e*f-23*C*a^2*b^2*c^2*f^2-102*C*a^2*
b^2*c*d*e*f-23*C*a^2*b^2*d^2*e^2+40*C*a*b^3*c^2*e*f+40*C*a*b^3*c*d*e^2-15*
C*b^4*c^2*e^2)/((x+a/b)*(b*d*f*x^2+b*c*f*x+b*d*e*x+b*c*e))^(1/2)+2*((B*b*d
*f-3*C*a*d*f+C*b*c*f+C*b*d*e)/b^4+1/15*d*f*(2*A*a*b^2*d*f-A*b^3*c*f-A*b^3*
d*e-7*B*a^2*b*d*f+6*B*a*b^2*c*f+6*B*a*b^2*d*e-5*B*b^3*c*e+12*C*a^3*d*f-11*
C*a^2*b*c*f-11*C*a^2*b*d*e+10*C*a*b^2*c*e)/b^4/(a^2*d*f-a*b*c*f-a*b*d*e+b^
2*c*e)-1/15/b^4*(a*d*f-b*c*f-b*d*e)*(2*A*a^2*b^2*d^2*f^2-2*A*a*b^3*c*d*f^2
-2*A*a*b^3*d^2*e*f+2*A*b^4*c^2*f^2-2*A*b^4*c*d*e*f+2*A*b^4*d^2*e^2+8*B*a^3
*b*d^2*f^2-13*B*a^2*b^2*c*d*f^2-13*B*a^2*b^2*d^2*e*f+3*B*a*b^3*c^2*f^2+...
```

3.65.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.86 (sec) , antiderivative size = 4721, normalized size of antiderivative = 4.90

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(7/2),x, algor
ithm="fricas")
```

output
```
-2/45*(3*((15*C*a^4*b^4*d^3 + (8*C*a^2*b^6 + 2*B*a*b^7 + 3*A*b^8)*c^2*d -
5*(5*C*a^3*b^5 + A*a*b^7)*c*d^2)*e^2*f - (5*(5*C*a^3*b^5 + A*a*b^7)*c^2*d
- 10*(7*C*a^4*b^4 - B*a^3*b^5 + A*a^2*b^6)*c*d^2 + (41*C*a^5*b^3 - 6*B*a^4
*b^4 + A*a^3*b^5)*d^3)*e*f^2 + (15*C*a^4*b^4*c^2*d - (41*C*a^5*b^3 - 6*B*a
^4*b^4 + A*a^3*b^5)*c*d^2 + (24*C*a^6*b^2 - 4*B*a^5*b^3 - A*a^4*b^4)*d^3)*
f^3 + ((15*C*b^8*c^2*d - 5*(8*C*a*b^7 - B*b^8)*c*d^2 + (23*C*a^2*b^6 - 3*B
*a*b^7 - 2*A*b^8)*d^3)*e^2*f - (5*(8*C*a*b^7 - B*b^8)*c^2*d - 2*(51*C*a^2*
b^6 - 11*B*a*b^7 + A*b^8)*c*d^2 + (58*C*a^3*b^5 - 13*B*a^2*b^6 - 2*A*a*b^7
)*d^3)*e*f^2 + ((23*C*a^2*b^6 - 3*B*a*b^7 - 2*A*b^8)*c^2*d - (58*C*a^3*b^5
 - 13*B*a^2*b^6 - 2*A*a*b^7)*c*d^2 + (33*C*a^4*b^4 - 8*B*a^3*b^5 - 2*A*a^2
*b^6)*d^3)*f^3)*x^2 + ((5*(4*C*a*b^7 + B*b^8)*c^2*d - (59*C*a^2*b^6 + B*a*
b^7 - A*b^8)*c*d^2 + 5*(7*C*a^3*b^5 - A*a*b^7)*d^3)*e^2*f - ((59*C*a^2*b^6
 + B*a*b^7 - A*b^8)*c^2*d - 20*(8*C*a^3*b^5 - B*a^2*b^6)*c*d^2 + (93*C*a^4
*b^4 - 13*B*a^3*b^5 - 7*A*a^2*b^6)*d^3)*e*f^2 + (5*(7*C*a^3*b^5 - A*a*b^7)
*c^2*d - (93*C*a^4*b^4 - 13*B*a^3*b^5 - 7*A*a^2*b^6)*c*d^2 + 3*(18*C*a^5*b
^3 - 3*B*a^4*b^4 - 2*A*a^3*b^5)*d^3)*f^3)*x)*sqrt(b*x + a)*sqrt(d*x + c)*s
qrt(f*x + e) - ((15*C*a^3*b^5*c^2*d - 5*(4*C*a^4*b^4 + B*a^3*b^5)*c*d^2 +
(7*C*a^5*b^3 + 3*B*a^4*b^4 + 2*A*a^3*b^5)*d^3)*e^3 + (15*C*a^3*b^5*c^3 - 1
0*(13*C*a^4*b^4 - 2*B*a^3*b^5)*c^2*d + (182*C*a^5*b^3 - 22*B*a^4*b^4 - 3*A
*a^3*b^5)*c*d^2 - (73*C*a^6*b^2 - 8*B*a^5*b^3 + 3*A*a^4*b^4)*d^3)*e^2*f...
```

3.65.6 Sympy [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{7/2}}\,dx = \int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{\frac{7}{2}}}\,dx$$

input
```
integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2)/(b*x+a)**(7/2),x)
```

output
```
Integral(sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2)/(a + b*x)**(7/2),
x)
```

3.65.7 Maxima [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{7/2}}\,dx=\int \frac{(Cx^2+Bx+A)\sqrt{dx+c}\sqrt{fx+e}}{(bx+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(7/2),x, algor
ithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(d*x + c)*sqrt(f*x + e)/(b*x + a)^(7/2), x
)
```

3.65.8 Giac [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{7/2}}\,dx=\int \frac{(Cx^2+Bx+A)\sqrt{dx+c}\sqrt{fx+e}}{(bx+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(7/2),x, algor
ithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(d*x + c)*sqrt(f*x + e)/(b*x + a)^(7/2), x
)
```

3.65.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{7/2}}\,dx=\int \frac{\sqrt{e+f\,x}\,\sqrt{c+d\,x}\,(C\,x^2+B\,x+A)}{(a+b\,x)^{7/2}}\,dx$$

input
```
int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x)^(7/2),x)
```

output
```
int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x)^(7/2), x
)
```

3.66 $\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{9/2}}\,dx$

3.66.1 Optimal result

Integrand size = 38, antiderivative size = 1716

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{9/2}}\,dx = \text{Too large to display}$$

output
```
-2/7*(A*b^2-a*(B*b-C*a))*(d*x+c)^(3/2)*(f*x+e)^(3/2)/b/(-a*d+b*c)/(-a*f+b*
e)/(b*x+a)^(7/2)+2/35*(6*a^3*C*d*f+a*b^2*(-8*A*d*f+3*B*c*f+3*B*d*e+14*C*c*
e)-b^3*(7*B*c*e-4*A*(c*f+d*e))+a^2*b*(B*d*f-10*C*(c*f+d*e)))*(f*x+e)^(3/2)
*(d*x+c)^(1/2)/b^2/(-a*d+b*c)/(-a*f+b*e)^2/(b*x+a)^(5/2)-2/105*(24*a^4*C*d
^2*f^2-a^3*b*d*f*(-4*B*d*f+43*C*c*f+61*C*d*e)-3*a*b^3*(d^2*e*(-3*A*f+B*e)+
2*c^2*f*(-B*f+7*C*e)+c*d*(5*A*f^2-5*B*e*f+28*C*e^2))-b^4*(4*A*d^2*e^2-c*d*
e*(-A*f+7*B*e)-c^2*(8*A*f^2-14*B*e*f+35*C*e^2))-3*a^2*b^2*(d*f*(-A*d*f+2*B
*c*f+3*B*d*e)-C*(5*c^2*f^2+37*c*d*e*f+15*d^2*e^2)))*(d*x+c)^(1/2)*(f*x+e)^
(1/2)/b^3/(-a*d+b*c)^2/(-a*f+b*e)^2/(b*x+a)^(3/2)+2/105*(48*a^5*C*d^3*f^3+
8*a^4*b*d^2*f^2*(B*d*f-16*C*(c*f+d*e))-b^5*(8*A*d^3*e^3-c*d^2*e^2*(5*A*f+1
4*B*e)+c^2*d*e*(-5*A*f^2+14*B*e*f+35*C*e^2)+c^3*f*(8*A*f^2-14*B*e*f+35*C*e
^2))-a*b^4*(d^3*e^2*(-19*A*f+6*B*e)-6*c^3*f^2*(-B*f+7*C*e)-c^2*d*f*(238*C*
e^2-19*f*(-A*f+B*e))-c*d^2*e*(42*C*e^2-f*(20*A*f+19*B*e)))+a^3*b^2*d*f*(C*
(103*c^2*f^2+344*c*d*e*f+103*d^2*e^2)+d*f*(6*A*d*f-19*B*(c*f+d*e)))-3*a^2*
b^3*(C*(5*c^3*f^3+94*c^2*d*e*f^2+94*c*d^2*e^2*f+5*d^3*e^3)+d*f*(3*A*d*f*(c
*f+d*e)-B*(3*c^2*f^2+16*c*d*e*f+3*d^2*e^2))))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/
b^3/(-a*d+b*c)^3/(-a*f+b*e)^3/(b*x+a)^(1/2)+2/105*(48*a^5*C*d^3*f^3+8*a^4*
b*d^2*f^2*(B*d*f-16*C*(c*f+d*e))-b^5*(8*A*d^3*e^3-c*d^2*e^2*(5*A*f+14*B*e)
+c^2*d*e*(-5*A*f^2+14*B*e*f+35*C*e^2)+c^3*f*(8*A*f^2-14*B*e*f+35*C*e^2))-a
*b^4*(d^3*e^2*(-19*A*f+6*B*e)-6*c^3*f^2*(-B*f+7*C*e)-c^2*d*f*(238*C*e^2...
```

3.66.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 36.94 (sec) , antiderivative size = 2437, normalized size of antiderivative = 1.42

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{9/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^(9/2),
x]
```

output
```
Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*((-2*(A*b^2 - a*b*B + a^2*C))/(7
*b^3*(a + b*x)^4) - (2*(7*b^3*B*c*e - 14*a*b^2*c*C*e + A*b^3*d*e - 8*a*b^2
*B*d*e + 15*a^2*b*C*d*e + A*b^3*c*f - 8*a*b^2*B*c*f + 15*a^2*b*c*C*f - 2*a
*A*b^2*d*f + 9*a^2*b*B*d*f - 16*a^3*C*d*f))/(35*b^3*(b*c - a*d)*(b*e - a*f
)*(a + b*x)^3) - (2*(35*b^4*c^2*C*e^2 + 7*b^4*B*c*d*e^2 - 84*a*b^3*c*C*d*e
^2 - 4*A*b^4*d^2*e^2 - 3*a*b^3*B*d^2*e^2 + 45*a^2*b^2*C*d^2*e^2 + 7*b^4*B*
c^2*e*f - 84*a*b^3*c^2*C*e*f + 2*A*b^4*c*d*e*f - 30*a*b^3*B*c*d*e*f + 198*
a^2*b^2*c*C*d*e*f + 6*a*A*b^3*d^2*e*f + 15*a^2*b^2*B*d^2*e*f - 106*a^3*b*C
*d^2*e*f - 4*A*b^4*c^2*f^2 - 3*a*b^3*B*c^2*f^2 + 45*a^2*b^2*c^2*C*f^2 + 6*
a*A*b^3*c*d*f^2 + 15*a^2*b^2*B*c*d*f^2 - 106*a^3*b*c*C*d*f^2 - 6*a^2*A*b^2
*d^2*f^2 - 8*a^3*b*B*d^2*f^2 + 57*a^4*C*d^2*f^2))/(105*b^3*(b*c - a*d)^2*(
b*e - a*f)^2*(a + b*x)^2) - (2*(35*b^5*c^2*C*d*e^3 - 14*b^5*B*c*d^2*e^3 -
42*a*b^4*c*C*d^2*e^3 + 8*A*b^5*d^3*e^3 + 6*a*b^4*B*d^3*e^3 + 15*a^2*b^3*C*
d^3*e^3 + 35*b^5*c^3*C*e^2*f + 14*b^5*B*c^2*d*e^2*f - 238*a*b^4*c^2*C*d*e^
2*f - 5*A*b^5*c*d^2*e^2*f + 19*a*b^4*B*c*d^2*e^2*f + 282*a^2*b^3*c*C*d^2*e
^2*f - 19*a*A*b^4*d^3*e^2*f - 9*a^2*b^3*B*d^3*e^2*f - 103*a^3*b^2*C*d^3*e^
2*f - 14*b^5*B*c^3*e*f^2 - 42*a*b^4*c^3*C*e*f^2 - 5*A*b^5*c^2*d*e*f^2 + 19
*a*b^4*B*c^2*d*e*f^2 + 282*a^2*b^3*c^2*C*d*e*f^2 + 20*a*A*b^4*c*d^2*e*f^2
- 48*a^2*b^3*B*c*d^2*e*f^2 - 344*a^3*b^2*c*C*d^2*e*f^2 + 9*a^2*A*b^3*d^3*e
*f^2 + 19*a^3*b^2*B*d^3*e*f^2 + 128*a^4*b*C*d^3*e*f^2 + 8*A*b^5*c^3*f^3...
```

3.66.3 Rubi [A] (verified)

Time = 4.10 (sec) , antiderivative size = 1770, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {2117, 27, 167, 27, 167, 27, 169, 27, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

3.66. $\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{9/2}}\,dx$

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)}{(a+bx)^{9/2}}\,dx$$

$$\Big\downarrow \; 2117$$

$$-\frac{2\int -\frac{\sqrt{c+dx}\sqrt{e+fx}\left(3C(de+cf)a^2-b(7cCe+3Bde+3Bcf-7Adf)a+b^2(7Bce-4A(de+cf))+b\left(\frac{6Cdfa^2}{b}-7Cdea-7cCfa+Bdfa+7bcCe-Abd\right.}{2b(a+bx)^{7/2}}}{7(bc-ad)(be-af)}$$
$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{7b(a+bx)^{7/2}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(3C(de+cf)a^2-b(7cCe+3Bde+3Bcf-7Adf)a+b^2(7Bce-4A(de+cf))+b\left(\frac{6Cdfa^2}{b}+Bdfa-7C(de+cf)a+b(7cCe-Adf)\right)x\right)}{(a+bx)^{7/2}}dx}{7b(bc-ad)(be-af)}$$
$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{7b(a+bx)^{7/2}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 167$$

$$2\int -\frac{\sqrt{e+fx}\left(6Cdf(de+3cf)a^3+b\left(Bdf(de+3cf)-5C\left(2d^2e^2+11cdfe+3c^2f^2\right)\right)a^2+b^2\left(6f(7Ce-Bf)c^2+d\left(49Ce^2-8Bfe+11Af^2\right)c+d^2e(3Be-8Af)\right)a+b^3\left(-\left(\left(35\right.\right.\right.\right.}{}$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{7b(a+bx)^{7/2}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2\sqrt{c+dx}(e+fx)^{3/2}\left(6a^3Cdf+a^2b(Bdf-10C(cf+de))+ab^2(-8Adf+3Bcf+3Bde+14cCe)-b^3(7Bce-4A(cf+de))\right)}{5b(a+bx)^{5/2}(be-af)}-\int\frac{\sqrt{e+fx}\left(6Cdf(de+3cf)a^3+\right.}{}}{}$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{7b(a+bx)^{7/2}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 167$$

$$\frac{\frac{2\left(6Cdfa^3+b(Bdf-10C(de+cf))a^2+b^2(14cCe+3Bde+3Bcf-8Adf)a-b^3(7Bce-4A(de+cf))\right)\sqrt{c+dx}(e+fx)^{3/2}}{5b(be-af)(a+bx)^{5/2}}-\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(24Cd^2f^2a^4-\right.}{}}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}(e+fx)^{3/2}}{7b(bc-ad)(be-af)(a+bx)^{7/2}}$$

3.66. $\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{9/2}}\,dx$

$\Big\downarrow$ 27

$$\frac{\frac{2\left(6Cdfa^3+b(Bdf-10C(de+cf))a^2+b^2(14cCe+3Bde+3Bcf-8Adf)a-b^3(7Bce-4A(de+cf))\right)\sqrt{c+dx}(e+fx)^{3/2}}{5b(be-af)(a+bx)^{5/2}}-\frac{2\left(24Cd^2f^2a^4-bdf(61Cde+43c\right.}{}}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}(e+fx)^{3/2}}{7b(bc-ad)(be-af)(a+bx)^{7/2}}$$

$\Big\downarrow$ 169

$$\frac{\frac{2\left(6Cdfa^3+b(Bdf-10C(de+cf))a^2+b^2(14cCe+3Bde+3Bcf-8Adf)a-b^3(7Bce-4A(de+cf))\right)\sqrt{c+dx}(e+fx)^{3/2}}{5b(be-af)(a+bx)^{5/2}}-\frac{2\left(24Cd^2f^2a^4-bdf(61Cde+43c\right.}{}}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}(e+fx)^{3/2}}{7b(bc-ad)(be-af)(a+bx)^{7/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{2\left(6Cdfa^3+b(Bdf-10C(de+cf))a^2+b^2(14cCe+3Bde+3Bcf-8Adf)a-b^3(7Bce-4A(de+cf))\right)\sqrt{c+dx}(e+fx)^{3/2}}{5b(be-af)(a+bx)^{5/2}}-\frac{2\left(24Cd^2f^2a^4-bdf(61Cde+43c\right.}{}}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}(e+fx)^{3/2}}{7b(bc-ad)(be-af)(a+bx)^{7/2}}$$

$\Big\downarrow$ 176

$$\frac{\frac{2\left(6Cdfa^3+b(Bdf-10C(de+cf))a^2+b^2(14cCe+3Bde+3Bcf-8Adf)a-b^3(7Bce-4A(de+cf))\right)\sqrt{c+dx}(e+fx)^{3/2}}{5b(be-af)(a+bx)^{5/2}}-\frac{2\left(24Cd^2f^2a^4-bdf(61Cde+43c\right.}{}}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}(e+fx)^{3/2}}{7b(bc-ad)(be-af)(a+bx)^{7/2}}$$

$\Big\downarrow$ 124

3.66. $\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)}{(a+bx)^{9/2}}\,dx$

$$\frac{\frac{2\left(6Cdfa^3+b(Bdf-10C(de+cf))a^2+b^2(14cCe+3Bde+3Bcf-8Adf)a-b^3(7Bce-4A(de+cf))\right)\sqrt{c+dx}(e+fx)^{3/2}}{5b(be-af)(a+bx)^{5/2}}-\frac{2\left(24Cd^2f^2a^4-bdf(61Cde+43c\right.}{}}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}(e+fx)^{3/2}}{7b(bc-ad)(be-af)(a+bx)^{7/2}}$$

$$\Big\downarrow \text{ 123}$$

$$\frac{\frac{2\left(6Cdfa^3+b(Bdf-10C(de+cf))a^2+b^2(14cCe+3Bde+3Bcf-8Adf)a-b^3(7Bce-4A(de+cf))\right)\sqrt{c+dx}(e+fx)^{3/2}}{5b(be-af)(a+bx)^{5/2}}-\frac{2\left(24Cd^2f^2a^4-bdf(61Cde+43c\right.}{}}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}(e+fx)^{3/2}}{7b(bc-ad)(be-af)(a+bx)^{7/2}}$$

$$\Big\downarrow \text{ 131}$$

$$\frac{\frac{2\left(6Cdfa^3+b(Bdf-10C(de+cf))a^2+b^2(14cCe+3Bde+3Bcf-8Adf)a-b^3(7Bce-4A(de+cf))\right)\sqrt{c+dx}(e+fx)^{3/2}}{5b(be-af)(a+bx)^{5/2}}-\frac{2\left(24Cd^2f^2a^4-bdf(61Cde+43c\right.}{}}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}(e+fx)^{3/2}}{7b(bc-ad)(be-af)(a+bx)^{7/2}}$$

$$\Big\downarrow \text{ 131}$$

$$\frac{\frac{2\left(6Cdfa^3+b(Bdf-10C(de+cf))a^2+b^2(14cCe+3Bde+3Bcf-8Adf)a-b^3(7Bce-4A(de+cf))\right)\sqrt{c+dx}(e+fx)^{3/2}}{5b(be-af)(a+bx)^{5/2}}-\frac{2\left(24Cd^2f^2a^4-bdf(61Cde+43c\right.}{}}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}(e+fx)^{3/2}}{7b(bc-ad)(be-af)(a+bx)^{7/2}}$$

$$\Big\downarrow \text{ 130}$$

3.66. $\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)}{(a+bx)^{9/2}}\,dx$

$$\frac{2\left(6Cdfa^3+b(Bdf-10C(de+cf))a^2+b^2(14cCe+3Bde+3Bcf-8Adf)a-b^3(7Bce-4A(de+cf))\right)\sqrt{c+dx}(e+fx)^{3/2}}{5b(be-af)(a+bx)^{5/2}} - \frac{2\left(24Cd^2f^2a^4-bdf(61Cde+43c\right.}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}(e+fx)^{3/2}}{7b(bc-ad)(be-af)(a+bx)^{7/2}}$$

input
```
Int[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^(9/2),x]
```

output
```
(-2*(A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(7*b*(b*c - a
*d)*(b*e - a*f)*(a + b*x)^(7/2)) + ((2*(6*a^3*C*d*f + a*b^2*(14*c*C*e + 3*
B*d*e + 3*B*c*f - 8*A*d*f) - b^3*(7*B*c*e - 4*A*(d*e + c*f)) + a^2*b*(B*d*
f - 10*C*(d*e + c*f)))*Sqrt[c + d*x]*(e + f*x)^(3/2))/(5*b*(b*e - a*f)*(a
+ b*x)^(5/2)) - ((2*(24*a^4*C*d^2*f^2 - a^3*b*d*f*(61*C*d*e + 43*c*C*f - 4
*B*d*f) - 3*a*b^3*(d^2*e*(B*e - 3*A*f) + 2*c^2*f*(7*C*e - B*f) + c*d*(28*C
*e^2 - 5*B*e*f + 5*A*f^2)) - b^4*(4*A*d^2*e^2 - c*d*e*(7*B*e - A*f) - c^2*
(35*C*e^2 - 14*B*e*f + 8*A*f^2)) - 3*a^2*b^2*(d*f*(3*B*d*e + 2*B*c*f - A*d
*f) - C*(15*d^2*e^2 + 37*c*d*e*f + 5*c^2*f^2)))*Sqrt[c + d*x]*Sqrt[e + f*x
])/(3*b*(b*c - a*d)*(a + b*x)^(3/2)) - ((2*(48*a^5*C*d^3*f^3 + 8*a^4*b*d^2
*f^2*(B*d*f - 16*C*(d*e + c*f)) - b^5*(8*A*d^3*e^3 - c*d^2*e^2*(14*B*e + 5
*A*f) + c^2*d*e*(35*C*e^2 + 14*B*e*f - 5*A*f^2) + c^3*f*(35*C*e^2 - 14*B*e
*f + 8*A*f^2)) - a*b^4*(d^3*e^2*(6*B*e - 19*A*f) - 6*c^3*f^2*(7*C*e - B*f)
 - c^2*d*f*(238*C*e^2 - 19*f*(B*e - A*f)) - c*d^2*e*(42*C*e^2 - f*(19*B*e
+ 20*A*f))) + a^3*b^2*d*f*(C*(103*d^2*e^2 + 344*c*d*e*f + 103*c^2*f^2) + d
*f*(6*A*d*f - 19*B*(d*e + c*f))) - 3*a^2*b^3*(C*(5*d^3*e^3 + 94*c*d^2*e^2*
f + 94*c^2*d*e*f^2 + 5*c^3*f^3) + d*f*(3*A*d*f*(d*e + c*f) - B*(3*d^2*e^2
+ 16*c*d*e*f + 3*c^2*f^2))))*Sqrt[c + d*x]*Sqrt[e + f*x])/((b*c - a*d)*(b*
e - a*f)*Sqrt[a + b*x]) - (d*f*((2*Sqrt[-(b*c) + a*d]*(48*a^5*C*d^3*f^3 +
8*a^4*b*d^2*f^2*(B*d*f - 16*C*(d*e + c*f)) - b^5*(8*A*d^3*e^3 - c*d^2*e...
```

3.66. $\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{9/2}}\,dx$

3.66.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

3.66. $\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{9/2}}\,dx$

rule 167
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2*p]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 2117
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Si
mp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*
(e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1
) - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x]
, x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n, 2*p]
```

3.66. $\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{9/2}} \, dx$

3.66.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3899 vs. 2(1646) = 3292.

Time = 5.38 (sec) , antiderivative size = 3900, normalized size of antiderivative = 2.27

method	result	size
elliptic	Expression too large to display	3900
default	Expression too large to display	65231

input
```
int((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(9/2),x,method=_RETU
RNVERBOSE)
```

output
```
((b*x+a)*(d*x+c)*(f*x+e))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*
(-2/7*(A*b^2-B*a*b+C*a^2)/b^7*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c
*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)/(x+a/b)^4+2/35*(2*A*a*b^2*d*f-A*b^3*c*f-
A*b^3*d*e-9*B*a^2*b*d*f+8*B*a*b^2*c*f+8*B*a*b^2*d*e-7*B*b^3*c*e+16*C*a^3*d
*f-15*C*a^2*b*c*f-15*C*a^2*b*d*e+14*C*a*b^2*c*e)/b^6/(a^2*d*f-a*b*c*f-a*b*
d*e+b^2*c*e)*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*
e*x+a*c*e)^(1/2)/(x+a/b)^3+2/105*(6*A*a^2*b^2*d^2*f^2-6*A*a*b^3*c*d*f^2-6*
A*a*b^3*d^2*e*f+4*A*b^4*c^2*f^2-2*A*b^4*c*d*e*f+4*A*b^4*d^2*e^2+8*B*a^3*b*
d^2*f^2-15*B*a^2*b^2*c*d*f^2-15*B*a^2*b^2*d^2*e*f+3*B*a*b^3*c^2*f^2+30*B*a
*b^3*c*d*e*f+3*B*a*b^3*d^2*e^2-7*B*b^4*c^2*e*f-7*B*b^4*c*d*e^2-57*C*a^4*d^
2*f^2+106*C*a^3*b*c*d*f^2+106*C*a^3*b*d^2*e*f-45*C*a^2*b^2*c^2*f^2-198*C*a
^2*b^2*c*d*e*f-45*C*a^2*b^2*d^2*e^2+84*C*a*b^3*c^2*e*f+84*C*a*b^3*c*d*e^2-
35*C*b^4*c^2*e^2)/b^5/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)^2*(b*d*f*x^3+a*d*f
*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)/(x+a/b)^2+2/
105*(b*d*f*x^2+b*c*f*x+b*d*e*x+b*c*e)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)^3/
b^4*(6*A*a^3*b^2*d^3*f^3-9*A*a^2*b^3*c*d^2*f^3-9*A*a^2*b^3*d^3*e*f^2+19*A*
a*b^4*c^2*d*f^3-20*A*a*b^4*c*d^2*e*f^2+19*A*a*b^4*d^3*e^2*f-8*A*b^5*c^3*f^
3+5*A*b^5*c^2*d*e*f^2+5*A*b^5*c*d^2*e^2*f-8*A*b^5*d^3*e^3+8*B*a^4*b*d^3*f^
3-19*B*a^3*b^2*c*d^2*f^3-19*B*a^3*b^2*d^3*e*f^2+9*B*a^2*b^3*c^2*d*f^3+48*B
*a^2*b^3*c*d^2*e*f^2+9*B*a^2*b^3*d^3*e^2*f-6*B*a*b^4*c^3*f^3-19*B*a*b^4...
```

3.66.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.01 (sec) , antiderivative size = 9150, normalized size of antiderivative = 5.33

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(9/2),x, algor
ithm="fricas")
```

output
```
Too large to include
```

3.66.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2)/(b*x+a)**(9/2),x)
```

output
```
Timed out
```

3.66.7 Maxima [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{9/2}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{dx+c}\sqrt{fx+e}}{(bx+a)^{\frac{9}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(9/2),x, algor
ithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(d*x + c)*sqrt(f*x + e)/(b*x + a)^(9/2), x
)
```

3.66.8 Giac [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{9/2}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{dx+c}\sqrt{fx+e}}{(bx+a)^{\frac{9}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(9/2),x, algor
ithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(d*x + c)*sqrt(f*x + e)/(b*x + a)^(9/2), x
)
```

3.66.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{9/2}}\,dx = \int \frac{\sqrt{e+f\,x}\,\sqrt{c+d\,x}\,(C\,x^2+B\,x+A)}{(a+b\,x)^{9/2}}\,dx$$

input
```
int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x)^(9/2),x)
```

output
```
int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x)^(9/2), x
)
```

3.67 $\int \frac{(a+bx)^{3/2}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$

3.67.1 Optimal result

Integrand size = 38, antiderivative size = 1235

$$\int \frac{(a+bx)^{3/2}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx =$$

$$-\frac{2\big(5bdf(7adf(5bcCe+3aCde+acCf-9Abdf)-(3bce+3ade+acf)(4aCdf+b(8Cde+6cCf-9Bdf)}{}$$

$$-\frac{2(7bdf(5bcCe+3aCde+acCf-9Abdf)-(6bde+4bcf-3adf)(4aCdf+b(8Cde+6cCf-9Bdf)))\sqrt{}}{315bd^3f^3}$$

$$-\frac{2(4aCdf+b(8Cde+6cCf-9Bdf))(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}}{63bd^2f^2}$$

$$+\frac{2C(a+bx)^{5/2}(c+dx)^{3/2}\sqrt{e+fx}}{9bdf}$$

$$+\frac{2\sqrt{-bc+ad}(8a^4Cd^4f^4+a^3bd^3f^3(11Cde-7cCf-18Bdf)-3a^2b^2d^2f^2(3df(4Bde-3Bcf-7Adf)-C}{}$$

$$+\frac{2\sqrt{-bc+ad}(be-af)(de-cf)\left(4a^3Cd^3f^3+3a^2bd^2f^2(3Cde-cCf-3Bdf)-3ab^2df(3df(16Bde+3Bc\right.}{}$$

output
```
-2/63*(4*a*C*d*f+b*(-9*B*d*f+6*C*c*f+8*C*d*e))*(b*x+a)^(3/2)*(d*x+c)^(3/2)
*(f*x+e)^(1/2)/b/d^2/f^2+2/9*C*(b*x+a)^(5/2)*(d*x+c)^(3/2)*(f*x+e)^(1/2)/b
/d/f-2/315*(7*b*d*f*(-9*A*b*d*f+C*a*c*f+3*C*a*d*e+5*C*b*c*e)-(-3*a*d*f+4*b
*c*f+6*b*d*e)*(4*a*C*d*f+b*(-9*B*d*f+6*C*c*f+8*C*d*e)))*(d*x+c)^(3/2)*(b*x
+a)^(1/2)*(f*x+e)^(1/2)/b/d^3/f^3-2/945*(5*b*d*f*(7*a*d*f*(-9*A*b*d*f+C*a*
c*f+3*C*a*d*e+5*C*b*c*e)-(a*c*f+3*a*d*e+3*b*c*e)*(4*a*C*d*f+b*(-9*B*d*f+6*
C*c*f+8*C*d*e)))+2*(1/2*a*d*f-b*(c*f+2*d*e))*(7*b*d*f*(-9*A*b*d*f+C*a*c*f+
3*C*a*d*e+5*C*b*c*e)-(-3*a*d*f+4*b*c*f+6*b*d*e)*(4*a*C*d*f+b*(-9*B*d*f+6*C
*c*f+8*C*d*e))))*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^2/d^3/f^4+2/3
15*(8*a^4*C*d^4*f^4+a^3*b*d^3*f^3*(-18*B*d*f-7*C*c*f+11*C*d*e)-3*a^2*b^2*d
^2*f^2*(3*d*f*(-7*A*d*f-3*B*c*f+4*B*d*e)-C*(-3*c^2*f^2-5*c*d*e*f+9*d^2*e^2
))-a*b^3*d*f*(2*C*(-16*c^3*f^3-18*c^2*d*e*f^2-33*c*d^2*e^2*f+92*d^3*e^3)+3
*d*f*(7*A*d*f*(-7*c*f+13*d*e)-B*(-19*c^2*f^2-29*c*d*e*f+72*d^2*e^2)))+b^4*
(C*(-16*c^4*f^4-16*c^3*d*e*f^3-21*c^2*d^2*e^2*f^2-40*c*d^3*e^3*f+128*d^4*e
^4)+3*d*f*(7*A*d*f*(-2*c^2*f^2-3*c*d*e*f+8*d^2*e^2)-B*(-8*c^3*f^3-9*c^2*d*
e*f^2-16*c*d^2*e^2*f+48*d^3*e^3))))*EllipticE(d^(1/2)*(b*x+a)^(1/2)/(a*d-b
*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(a*d-b*c)^(1/2)*(b*(d*x+c)/(-
a*d+b*c))^(1/2)*(f*x+e)^(1/2)/b^3/d^(7/2)/f^5/(d*x+c)^(1/2)/(b*(f*x+e)/(-a
*f+b*e))^(1/2)+2/315*(-a*f+b*e)*(-c*f+d*e)*(4*a^3*C*d^3*f^3+3*a^2*b*d^2*f^
2*(-3*B*d*f-C*c*f+3*C*d*e)-3*a*b^2*d*f*(3*d*f*(-21*A*d*f+3*B*c*f+16*B*d...
```

3.67.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 34.13 (sec) , antiderivative size = 1470, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx)^{3/2}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx =$$

$$-\frac{2\left(-b^2\sqrt{-a+\frac{bc}{d}}(8a^4Cd^4f^4+a^3bd^3f^3(11Cde-7cCf-18Bdf)+3a^2b^2d^2f^2(3df(-4Bde+3Bcf+7Aa\right.}{}$$

input
```
Integrate[((a + b*x)^(3/2)*Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],
x]
```

output
```
(-2*(-(b^2*Sqrt[-a + (b*c)/d]*(8*a^4*C*d^4*f^4 + a^3*b*d^3*f^3*(11*C*d*e -
 7*c*C*f - 18*B*d*f) + 3*a^2*b^2*d^2*f^2*(3*d*f*(-4*B*d*e + 3*B*c*f + 7*A*
d*f) + C*(9*d^2*e^2 - 5*c*d*e*f - 3*c^2*f^2)) + a*b^3*d*f*(C*(-184*d^3*e^3
 + 66*c*d^2*e^2*f + 36*c^2*d*e*f^2 + 32*c^3*f^3) - 3*d*f*(7*A*d*f*(13*d*e
- 7*c*f) + B*(-72*d^2*e^2 + 29*c*d*e*f + 19*c^2*f^2))) + b^4*(C*(128*d^4*e
^4 - 40*c*d^3*e^3*f - 21*c^2*d^2*e^2*f^2 - 16*c^3*d*e*f^3 - 16*c^4*f^4) +
3*d*f*(7*A*d*f*(8*d^2*e^2 - 3*c*d*e*f - 2*c^2*f^2) + B*(-48*d^3*e^3 + 16*c
*d^2*e^2*f + 9*c^2*d*e*f^2 + 8*c^3*f^3))))*(c + d*x)*(e + f*x)) + b^2*Sqrt
[-a + (b*c)/d]*d*f*(a + b*x)*(c + d*x)*(e + f*x)*(4*a^3*C*d^3*f^3 - 3*a^2*
b*d^2*f^2*(3*B*d*f + C*(-2*d*e + c*f + d*f*x)) - a*b^2*d*f*(9*d*f*(14*A*d*
f + B*(-11*d*e + 3*c*f + 8*d*f*x)) + C*(-15*c^2*f^2 + c*d*f*(-19*e + 11*f*
x) + d^2*(84*e^2 - 61*e*f*x + 50*f^2*x^2))) + b^3*(C*(-8*c^3*f^3 + 3*c^2*d
*f^2*(-3*e + 2*f*x) + c*d^2*f*(-12*e^2 + 7*e*f*x - 5*f^2*x^2) + d^3*(64*e^
3 - 48*e^2*f*x + 40*e*f^2*x^2 - 35*f^3*x^3)) - 3*d*f*(7*A*d*f*(-4*d*e + c*
f + 3*d*f*x) + B*(-4*c^2*f^2 + c*d*f*(-5*e + 3*f*x) + 3*d^2*(8*e^2 - 6*e*f
*x + 5*f^2*x^2))))) - I*(b*c - a*d)*f*(8*a^4*C*d^4*f^4 + a^3*b*d^3*f^3*(11
*C*d*e - 7*c*C*f - 18*B*d*f) + 3*a^2*b^2*d^2*f^2*(3*d*f*(-4*B*d*e + 3*B*c*
f + 7*A*d*f) + C*(9*d^2*e^2 - 5*c*d*e*f - 3*c^2*f^2)) + a*b^3*d*f*(C*(-184
*d^3*e^3 + 66*c*d^2*e^2*f + 36*c^2*d*e*f^2 + 32*c^3*f^3) - 3*d*f*(7*A*d*f*
(13*d*e - 7*c*f) + B*(-72*d^2*e^2 + 29*c*d*e*f + 19*c^2*f^2))) + b^4*(C...
```

3.67.3 Rubi [A] (verified)

Time = 2.93 (sec) , antiderivative size = 1268, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {2118, 27, 171, 27, 171, 27, 171, 27, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/2}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}dx$$

$$\Big\downarrow \; 2118$$

$$\frac{2\int -\frac{b(a+bx)^{3/2}\sqrt{c+dx}(5bcCe+3aCde+acCf-9Abdf+(4aCdf+b(8Cde+6cCf-9Bdf))x)}{2\sqrt{e+fx}}dx}{9b^2df}+\frac{2C(a+bx)^{5/2}(c+dx)^{3/2}\sqrt{e+fx}}{9bdf}$$

$$\Big\downarrow \; 27$$

$$\frac{2C(a+bx)^{5/2}(c+dx)^{3/2}\sqrt{e+fx}}{9bdf}-\frac{\int \frac{(a+bx)^{3/2}\sqrt{c+dx}(5bcCe+3aCde+acCf-9Abdf+(4aCdf+b(8Cde+6cCf-9Bdf))x)}{\sqrt{e+fx}}dx}{9bdf}$$

$\downarrow$ 171

$$\frac{2C(a+bx)^{5/2}(c+dx)^{3/2}\sqrt{e+fx}}{9bdf}-$$

$$\frac{\frac{2\int \frac{\sqrt{a+bx}\sqrt{c+dx}(7adf(5bcCe+3aCde+acCf-9Abdf)-(3bce+3ade+acf)(4aCdf+b(8Cde+6cCf-9Bdf))+(7bdf(5bcCe+3aCde+acCf-9Abdf)-(6bde+4bcf-3ad}{2\sqrt{e+fx}}}{7df}}{9bdf}$$

$\downarrow$ 27

$$\frac{2C(a+bx)^{5/2}(c+dx)^{3/2}\sqrt{e+fx}}{9bdf}-$$

$$\frac{\frac{\int \frac{\sqrt{a+bx}\sqrt{c+dx}(7adf(5bcCe+3aCde+acCf-9Abdf)-(3bce+3ade+acf)(4aCdf+b(8Cde+6cCf-9Bdf))+(7bdf(5bcCe+3aCde+acCf-9Abdf)-(6bde+4bcf-3adf)}{\sqrt{e+fx}}}{7df}}{9bdf}$$

$\downarrow$ 171

$$\frac{2C(a+bx)^{5/2}(c+dx)^{3/2}\sqrt{e+fx}}{9bdf}-$$

$$2\int \frac{\sqrt{c+dx}\Big(5adf(7adf(5bcCe+3aCde+acCf-9Abdf)-(3bce+3ade+acf)(4aCdf+b(8Cde+6cCf-9Bdf)))-(bce+3ade+acf)(7bdf(5bcCe+3aCde+acCf-9Abdf)-(}{}$$

$\downarrow$ 27

$$\frac{2C(a+bx)^{5/2}(c+dx)^{3/2}\sqrt{e+fx}}{9bdf}-$$

$$\int \frac{\sqrt{c+dx}\Big(5adf(7adf(5bcCe+3aCde+acCf-9Abdf)-(3bce+3ade+acf)(4aCdf+b(8Cde+6cCf-9Bdf)))-(bce+3ade+acf)(7bdf(5bcCe+3aCde+acCf-9Abdf)-(6}{}$$

$\downarrow$ 171

$$\frac{2C(a+bx)^{5/2}(c+dx)^{3/2}\sqrt{e+fx}}{9bdf}-$$

$$\frac{2(4aCdf+b(8Cde+6cCf-9Bdf))(a+bx)^{3/2}\sqrt{e+fx}(c+dx)^{3/2}}{7df}+\frac{\frac{2(7bdf(5bcCe+3aCde+acCf-9Abdf)-(6bde+4bcf-3adf)(4aCdf+b(8Cde+6cCf-9Bdf}{5df}}{}$$

$\downarrow$ 27

3.67. $\int \frac{(a+bx)^{3/2}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$

$$\frac{2C(a+bx)^{5/2}(c+dx)^{3/2}\sqrt{e+fx}}{9bdf}-$$

$$\frac{\frac{2(4aCdf+b(8Cde+6cCf-9Bdf))(a+bx)^{3/2}\sqrt{e+fx}(c+dx)^{3/2}}{7df}+\frac{2(7bdf(5bcCe+3aCde+acCf-9Abdf)-(6bde+4bcf-3adf)(4aCdf+b(8Cde+6cCf-9Bdf}{5df}}{}$$

$\downarrow$ 176

$$\frac{2C(a+bx)^{5/2}(c+dx)^{3/2}\sqrt{e+fx}}{9bdf}-$$

$$\frac{\frac{2(4aCdf+b(8Cde+6cCf-9Bdf))(a+bx)^{3/2}\sqrt{e+fx}(c+dx)^{3/2}}{7df}+\frac{2(7bdf(5bcCe+3aCde+acCf-9Abdf)-(6bde+4bcf-3adf)(4aCdf+b(8Cde+6cCf-9Bdf}{5df}}{}$$

$\downarrow$ 124

$$\frac{2C(a+bx)^{5/2}(c+dx)^{3/2}\sqrt{e+fx}}{9bdf}-$$

$$\frac{\frac{2(4aCdf+b(8Cde+6cCf-9Bdf))(a+bx)^{3/2}\sqrt{e+fx}(c+dx)^{3/2}}{7df}+\frac{2(7bdf(5bcCe+3aCde+acCf-9Abdf)-(6bde+4bcf-3adf)(4aCdf+b(8Cde+6cCf-9Bdf}{5df}}{}$$

$\downarrow$ 123

$$\frac{2C(a+bx)^{5/2}(c+dx)^{3/2}\sqrt{e+fx}}{9bdf}-$$

$$\frac{\frac{2(4aCdf+b(8Cde+6cCf-9Bdf))(a+bx)^{3/2}\sqrt{e+fx}(c+dx)^{3/2}}{7df}+\frac{2(7bdf(5bcCe+3aCde+acCf-9Abdf)-(6bde+4bcf-3adf)(4aCdf+b(8Cde+6cCf-9Bdf}{5df}}{}$$

$\downarrow$ 131

$$\frac{2C(a+bx)^{5/2}(c+dx)^{3/2}\sqrt{e+fx}}{9bdf}-$$

$$\frac{\frac{2(4aCdf+b(8Cde+6cCf-9Bdf))(a+bx)^{3/2}\sqrt{e+fx}(c+dx)^{3/2}}{7df}+\frac{2(7bdf(5bcCe+3aCde+acCf-9Abdf)-(6bde+4bcf-3adf)(4aCdf+b(8Cde+6cCf-9Bdf}{5df}}{}$$

$\downarrow$ 131

3.67. $\int \frac{(a+bx)^{3/2}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$

$$\frac{2C(a+bx)^{5/2}(c+dx)^{3/2}\sqrt{e+fx}}{9bdf}-$$

$$\frac{\frac{2(4aCdf+b(8Cde+6cCf-9Bdf))(a+bx)^{3/2}\sqrt{e+fx}(c+dx)^{3/2}}{7df}+\frac{2(7bdf(5bcCe+3aCde+acCf-9Abdf)-(6bde+4bcf-3adf)(4aCdf+b(8Cde+6cCf-9Bdf}{5df}}{}$$

$\downarrow$ 130

$$\frac{2C(a+bx)^{5/2}(c+dx)^{3/2}\sqrt{e+fx}}{9bdf}-$$

$$\frac{\frac{2(4aCdf+b(8Cde+6cCf-9Bdf))(a+bx)^{3/2}\sqrt{e+fx}(c+dx)^{3/2}}{7df}+\frac{2(7bdf(5bcCe+3aCde+acCf-9Abdf)-(6bde+4bcf-3adf)(4aCdf+b(8Cde+6cCf-9Bdf}{5df}}{}$$

input
```
Int[((a + b*x)^(3/2)*Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]
```

output
```
(2*C*(a + b*x)^(5/2)*(c + d*x)^(3/2)*Sqrt[e + f*x])/(9*b*d*f) - ((2*(4*a*C
*d*f + b*(8*C*d*e + 6*c*C*f - 9*B*d*f))*(a + b*x)^(3/2)*(c + d*x)^(3/2)*Sq
rt[e + f*x])/(7*d*f) + ((2*(7*b*d*f*(5*b*c*C*e + 3*a*C*d*e + a*c*C*f - 9*A
*b*d*f) - (6*b*d*e + 4*b*c*f - 3*a*d*f)*(4*a*C*d*f + b*(8*C*d*e + 6*c*C*f
- 9*B*d*f)))*Sqrt[a + b*x]*(c + d*x)^(3/2)*Sqrt[e + f*x])/(5*d*f) + ((2*(5
*b*d*f*(7*a*d*f*(5*b*c*C*e + 3*a*C*d*e + a*c*C*f - 9*A*b*d*f) - (3*b*c*e +
 3*a*d*e + a*c*f)*(4*a*C*d*f + b*(8*C*d*e + 6*c*C*f - 9*B*d*f))) + 2*((a*d
*f)/2 - b*(2*d*e + c*f))*(7*b*d*f*(5*b*c*C*e + 3*a*C*d*e + a*c*C*f - 9*A*b
*d*f) - (6*b*d*e + 4*b*c*f - 3*a*d*f)*(4*a*C*d*f + b*(8*C*d*e + 6*c*C*f -
9*B*d*f))))*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])/(3*b*f) + ((-6*Sqrt
[-(b*c) + a*d]*(8*a^4*C*d^4*f^4 + a^3*b*d^3*f^3*(11*C*d*e - 7*c*C*f - 18*B
*d*f) - 3*a^2*b^2*d^2*f^2*(3*d*f*(4*B*d*e - 3*B*c*f - 7*A*d*f) - C*(9*d^2*
e^2 - 5*c*d*e*f - 3*c^2*f^2)) - a*b^3*d*f*(C*(184*d^3*e^3 - 66*c*d^2*e^2*f
 - 36*c^2*d*e*f^2 - 32*c^3*f^3) + 3*d*f*(7*A*d*f*(13*d*e - 7*c*f) - B*(72*
d^2*e^2 - 29*c*d*e*f - 19*c^2*f^2))) + b^4*(C*(128*d^4*e^4 - 40*c*d^3*e^3*
f - 21*c^2*d^2*e^2*f^2 - 16*c^3*d*e*f^3 - 16*c^4*f^4) + 3*d*f*(7*A*d*f*(8*
d^2*e^2 - 3*c*d*e*f - 2*c^2*f^2) - B*(48*d^3*e^3 - 16*c*d^2*e^2*f - 9*c^2*
d*e*f^2 - 8*c^3*f^3))))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*Elli
pticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/
(d*(b*e - a*f))])/(b*Sqrt[d]*f*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - ...
```

3.67. $\int \frac{(a+bx)^{3/2}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$

3.67.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

3.67. $\int \frac{(a+bx)^{3/2}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

3.67.4 Maple [A] (verified)

Time = 3.50 (sec) , antiderivative size = 2108, normalized size of antiderivative = 1.71

method	result	size
elliptic	Expression too large to display	2108
default	Expression too large to display	15736

input
```
int((b*x+a)^(3/2)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
((b*x+a)*(d*x+c)*(f*x+e))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*
(2/9*C*b/f*x^3*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*
c*e*x+a*c*e)^(1/2)+2/7*(b^2*B*d+2*C*a*b*d+C*b^2*c-2/9*C*b/f*(4*a*d*f+4*b*c
*f+4*b*d*e))/b/d/f*x^2*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*
d*e*x+b*c*e*x+a*c*e)^(1/2)+2/5*(A*b^2*d+2*B*a*b*d+B*b^2*c+C*a^2*d+2*C*a*b*
c-2/9*C*b/f*(7/2*a*c*f+7/2*a*d*e+7/2*b*c*e)-2/7*(b^2*B*d+2*C*a*b*d+C*b^2*c
-2/9*C*b/f*(4*a*d*f+4*b*c*f+4*b*d*e))/b/d/f*(3*a*d*f+3*b*c*f+3*b*d*e))/b/d
/f*x*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*
e)^(1/2)+2/3*(2*A*a*b*d+b^2*A*c+B*a^2*d+2*B*b*c*a+C*a^2*c-2/3*C*b/f*a*c*e-
2/7*(b^2*B*d+2*C*a*b*d+C*b^2*c-2/9*C*b/f*(4*a*d*f+4*b*c*f+4*b*d*e))/b/d/f*
(5/2*a*c*f+5/2*a*d*e+5/2*b*c*e)-2/5*(A*b^2*d+2*B*a*b*d+B*b^2*c+C*a^2*d+2*C
*a*b*c-2/9*C*b/f*(7/2*a*c*f+7/2*a*d*e+7/2*b*c*e)-2/7*(b^2*B*d+2*C*a*b*d+C*
b^2*c-2/9*C*b/f*(4*a*d*f+4*b*c*f+4*b*d*e))/b/d/f*(3*a*d*f+3*b*c*f+3*b*d*e)
)/b/d/f*(2*a*d*f+2*b*c*f+2*b*d*e))/b/d/f*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*
d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)+2*(A*a^2*c-2/5*(A*b^2*d+2*B*a
*b*d+B*b^2*c+C*a^2*d+2*C*a*b*c-2/9*C*b/f*(7/2*a*c*f+7/2*a*d*e+7/2*b*c*e)-2
/7*(b^2*B*d+2*C*a*b*d+C*b^2*c-2/9*C*b/f*(4*a*d*f+4*b*c*f+4*b*d*e))/b/d/f*(
3*a*d*f+3*b*c*f+3*b*d*e))/b/d/f*a*c*e-2/3*(2*A*a*b*d+b^2*A*c+B*a^2*d+2*B*b
*c*a+C*a^2*c-2/3*C*b/f*a*c*e-2/7*(b^2*B*d+2*C*a*b*d+C*b^2*c-2/9*C*b/f*(4*a
*d*f+4*b*c*f+4*b*d*e))/b/d/f*(5/2*a*c*f+5/2*a*d*e+5/2*b*c*e)-2/5*(A*b^2...
```

3.67.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 1931, normalized size of antiderivative = 1.56

$$\int \frac{(a+bx)^{3/2}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(3/2)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algor
ithm="fricas")
```

output
```
2/945*(3*(35*C*b^5*d^5*f^5*x^3 - 64*C*b^5*d^5*e^3*f^2 + 12*(C*b^5*c*d^4 +
(7*C*a*b^4 + 6*B*b^5)*d^5)*e^2*f^3 + (9*C*b^5*c^2*d^3 - (19*C*a*b^4 + 15*B
*b^5)*c*d^4 - 3*(2*C*a^2*b^3 + 33*B*a*b^4 + 28*A*b^5)*d^5)*e*f^4 + (8*C*b^
5*c^3*d^2 - 3*(5*C*a*b^4 + 4*B*b^5)*c^2*d^3 + 3*(C*a^2*b^3 + 9*B*a*b^4 + 7
*A*b^5)*c*d^4 - (4*C*a^3*b^2 - 9*B*a^2*b^3 - 126*A*a*b^4)*d^5)*f^5 - 5*(8*
C*b^5*d^5*e*f^4 - (C*b^5*c*d^4 + (10*C*a*b^4 + 9*B*b^5)*d^5)*f^5)*x^2 + (4
8*C*b^5*d^5*e^2*f^3 - (7*C*b^5*c*d^4 + (61*C*a*b^4 + 54*B*b^5)*d^5)*e*f^4
- (6*C*b^5*c^2*d^3 - (11*C*a*b^4 + 9*B*b^5)*c*d^4 - 3*(C*a^2*b^3 + 24*B*a*
b^4 + 21*A*b^5)*d^5)*f^5)*x)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e) - (
128*C*b^5*d^5*e^5 - 8*(13*C*b^5*c*d^4 + (31*C*a*b^4 + 18*B*b^5)*d^5)*e^4*f
 - (25*C*b^5*c^2*d^3 - 2*(113*C*a*b^4 + 60*B*b^5)*c*d^4 - (95*C*a^2*b^3 +
288*B*a*b^4 + 168*A*b^5)*d^5)*e^3*f^2 - (10*C*b^5*c^3*d^2 - 15*(3*C*a*b^4
+ 2*B*b^5)*c^2*d^3 + 3*(37*C*a^2*b^3 + 91*B*a*b^4 + 49*A*b^5)*c*d^4 - (20*
C*a^3*b^2 - 117*B*a^2*b^3 - 357*A*a*b^4)*d^5)*e^2*f^3 - (8*C*b^5*c^4*d - (
22*C*a*b^4 + 15*B*b^5)*c^3*d^2 + 3*(5*C*a^2*b^3 + 21*B*a*b^4 + 14*A*b^5)*c
^2*d^3 + 7*(2*C*a^3*b^2 - 21*B*a^2*b^3 - 54*A*a*b^4)*c*d^4 - (7*C*a^4*b -
27*B*a^3*b^2 + 168*A*a^2*b^3)*d^5)*e*f^4 - (16*C*b^5*c^5 - 8*(5*C*a*b^4 +
3*B*b^5)*c^4*d + (22*C*a^2*b^3 + 69*B*a*b^4 + 42*A*b^5)*c^3*d^2 + (7*C*a^3
*b^2 - 51*B*a^2*b^3 - 168*A*a*b^4)*c^2*d^3 + (11*C*a^4*b - 36*B*a^3*b^2 +
357*A*a^2*b^3)*c*d^4 - (8*C*a^5 - 18*B*a^4*b + 63*A*a^3*b^2)*d^5)*f^5)*...
```

3.67.6 Sympy [F]

$$\int \frac{(a+bx)^{3/2}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \int \frac{(a+bx)^{\frac{3}{2}}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$$

input
```
integrate((b*x+a)**(3/2)*(C*x**2+B*x+A)*(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

output
```
Integral((a + b*x)**(3/2)*sqrt(c + d*x)*(A + B*x + C*x**2)/sqrt(e + f*x),
x)
```

3.67.7 Maxima [F]

$$\int \frac{(a+bx)^{3/2}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \int \frac{(Cx^2+Bx+A)(bx+a)^{\frac{3}{2}}\sqrt{dx+c}}{\sqrt{fx+e}}\,dx$$

input
```
integrate((b*x+a)^(3/2)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algor
ithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*(b*x + a)^(3/2)*sqrt(d*x + c)/sqrt(f*x + e), x
)
```

3.67.8 Giac [F]

$$\int \frac{(a+bx)^{3/2}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \int \frac{(Cx^2+Bx+A)(bx+a)^{\frac{3}{2}}\sqrt{dx+c}}{\sqrt{fx+e}}\,dx$$

input
```
integrate((b*x+a)^(3/2)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algor
ithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*(b*x + a)^(3/2)*sqrt(d*x + c)/sqrt(f*x + e), x
)
```

3.67.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \int \frac{(a+b\,x)^{3/2}\,\sqrt{c+d\,x}\,(C\,x^2+B\,x+A)}{\sqrt{e+f\,x}}\,dx$$

input
```
int(((a + b*x)^(3/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(e + f*x)^(1/2),x)
```

output
```
int(((a + b*x)^(3/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(e + f*x)^(1/2), x
)
```

3.68 $\int \frac{\sqrt{a+bx}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$

3.68.1 Optimal result

Integrand size = 38, antiderivative size = 766

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx =$$

$$-\frac{2(5bdf(3bcCe+3aCde+acCf-7Abdf)+(adf-2b(2de+cf))(4aCdf+b(6Cde+4cCf-7Bdf)))}{105b^2d^2f^3}$$

$$-\frac{2(4aCdf+b(6Cde+4cCf-7Bdf))\sqrt{a+bx}(c+dx)^{3/2}\sqrt{e+fx}}{35bd^2f^2}$$

$$+\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}}{7bdf}$$

$$-\frac{2\sqrt{-bc+ad}(3bdf(5adf(3bcCe+3aCde+acCf-7Abdf)-(bce+3ade+acf)(4aCdf+b(6Cde+4}{}$$

$$+\frac{2\sqrt{-bc+ad}(be-af)(de-cf)\left(4a^2Cd^2f^2+abdf(8Cde-2cCf-7Bdf)-b^2(7df(8Bde+Bcf-10}{105b^3d^{5/2}f^4\sqrt{c+d}}$$

output
```
2/7*C*(b*x+a)^(3/2)*(d*x+c)^(3/2)*(f*x+e)^(1/2)/b/d/f-2/35*(4*a*C*d*f+b*(-
7*B*d*f+4*C*c*f+6*C*d*e))*(d*x+c)^(3/2)*(b*x+a)^(1/2)*(f*x+e)^(1/2)/b/d^2/
f^2-2/105*(5*b*d*f*(-7*A*b*d*f+C*a*c*f+3*C*a*d*e+3*C*b*c*e)+(a*d*f-2*b*(c*
f+2*d*e))*(4*a*C*d*f+b*(-7*B*d*f+4*C*c*f+6*C*d*e)))*(b*x+a)^(1/2)*(d*x+c)^
(1/2)*(f*x+e)^(1/2)/b^2/d^2/f^3-2/105*(3*b*d*f*(5*a*d*f*(-7*A*b*d*f+C*a*c*
f+3*C*a*d*e+3*C*b*c*e)-(a*c*f+3*a*d*e+b*c*e)*(4*a*C*d*f+b*(-7*B*d*f+4*C*c*
f+6*C*d*e)))+2*(1/2*b*c*f-d*(a*f+b*e))*(5*b*d*f*(-7*A*b*d*f+C*a*c*f+3*C*a*
d*e+3*C*b*c*e)+(a*d*f-2*b*(c*f+2*d*e))*(4*a*C*d*f+b*(-7*B*d*f+4*C*c*f+6*C*
d*e))))*EllipticE(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-
a*f+b*e))^(1/2))*(a*d-b*c)^(1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(f*x+e)^(1/2
)/b^3/d^(5/2)/f^4/(d*x+c)^(1/2)/(b*(f*x+e)/(-a*f+b*e))^(1/2)+2/105*(-a*f+b
*e)*(-c*f+d*e)*(4*a^2*C*d^2*f^2+a*b*d*f*(-7*B*d*f-2*C*c*f+8*C*d*e)-b^2*(7*
d*f*(-10*A*d*f+B*c*f+8*B*d*e)-4*C*(c^2*f^2+2*c*d*e*f+12*d^2*e^2)))*Ellipti
cF(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2)
)*(a*d-b*c)^(1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(b*(f*x+e)/(-a*f+b*e))^(1/2
)/b^3/d^(5/2)/f^4/(d*x+c)^(1/2)/(f*x+e)^(1/2)
```

3.68.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 28.61 (sec) , antiderivative size = 922, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$$

$$= \frac{2\left(b^2\sqrt{-a+\frac{bc}{d}}(8a^3Cd^3f^3+a^2bd^2f^2(9Cde-5cCf-14Bdf)+ab^2df(7df(-3Bde+2Bcf+5Adf)+C\right.}{}$$

input
```
Integrate[(Sqrt[a + b*x]*Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]
```

output
```
(2*(b^2*Sqrt[-a + (b*c)/d]*(8*a^3*C*d^3*f^3 + a^2*b*d^2*f^2*(9*C*d*e - 5*c
*C*f - 14*B*d*f) + a*b^2*d*f*(7*d*f*(-3*B*d*e + 2*B*c*f + 5*A*d*f) + C*(16
*d^2*e^2 - 8*c*d*e*f - 5*c^2*f^2)) + b^3*(C*(-48*d^3*e^3 + 16*c*d^2*e^2*f
+ 9*c^2*d*e*f^2 + 8*c^3*f^3) + 7*d*f*(5*A*d*f*(-2*d*e + c*f) + B*(8*d^2*e^
2 - 3*c*d*e*f - 2*c^2*f^2))))*(c + d*x)*(e + f*x) + b^2*Sqrt[-a + (b*c)/d]
*d*f*(a + b*x)*(c + d*x)*(e + f*x)*(-4*a^2*C*d^2*f^2 + a*b*d*f*(7*B*d*f +
C*(-5*d*e + 2*c*f + 3*d*f*x)) + b^2*(7*d*f*(5*A*d*f + B*(-4*d*e + c*f + 3*
d*f*x)) + C*(-4*c^2*f^2 + c*d*f*(-5*e + 3*f*x) + 3*d^2*(8*e^2 - 6*e*f*x +
5*f^2*x^2)))) + I*(b*c - a*d)*f*(8*a^3*C*d^3*f^3 + a^2*b*d^2*f^2*(9*C*d*e
- 5*c*C*f - 14*B*d*f) + a*b^2*d*f*(7*d*f*(-3*B*d*e + 2*B*c*f + 5*A*d*f) +
C*(16*d^2*e^2 - 8*c*d*e*f - 5*c^2*f^2)) + b^3*(C*(-48*d^3*e^3 + 16*c*d^2*e
^2*f + 9*c^2*d*e*f^2 + 8*c^3*f^3) + 7*d*f*(5*A*d*f*(-2*d*e + c*f) + B*(8*d
^2*e^2 - 3*c*d*e*f - 2*c^2*f^2))))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(
a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticE[I*ArcSinh[Sqrt[-a +
 (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)] - I*b*(b*c - a*
d)*f*(d*e - c*f)*(4*a^2*C*d^2*f^2 + a*b*d*f*(5*C*d*e + c*C*f - 7*B*d*f) -
b^2*(7*d*f*(-4*B*d*e - 2*B*c*f + 5*A*d*f) + C*(24*d^2*e^2 + 13*c*d*e*f + 8
*c^2*f^2)))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e +
 f*x))/(f*(a + b*x))]*EllipticF[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]
], (b*d*e - a*d*f)/(b*c*f - a*d*f)]))/(105*b^4*Sqrt[-a + (b*c)/d]*d^3*f...
```

3.68.3 Rubi [A] (verified)

Time = 1.54 (sec) , antiderivative size = 792, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2118, 27, 171, 27, 171, 27, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{\sqrt{e+fx}}dx$$

$$\Big\downarrow \ 2118$$

$$\frac{2\int -\frac{b\sqrt{a+bx}\sqrt{c+dx}(3bcCe+3aCde+acCf-7Abdf+(4aCdf+b(6Cde+4cCf-7Bdf))x)}{2\sqrt{e+fx}}dx}{7b^2df}+\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}}{7bdf}$$

$$\Big\downarrow \ 27$$

$$\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}}{7bdf}-\frac{\int \frac{\sqrt{a+bx}\sqrt{c+dx}(3bcCe+3aCde+acCf-7Abdf+(4aCdf+b(6Cde+4cCf-7Bdf))x)}{\sqrt{e+fx}}dx}{7bdf}$$

$\downarrow$ 171

$$\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}}{7bdf}-$$

$$2\int \frac{\sqrt{c+dx}(5adf(3bcCe+3aCde+acCf-7Abdf)-(bce+3ade+acf)(4aCdf+b(6Cde+4cCf-7Bdf))+(5bdf(3bcCe+3aCde+acCf-7Abdf)+(adf-2b(2de+cf))(4aCdf}{2\sqrt{a+bx}\sqrt{e+fx}} \quad 5df \quad 7bdf$$

$\downarrow$ 27

$$\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}}{7bdf}-$$

$$\int \frac{\sqrt{c+dx}(5adf(3bcCe+3aCde+acCf-7Abdf)-(bce+3ade+acf)(4aCdf+b(6Cde+4cCf-7Bdf))+(5bdf(3bcCe+3aCde+acCf-7Abdf)+(adf-2b(2de+cf))(4aCdf+}{\sqrt{a+bx}\sqrt{e+fx}} \quad 5df \quad 7bdf$$

$\downarrow$ 171

$$\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}}{7bdf}-$$

$$2\int \frac{3bcf(5adf(3bcCe+3aCde+acCf-7Abdf)-(bce+3ade+acf)(4aCdf+b(6Cde+4cCf-7Bdf)))-(bce+ade+acf)(5bdf(3bcCe+3aCde+acCf-7Abdf)+(adf-2b(2d}{}$$

$\downarrow$ 27

$$\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}}{7bdf}-$$

$$\int \frac{3bcf(5adf(3bcCe+3aCde+acCf-7Abdf)-(bce+3ade+acf)(4aCdf+b(6Cde+4cCf-7Bdf)))-(bce+ade+acf)(5bdf(3bcCe+3aCde+acCf-7Abdf)+(adf-2b(2de-}{}$$

$\downarrow$ 176

$$\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}}{7bdf}-$$

$$\frac{\left(3bdf(5adf(acCf+3aCde-7Abdf+3bcCe)-(acf+3ade+bce)(4aCdf+b(-7Bdf+4cCf+6Cde)))+2\left(\frac{bcf}{2}-d(af+be)\right)(5bdf(acCf+3aCde-7Abdf+3bcCe)+(adf-2b(\right.}{f}$$

$\downarrow$ 124

3.68. $\int \frac{\sqrt{a+bx}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$

$$\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}}{7bdf}-$$

$\sqrt{e+fx}\sqrt{\frac{b(c+dx)}{bc-ad}}\left(3bdf(5adf(acCf+3aCde-7Abdf+3bcCe)-(acf+3ade+bce)(4aCdf+b(-7Bdf+4cCf+6Cde)))+2\left(\frac{bcf}{2}-d(af+be)\right)(5bdf(acCf+3aCde-7Abdf+\right.$

$f\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}$

$\downarrow$ 123

$$\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}}{7bdf}-$$

$2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}\left(3bdf(5adf(acCf+3aCde-7Abdf+3bcCe)-(acf+3ade+bce)(4aCdf+b(-7Bdf+4cCf+6Cde)))+2\left(\frac{bcf}{2}-d(af+be)\right)(5bdf(acCf+3aCde\right.$

$b\sqrt{d}f\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}$

$\downarrow$ 131

$$\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}}{7bdf}-$$

$2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}\left(3bdf(5adf(acCf+3aCde-7Abdf+3bcCe)-(acf+3ade+bce)(4aCdf+b(-7Bdf+4cCf+6Cde)))+2\left(\frac{bcf}{2}-d(af+be)\right)(5bdf(acCf+3aCde\right.$

$b\sqrt{d}f\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}$

$\downarrow$ 131

$$\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}}{7bdf}-$$

$\frac{2(4aCdf+b(6Cde+4cCf-7Bdf))\sqrt{a+bx}\sqrt{e+fx}(c+dx)^{3/2}}{5df}+\frac{2\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}(5bdf(3bcCe+3aCde+acCf-7Abdf)+(adf-2b(2de+cf))(4aCdf+b(6}{3bf}$

$\downarrow$ 130

$$\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}}{7bdf}-$$

$2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}\left(3bdf(5adf(acCf+3aCde-7Abdf+3bcCe)-(acf+3ade+bce)(4aCdf+b(-7Bdf+4cCf+6Cde)))+2\left(\frac{bcf}{2}-d(af+be)\right)(5bdf(acCf+3aCde\right.$

$b\sqrt{d}f\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}$

3.68. $\int\frac{\sqrt{a+bx}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$

input
```
Int[(Sqrt[a + b*x]*Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]
```

output
```
(2*C*(a + b*x)^(3/2)*(c + d*x)^(3/2)*Sqrt[e + f*x])/(7*b*d*f) - ((2*(4*a*C
*d*f + b*(6*C*d*e + 4*c*C*f - 7*B*d*f))*Sqrt[a + b*x]*(c + d*x)^(3/2)*Sqrt
[e + f*x])/(5*d*f) + ((2*(5*b*d*f*(3*b*c*C*e + 3*a*C*d*e + a*c*C*f - 7*A*b
*d*f) + (a*d*f - 2*b*(2*d*e + c*f))*(4*a*C*d*f + b*(6*C*d*e + 4*c*C*f - 7*
B*d*f)))*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])/(3*b*f) + ((2*Sqrt[-(b
*c) + a*d]*(3*b*d*f*(5*a*d*f*(3*b*c*C*e + 3*a*C*d*e + a*c*C*f - 7*A*b*d*f)
 - (b*c*e + 3*a*d*e + a*c*f)*(4*a*C*d*f + b*(6*C*d*e + 4*c*C*f - 7*B*d*f))
) + 2*((b*c*f)/2 - d*(b*e + a*f))*(5*b*d*f*(3*b*c*C*e + 3*a*C*d*e + a*c*C*
f - 7*A*b*d*f) + (a*d*f - 2*b*(2*d*e + c*f))*(4*a*C*d*f + b*(6*C*d*e + 4*c
*C*f - 7*B*d*f))))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE
[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b
*e - a*f))])/(b*Sqrt[d]*f*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]) -
 (2*Sqrt[-(b*c) + a*d]*(b*e - a*f)*(d*e - c*f)*(4*a^2*C*d^2*f^2 + a*b*d*f*
(8*C*d*e - 2*c*C*f - 7*B*d*f) - b^2*(7*d*f*(8*B*d*e + B*c*f - 10*A*d*f) -
4*C*(12*d^2*e^2 + 2*c*d*e*f + c^2*f^2)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*S
qrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sq
rt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(b*Sqrt[d]*f*Sqrt[c +
 d*x]*Sqrt[e + f*x]))/(3*b*f))/(5*d*f))/(7*b*d*f)
```

3.68.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

3.68. $\int \frac{\sqrt{a+bx}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

3.68.4 Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 1205, normalized size of antiderivative = 1.57

method	result	size
elliptic	Expression too large to display	1205
default	Expression too large to display	10271

input
```
int((b*x+a)^(1/2)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
((b*x+a)*(d*x+c)*(f*x+e))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*
(2/7/f*C*x^2*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*
e*x+a*c*e)^(1/2)+2/5*(B*b*d+C*a*d+C*b*c-2/7/f*C*(3*a*d*f+3*b*c*f+3*b*d*e))
/b/d/f*x*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+
a*c*e)^(1/2)+2/3*(A*b*d+B*a*d+B*b*c+C*a*c-2/7/f*C*(5/2*a*c*f+5/2*a*d*e+5/2
*b*c*e)-2/5*(B*b*d+C*a*d+C*b*c-2/7/f*C*(3*a*d*f+3*b*c*f+3*b*d*e))/b/d/f*(2
*a*d*f+2*b*c*f+2*b*d*e))/b/d/f*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*
c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)+2*(A*a*c-2/5*(B*b*d+C*a*d+C*b*c-2/7/f*C
*(3*a*d*f+3*b*c*f+3*b*d*e))/b/d/f*a*c*e-2/3*(A*b*d+B*a*d+B*b*c+C*a*c-2/7/f
*C*(5/2*a*c*f+5/2*a*d*e+5/2*b*c*e)-2/5*(B*b*d+C*a*d+C*b*c-2/7/f*C*(3*a*d*f
+3*b*c*f+3*b*d*e))/b/d/f*(2*a*d*f+2*b*c*f+2*b*d*e))/b/d/f*(1/2*a*c*f+1/2*a
*d*e+1/2*b*c*e))*(e/f-c/d)*((x+e/f)/(e/f-c/d))^(1/2)*((x+a/b)/(-e/f+a/b))^
(1/2)*((x+c/d)/(-e/f+c/d))^(1/2)/(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+
a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*EllipticF(((x+e/f)/(e/f-c/d))^(1/2),(
(-e/f+c/d)/(-e/f+a/b))^(1/2))+2*(A*a*d+A*b*c+B*a*c-4/7/f*C*a*c*e-2/5*(B*b*
d+C*a*d+C*b*c-2/7/f*C*(3*a*d*f+3*b*c*f+3*b*d*e))/b/d/f*(3/2*a*c*f+3/2*a*d*
e+3/2*b*c*e)-2/3*(A*b*d+B*a*d+B*b*c+C*a*c-2/7/f*C*(5/2*a*c*f+5/2*a*d*e+5/2
*b*c*e)-2/5*(B*b*d+C*a*d+C*b*c-2/7/f*C*(3*a*d*f+3*b*c*f+3*b*d*e))/b/d/f*(2
*a*d*f+2*b*c*f+2*b*d*e))/b/d/f*(a*d*f+b*c*f+b*d*e))*(e/f-c/d)*((x+e/f)/(e/
f-c/d))^(1/2)*((x+a/b)/(-e/f+a/b))^(1/2)*((x+c/d)/(-e/f+c/d))^(1/2)/(b*...
```

3.68.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.18 (sec) , antiderivative size = 1392, normalized size of antiderivative = 1.82

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(1/2)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algor
ithm="fricas")
```

output
```
2/315*(3*(15*C*b^4*d^4*f^4*x^2 + 24*C*b^4*d^4*e^2*f^2 - (5*C*b^4*c*d^3 + (
5*C*a*b^3 + 28*B*b^4)*d^4)*e*f^3 - (4*C*b^4*c^2*d^2 - (2*C*a*b^3 + 7*B*b^4
)*c*d^3 + (4*C*a^2*b^2 - 7*B*a*b^3 - 35*A*b^4)*d^4)*f^4 - 3*(6*C*b^4*d^4*e
*f^3 - (C*b^4*c*d^3 + (C*a*b^3 + 7*B*b^4)*d^4)*f^4)*x)*sqrt(b*x + a)*sqrt(
d*x + c)*sqrt(f*x + e) + (48*C*b^4*d^4*e^4 - 8*(5*C*b^4*c*d^3 + (5*C*a*b^3
 + 7*B*b^4)*d^4)*e^3*f - (10*C*b^4*c^2*d^2 - 7*(6*C*a*b^3 + 7*B*b^4)*c*d^3
 + (10*C*a^2*b^2 - 49*B*a*b^3 - 70*A*b^4)*d^4)*e^2*f^2 - (5*C*b^4*c^3*d -
7*(C*a*b^3 + 2*B*b^4)*c^2*d^2 - 7*(C*a^2*b^2 - 8*B*a*b^3 - 10*A*b^4)*c*d^3
 + (5*C*a^3*b - 14*B*a^2*b^2 + 70*A*a*b^3)*d^4)*e*f^3 - (8*C*b^4*c^4 - (9*
C*a*b^3 + 14*B*b^4)*c^3*d - (4*C*a^2*b^2 - 21*B*a*b^3 - 35*A*b^4)*c^2*d^2
- (9*C*a^3*b - 21*B*a^2*b^2 + 140*A*a*b^3)*c*d^3 + (8*C*a^4 - 14*B*a^3*b +
 35*A*a^2*b^2)*d^4)*f^4)*sqrt(b*d*f)*weierstrassPInverse(4/3*(b^2*d^2*e^2
- (b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^
2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d
- 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*
c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3), 1/3*(3*b*d*f*x + b*d*e + (b*c + a*d
)*f)/(b*d*f)) + 3*(48*C*b^4*d^4*e^3*f - 8*(2*C*b^4*c*d^3 + (2*C*a*b^3 + 7*
B*b^4)*d^4)*e^2*f^2 - (9*C*b^4*c^2*d^2 - (8*C*a*b^3 + 21*B*b^4)*c*d^3 + (9
*C*a^2*b^2 - 21*B*a*b^3 - 70*A*b^4)*d^4)*e*f^3 - (8*C*b^4*c^3*d - (5*C*a*b
^3 + 14*B*b^4)*c^2*d^2 - (5*C*a^2*b^2 - 14*B*a*b^3 - 35*A*b^4)*c*d^3 + ...
```

3.68.6 Sympy [F]

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \int \frac{\sqrt{a+bx}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$$

input
```
integrate((b*x+a)**(1/2)*(C*x**2+B*x+A)*(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x)*sqrt(c + d*x)*(A + B*x + C*x**2)/sqrt(e + f*x), x)
```

3.68.7 Maxima [F]

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{bx+a}\sqrt{dx+c}}{\sqrt{fx+e}}\,dx$$

input
```
integrate((b*x+a)^(1/2)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algor
ithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(b*x + a)*sqrt(d*x + c)/sqrt(f*x + e), x)
```

3.68.8 Giac [F]

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{bx+a}\sqrt{dx+c}}{\sqrt{fx+e}}\,dx$$

input
```
integrate((b*x+a)^(1/2)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algor
ithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(b*x + a)*sqrt(d*x + c)/sqrt(f*x + e), x)
```

3.68.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \int \frac{\sqrt{a+b\,x}\,\sqrt{c+d\,x}\,(C\,x^2+B\,x+A)}{\sqrt{e+f\,x}}\,dx$$

input
```
int(((a + b*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(e + f*x)^(1/2),x)
```

output
```
int(((a + b*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(e + f*x)^(1/2), x
)
```

3.69 $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{e+fx}}\,dx$

3.69.1 Optimal result

Integrand size = 38, antiderivative size = 527

$$\begin{aligned}
&\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{e+fx}}\,dx \\
&= -\frac{2(4aCdf+b(4Cde+2cCf-5Bdf))\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{15b^2df^2} \\
&\quad+\frac{2C\sqrt{a+bx}(c+dx)^{3/2}\sqrt{e+fx}}{5bdf} \\
&\quad-\frac{2\sqrt{-bc+ad}(3bdf(bcCe+3aCde+acCf-5Abdf)-(2bde-bcf+2adf)(4aCdf+b(4Cde+2cCf-}{15b^3d^{3/2}f^3\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}} \\
&\quad-\frac{2\sqrt{-bc+ad}(de-cf)\left(4a^2Cdf^2+abf(3Cde-cCf-5Bdf)-b^2(5df(2Be-3Af)-Ce(8de+cf))\right)}{15b^3d^{3/2}f^3\sqrt{c+dx}\sqrt{e+fx}}
\end{aligned}$$

output
```
2/5*C*(d*x+c)^(3/2)*(b*x+a)^(1/2)*(f*x+e)^(1/2)/b/d/f-2/15*(4*a*C*d*f+b*(-
5*B*d*f+2*C*c*f+4*C*d*e))*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^2/d/
f^2-2/15*(3*b*d*f*(-5*A*b*d*f+C*a*c*f+3*C*a*d*e+C*b*c*e)-(2*a*d*f-b*c*f+2*
b*d*e)*(4*a*C*d*f+b*(-5*B*d*f+2*C*c*f+4*C*d*e)))*EllipticE(d^(1/2)*(b*x+a)
^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(a*d-b*c)^(1/2)*
(b*(d*x+c)/(-a*d+b*c))^(1/2)*(f*x+e)^(1/2)/b^3/d^(3/2)/f^3/(d*x+c)^(1/2)/(
b*(f*x+e)/(-a*f+b*e))^(1/2)-2/15*(-c*f+d*e)*(4*a^2*C*d*f^2+a*b*f*(-5*B*d*f
-C*c*f+3*C*d*e)-b^2*(5*d*f*(-3*A*f+2*B*e)-C*e*(c*f+8*d*e)))*EllipticF(d^(1
/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(a*d-
b*c)^(1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(b*(f*x+e)/(-a*f+b*e))^(1/2)/b^3/d
^(3/2)/f^3/(d*x+c)^(1/2)/(f*x+e)^(1/2)
```

3.69.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 26.83 (sec) , antiderivative size = 562, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{e+fx}}\,dx$$

$$= \frac{2\sqrt{a+bx}\left(\frac{b^2(8a^2Cd^2f^2+abdf(7Cde-3cCf-10Bdf)+b^2(5df(-2Bde+Bcf+3Adf)+C(8d^2e^2-3cdef-2c^2f^2)))(c+dx)(e+fx)}{a+bx}+b^2df(\right.}{}$$

input
```
Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2))/(Sqrt[a + b*x]*Sqrt[e + f*x]),
x]
```

output

```
(2*Sqrt[a + b*x]*((b^2*(8*a^2*C*d^2*f^2 + a*b*d*f*(7*C*d*e - 3*c*C*f - 10*
B*d*f) + b^2*(5*d*f*(-2*B*d*e + B*c*f + 3*A*d*f) + C*(8*d^2*e^2 - 3*c*d*e*
f - 2*c^2*f^2)))*(c + d*x)*(e + f*x))/(a + b*x) + b^2*d*f*(c + d*x)*(e + f
*x)*(5*b*B*d*f - 4*a*C*d*f + b*C*(-4*d*e + c*f + 3*d*f*x)) + (I*(b*c - a*d
)*f*(8*a^2*C*d^2*f^2 + a*b*d*f*(7*C*d*e - 3*c*C*f - 10*B*d*f) + b^2*(5*d*f
*(-2*B*d*e + B*c*f + 3*A*d*f) + C*(8*d^2*e^2 - 3*c*d*e*f - 2*c^2*f^2)))*Sq
rt[a + b*x]*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b
*x))]*EllipticE[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*
f)/(b*c*f - a*d*f)])/Sqrt[-a + (b*c)/d] + I*b*Sqrt[-a + (b*c)/d]*d*f*(d*e
- c*f)*(5*b*B*d*f - 4*a*C*d*f - 2*b*C*(2*d*e + c*f))*Sqrt[a + b*x]*Sqrt[(b
*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticF[I*A
rcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)]
))/(15*b^4*d^2*f^3*Sqrt[c + d*x]*Sqrt[e + f*x])
```

3.69.3 Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2118, 27, 171, 27, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}\sqrt{e+fx}}\,dx$$

$$\Big\downarrow \ 2118$$

$$\frac{2\int -\frac{b\sqrt{c+dx}(bcCe+3aCde+acCf-5Abdf+(4aCdf+b(4Cde+2cCf-5Bdf))x)}{2\sqrt{a+bx}\sqrt{e+fx}}dx}{5b^2df}+\frac{2C\sqrt{a+bx}(c+dx)^{3/2}\sqrt{e+fx}}{5bdf}$$

$$\Big\downarrow \ 27$$

$$\frac{2C\sqrt{a+bx}(c+dx)^{3/2}\sqrt{e+fx}}{5bdf}-\frac{\int \frac{\sqrt{c+dx}(bcCe+3aCde+acCf-5Abdf+(4aCdf+b(4Cde+2cCf-5Bdf))x)}{\sqrt{a+bx}\sqrt{e+fx}}dx}{5bdf}$$

$$\Big\downarrow \ 171$$

$$\frac{2C\sqrt{a+bx}(c+dx)^{3/2}\sqrt{e+fx}}{5bdf}-$$
$$\frac{\frac{2\int \frac{3bcf(bcCe+3aCde+acCf-5Abdf)-(bce+ade+acf)(4aCdf+b(4Cde+2cCf-5Bdf))+(3bdf(bcCe+3aCde+acCf-5Abdf)-(2bde-bcf+2adf)(4aCdf+b(4Cde+2}{2\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}}{3bf}}{5bdf}$$

$\downarrow$ 27

$$\frac{2C\sqrt{a+bx}(c+dx)^{3/2}\sqrt{e+fx}}{5bdf} - $$

$$\frac{\frac{\int \frac{3bcf(bcCe+3aCde+acCf-5Abdf)-(bce+ade+acf)(4aCdf+b(4Cde+2cCf-5Bdf))+(3bdf(bcCe+3aCde+acCf-5Abdf)-(2bde-bcf+2adf)(4aCdf+b(4Cde+2c}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}}{3bf}}{5bdf}$$

$\downarrow$ 176

$$\frac{2C\sqrt{a+bx}(c+dx)^{3/2}\sqrt{e+fx}}{5bdf} - $$

$$\frac{\frac{\frac{(de-cf)\left(4a^2Cdf^2+abf(-5Bdf-cCf+3Cde)-\left(b^2(5df(2Be-3Af)-Ce(cf+8de))\right)\right)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx}{f}+\frac{(3bdf(acCf+3aCde-5Abdf+bcCe)-(2adf-bcf+}{}}{3bf}}{5bdf}$$

$\downarrow$ 124

$$\frac{2C\sqrt{a+bx}(c+dx)^{3/2}\sqrt{e+fx}}{5bdf} - $$

$$\frac{\frac{\frac{(de-cf)\left(4a^2Cdf^2+abf(-5Bdf-cCf+3Cde)-\left(b^2(5df(2Be-3Af)-Ce(cf+8de))\right)\right)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx}{f}+\frac{\sqrt{e+fx}\sqrt{\frac{b(c+dx)}{bc-ad}}(3bdf(acCf+3aCde-5Abdf+b}{}}{3bf}}{5bdf}$$

$\downarrow$ 123

$$\frac{2C\sqrt{a+bx}(c+dx)^{3/2}\sqrt{e+fx}}{5bdf} - $$

$$\frac{\frac{(de-cf)\left(4a^2Cdf^2+abf(-5Bdf-cCf+3Cde)-\left(b^2(5df(2Be-3Af)-Ce(cf+8de))\right)\right)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx}{f}+\frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}(3bdf(acCf+3aCde-}{}}{3bf}$$

$\downarrow$ 131

$$\frac{2C\sqrt{a+bx}(c+dx)^{3/2}\sqrt{e+fx}}{5bdf} - $$

$$\frac{\frac{(de-cf)\sqrt{\frac{b(c+dx)}{bc-ad}}\left(4a^2Cdf^2+abf(-5Bdf-cCf+3Cde)-\left(b^2(5df(2Be-3Af)-Ce(cf+8de))\right)\right)\int \frac{1}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}\sqrt{e+fx}}dx}{f\sqrt{c+dx}}+\frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}}{}}{3bf}$$

$\downarrow$ 131

3.69. $\displaystyle \int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{e+fx}}\,dx$

$$\frac{2C\sqrt{a+bx}(c+dx)^{3/2}\sqrt{e+fx}}{5bdf} - \frac{\frac{(de-cf)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\left(4a^2Cdf^2+abf(-5Bdf-cCf+3Cde)-\left(b^2(5df(2Be-3Af)-Ce(cf+8de))\right)\right)\int\frac{1}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}\sqrt{\frac{be}{be-af}+\frac{bfx}{be-af}}}dx}{f\sqrt{c+dx}\sqrt{e+fx}} + \frac{2\sqrt{e+\cdots}}{\cdots}}{3bf}$$

$\downarrow$ 130

$$\frac{2C\sqrt{a+bx}(c+dx)^{3/2}\sqrt{e+fx}}{5bdf} - \frac{\frac{2\sqrt{ad-bc}(de-cf)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\left(4a^2Cdf^2+abf(-5Bdf-cCf+3Cde)-\left(b^2(5df(2Be-3Af)-Ce(cf+8de))\right)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{ad-bc}}\right),\frac{(bc-ad)f}{d(be-af)}\right)}{b\sqrt{d}f\sqrt{c+dx}\sqrt{e+fx}} + \frac{2\cdots}{\cdots}}{3bf}$$

input

```
Int[(Sqrt[c + d*x]*(A + B*x + C*x^2))/(Sqrt[a + b*x]*Sqrt[e + f*x]),x]
```

output

```
(2*C*Sqrt[a + b*x]*(c + d*x)^(3/2)*Sqrt[e + f*x])/(5*b*d*f) - ((2*(4*a*C*d
*f + b*(4*C*d*e + 2*c*C*f - 5*B*d*f))*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
 f*x])/(3*b*f) + ((2*Sqrt[-(b*c) + a*d]*(3*b*d*f*(b*c*C*e + 3*a*C*d*e + a*
c*C*f - 5*A*b*d*f) - (2*b*d*e - b*c*f + 2*a*d*f)*(4*a*C*d*f + b*(4*C*d*e +
 2*c*C*f - 5*B*d*f)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*Ellipt
icE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d
*(b*e - a*f))])/(b*Sqrt[d]*f*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]
) + (2*Sqrt[-(b*c) + a*d]*(d*e - c*f)*(4*a^2*C*d*f^2 + a*b*f*(3*C*d*e - c*
C*f - 5*B*d*f) - b^2*(5*d*f*(2*B*e - 3*A*f) - C*e*(8*d*e + c*f)))*Sqrt[(b*
(c + d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(
Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f)
)])/(b*Sqrt[d]*f*Sqrt[c + d*x]*Sqrt[e + f*x]))/(3*b*f))/(5*b*d*f)
```

3.69. $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{e+fx}}\,dx$

3.69.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

3.69. $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{e+fx}}\,dx$

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

3.69.4 Maple [A] (verified)

Time = 1.95 (sec) , antiderivative size = 812, normalized size of antiderivative = 1.54

method	result
elliptic	$\frac{\sqrt{(bx+a)(dx+c)(fx+e)}\left(\frac{2Cx\sqrt{bdf\,x^3+adf\,x^2+bcf\,x^2+bde\,x^2+acfx+adex+bcex+ace}}{5fb}+\frac{2\left(Bd+Cc-\frac{2C(2adf+2bcf+2bde)}{5fb}\right)\sqrt{bdf\,x^3+adf\,x^2 \ldots}}{3bdf} \ldots\right.}{\ldots}$
default	Expression too large to display

input
```
int((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(1/2)/(f*x+e)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
((b*x+a)*(d*x+c)*(f*x+e))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*
(2/5*C/f/b*x*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*
e*x+a*c*e)^(1/2)+2/3*(B*d+C*c-2/5*C/f/b*(2*a*d*f+2*b*c*f+2*b*d*e))/b/d/f*(
b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/
2)+2*(A*c-2/5*C/f/b*a*c*e-2/3*(B*d+C*c-2/5*C/f/b*(2*a*d*f+2*b*c*f+2*b*d*e)
)/b/d/f*(1/2*a*c*f+1/2*a*d*e+1/2*b*c*e))*(e/f-c/d)*((x+e/f)/(e/f-c/d))^(1/
2)*((x+a/b)/(-e/f+a/b))^(1/2)*((x+c/d)/(-e/f+c/d))^(1/2)/(b*d*f*x^3+a*d*f*
x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*EllipticF(((x
+e/f)/(e/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1/2))+2*(A*d+B*c-2/5*C/f/b
*(3/2*a*c*f+3/2*a*d*e+3/2*b*c*e)-2/3*(B*d+C*c-2/5*C/f/b*(2*a*d*f+2*b*c*f+2
*b*d*e))/b/d/f*(a*d*f+b*c*f+b*d*e))*(e/f-c/d)*((x+e/f)/(e/f-c/d))^(1/2)*((
x+a/b)/(-e/f+a/b))^(1/2)*((x+c/d)/(-e/f+c/d))^(1/2)/(b*d*f*x^3+a*d*f*x^2+b
*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*((-e/f+a/b)*Ellipt
icE(((x+e/f)/(e/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1/2))-a/b*EllipticF
(((x+e/f)/(e/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1/2))))
```

3.69.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 1036, normalized size of antiderivative = 1.97

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{e+fx}}\,dx$$
$$= \frac{2\left(3\,(3\,Cb^3d^3f^3x - 4\,Cb^3d^3ef^2 + (Cb^3cd^2 - (4\,Cab^2 - 5\,Bb^3)d^3)f^3)\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e} - (8\,Cb^3d^3\right.}{}$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(1/2)/(f*x+e)^(1/2),x, algor
ithm="fricas")
```

output
```
2/45*(3*(3*C*b^3*d^3*f^3*x - 4*C*b^3*d^3*e*f^2 + (C*b^3*c*d^2 - (4*C*a*b^2
 - 5*B*b^3)*d^3)*f^3)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e) - (8*C*b^3
*d^3*e^3 - (7*C*b^3*c*d^2 - (3*C*a*b^2 - 10*B*b^3)*d^3)*e^2*f - (2*C*b^3*c
^2*d + 2*(C*a*b^2 - 5*B*b^3)*c*d^2 - (3*C*a^2*b - 5*B*a*b^2 + 15*A*b^3)*d^
3)*e*f^2 - (2*C*b^3*c^3 + (2*C*a*b^2 - 5*B*b^3)*c^2*d + (7*C*a^2*b - 10*B*
a*b^2 + 30*A*b^3)*c*d^2 - (8*C*a^3 - 10*B*a^2*b + 15*A*a*b^2)*d^3)*f^3)*sq
rt(b*d*f)*weierstrassPInverse(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*e*f +
 (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 -
3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3
)*e*f^2 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3)/(b^
3*d^3*f^3), 1/3*(3*b*d*f*x + b*d*e + (b*c + a*d)*f)/(b*d*f)) - 3*(8*C*b^3*
d^3*e^2*f - (3*C*b^3*c*d^2 - (7*C*a*b^2 - 10*B*b^3)*d^3)*e*f^2 - (2*C*b^3*
c^2*d + (3*C*a*b^2 - 5*B*b^3)*c*d^2 - (8*C*a^2*b - 10*B*a*b^2 + 15*A*b^3)*
d^3)*f^3)*sqrt(b*d*f)*weierstrassZeta(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d^
2)*e*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3*d^
3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a
^2*b*d^3)*e*f^2 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*
f^3)/(b^3*d^3*f^3), weierstrassPInverse(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*
d^2)*e*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3*
d^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d^...
```

3.69.6 Sympy [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{e+fx}}\,dx = \int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{e+fx}}\,dx$$

input
```
integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)/(b*x+a)**(1/2)/(f*x+e)**(1/2),x)
```

output
```
Integral(sqrt(c + d*x)*(A + B*x + C*x**2)/(sqrt(a + b*x)*sqrt(e + f*x)), x
)
```

3.69.7 Maxima [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{e+fx}}\,dx=\int \frac{(Cx^2+Bx+A)\sqrt{dx+c}}{\sqrt{bx+a}\sqrt{fx+e}}\,dx$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(1/2)/(f*x+e)^(1/2),x, algor
ithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(d*x + c)/(sqrt(b*x + a)*sqrt(f*x + e)), x
)
```

3.69.8 Giac [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{e+fx}}\,dx=\int \frac{(Cx^2+Bx+A)\sqrt{dx+c}}{\sqrt{bx+a}\sqrt{fx+e}}\,dx$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(1/2)/(f*x+e)^(1/2),x, algor
ithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(d*x + c)/(sqrt(b*x + a)*sqrt(f*x + e)), x
)
```

3.69.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{e+fx}}\,dx=\int \frac{\sqrt{c+d\,x}\,\left(C\,x^2+B\,x+A\right)}{\sqrt{e+f\,x}\,\sqrt{a+b\,x}}\,dx$$

input
```
int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)^(1/2)),
x)
```

output
```
int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)^(1/2)),
 x)
```

3.70 $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{3/2}\sqrt{e+fx}}\,dx$

3.70.1 Optimal result

Integrand size = 38, antiderivative size = 540

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{3/2}\sqrt{e+fx}}\,dx = \frac{2(4a^2Cdf+b^2(cCe+3Adf)-ab(Cde+cCf+3Bdf))\sqrt{a+bx}\sqrt{c+dx}}{3b^2(bc-ad)f(be-af)}$$
$$-\frac{2(Ab^2-a(bB-aC))(c+dx)^{3/2}\sqrt{e+fx}}{b(bc-ad)(be-af)\sqrt{a+bx}}$$
$$+\frac{2\sqrt{-bc+ad}(8a^2Cdf^2-abf(3Cde+cCf+6Bdf)+b^2(3df(Be+Af)-Ce(2de-cf)))\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{e+f}}{3b^3\sqrt{d}f^2(be-af)\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}$$
$$+\frac{2\sqrt{-bc+ad}(de-cf)(2bCe-3bBf+4aCf)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{-bc+ad}}\right),\frac{(bc-ad)f}{d(be-af)}\right)}{3b^3\sqrt{d}f^2\sqrt{c+dx}\sqrt{e+fx}}$$

output
```
-2*(A*b^2-a*(B*b-C*a))*(d*x+c)^(3/2)*(f*x+e)^(1/2)/b/(-a*d+b*c)/(-a*f+b*e)
/(b*x+a)^(1/2)+2/3*(4*a^2*C*d*f+b^2*(3*A*d*f+C*c*e)-a*b*(3*B*d*f+C*c*f+C*d
*e))*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^2/(-a*d+b*c)/f/(-a*f+b*e)
+2/3*(8*a^2*C*d*f^2-a*b*f*(6*B*d*f+C*c*f+3*C*d*e)+b^2*(3*d*f*(A*f+B*e)-C*e
*(-c*f+2*d*e)))*EllipticE(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c
)*f/d/(-a*f+b*e))^(1/2))*(a*d-b*c)^(1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(f*x
+e)^(1/2)/b^3/f^2/(-a*f+b*e)/d^(1/2)/(d*x+c)^(1/2)/(b*(f*x+e)/(-a*f+b*e))^
(1/2)+2/3*(-c*f+d*e)*(-3*B*b*f+4*C*a*f+2*C*b*e)*EllipticF(d^(1/2)*(b*x+a)^
(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(a*d-b*c)^(1/2)*(
b*(d*x+c)/(-a*d+b*c))^(1/2)*(b*(f*x+e)/(-a*f+b*e))^(1/2)/b^3/f^2/d^(1/2)/(
d*x+c)^(1/2)/(f*x+e)^(1/2)
```

3.70.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.02 (sec) , antiderivative size = 551, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{3/2}\sqrt{e+fx}}\,dx =$$
$$-\frac{2\left(b^2\sqrt{-a+\frac{bc}{d}}(-8a^2Cdf^2+abf(3Cde+cCf+6Bdf)+b^2(-3df(Be+Af)+Ce(2de-cf)))\,(c+dx)(\right.}{}$$

input
```
Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^(3/2)*Sqrt[e + f*x]
),x]
```

output
```
(-2*(b^2*Sqrt[-a + (b*c)/d]*(-8*a^2*C*d*f^2 + a*b*f*(3*C*d*e + c*C*f + 6*B
*d*f) + b^2*(-3*d*f*(B*e + A*f) + C*e*(2*d*e - c*f)))*(c + d*x)*(e + f*x)
+ b^2*Sqrt[-a + (b*c)/d]*d*f*(c + d*x)*(e + f*x)*(3*(A*b^2 + a*(-(b*B) + a
*C))*f - C*(b*e - a*f)*(a + b*x)) - I*(b*c - a*d)*f*(8*a^2*C*d*f^2 - a*b*f
*(3*C*d*e + c*C*f + 6*B*d*f) + b^2*(3*d*f*(B*e + A*f) + C*e*(-2*d*e + c*f)
))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f
*(a + b*x))]*EllipticE[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e
 - a*d*f)/(b*c*f - a*d*f)] - I*b*f*(d*e - c*f)*(4*a^2*C*d*f + b^2*(c*C*e +
 3*A*d*f) - a*b*(C*d*e + c*C*f + 3*B*d*f))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*
x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticF[I*ArcSinh[S
qrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)]))/(3*b^
4*Sqrt[-a + (b*c)/d]*d*f^2*(b*e - a*f)*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x])
```

3.70.3 Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 555, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2117, 27, 171, 27, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{3/2}\sqrt{e+fx}}\,dx$$

$$\Big\downarrow \text{2117}$$

$$-\frac{2\int -\frac{\sqrt{c+dx}\left(C(3de+cf)a^2-b(cCe+3Bde+Bcf-Adf)a+b^2(Bc+2Ad)e+b\left(\frac{4Cdfa^2}{b}-(Cde+cCf+3Bdf)a+b(cCe+3Adf)\right)x\right)}{2b\sqrt{a+bx}\sqrt{e+fx}}dx}{(bc-ad)(be-af)}-\frac{2(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{c+dx}\left(C(3de+cf)a^2-b(cCe+3Bde+Bcf-Adf)a+b^2(Bc+2Ad)e+b\left(\frac{4Cdfa^2}{b}-(Cde+cCf+3Bdf)a+b(cCe+3Adf)\right)x\right)}{\sqrt{a+bx}\sqrt{e+fx}}dx}{b(bc-ad)(be-af)}-\frac{2(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$$\Big\downarrow \text{171}$$

$$\frac{\frac{2\int \frac{(bc-ad)\left(4Cf(de+cf)a^2-b(3Bf(de+cf)+Ce(de+3cf))a-b^2e(cCe-3Bcf-3Adf)+\left((3df(Be+Af)-Ce(2de-cf))b^2-af(3Cde+cCf+6Bdf)b+8a^2Cdf^2\right)x\right)}{2\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx}{3bf}}{b(bc-ad)(be-af)}$$

$$\frac{2(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{(bc-ad)\int \frac{4Cf(de+cf)a^2-b(3Bf(de+cf)+Ce(de+3cf))a-b^2e(cCe-3Bcf-3Adf)+\left((3df(Be+Af)-Ce(2de-cf))b^2-af(3Cde+cCf+6Bdf)b+8a^2Cdf^2\right)x}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx}{3bf}+}{b(bc-ad)(be-af)}$$

$$\frac{2(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$$\Big\downarrow \text{176}$$

$$\frac{\frac{(bc-ad)\left(\frac{\left(8a^2Cdf^2-abf(6Bdf+cCf+3Cde)+b^2(3df(Af+Be)-Ce(2de-cf))\right)\int \frac{\sqrt{e+fx}}{\sqrt{a+bx}\sqrt{c+dx}}dx}{f}+\frac{(be-af)(de-cf)(4aCf-3bBf+2bCe)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+}}}{f}\right.}{3bf}}{b(bc-ad)(be-af)}$$

$$\frac{2(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$$\Big\downarrow \text{124}$$

3.70. $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{3/2}\sqrt{e+fx}}\,dx$

$$\frac{(bc-ad)\left(\frac{\sqrt{e+fx}\sqrt{\frac{b(c+dx)}{bc-ad}}\left(8a^2Cdf^2-abf(6Bdf+cCf+3Cde)+b^2(3df(Af+Be)-Ce(2de-cf))\right)\int\frac{\sqrt{\frac{be}{be-af}+\frac{bfx}{be-af}}}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}}dx}{f\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}+\frac{(be-af)(de-cf)(4aCf-3bBf\cdots}{\cdots}\right.}{3bf}$$

$$b(bc-ad)(be-af)$$

$$\frac{2(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$\downarrow$ 123

$$\frac{(bc-ad)\left(\frac{(be-af)(de-cf)(4aCf-3bBf+2bCe)\int\frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx}{f}+\frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}\left(8a^2Cdf^2-abf(6Bdf+cCf+3Cde)+b^2(3df(Af+Be)-C\cdots\right.}{b\sqrt{d}f\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}\right.}{3bf}$$

$$b(bc-ad)(be-af\cdots$$

$$\frac{2(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$\downarrow$ 131

$$\frac{(bc-ad)\left(\frac{(be-af)(de-cf)\sqrt{\frac{b(c+dx)}{bc-ad}}(4aCf-3bBf+2bCe)\int\frac{1}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}\sqrt{e+fx}}dx}{f\sqrt{c+dx}}+\frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}\left(8a^2Cdf^2-abf(6Bdf+cCf+3Cde\cdots\right.}{b\sqrt{d}f\sqrt{\cdots}}\right.}{3bf}$$

$$b(bc-ad)(\cdots$$

$$\frac{2(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$\downarrow$ 131

$$\frac{(bc-ad)\left(\frac{(be-af)(de-cf)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}(4aCf-3bBf+2bCe)\int\frac{1}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}\sqrt{\frac{be}{be-af}+\frac{bfx}{be-af}}}dx}{f\sqrt{c+dx}\sqrt{e+fx}}+\frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}\left(8a^2Cdf^2-\cdots\right.}{\cdots}\right.}{3bf}$$

$$b(bc\cdots$$

$$\frac{2(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$\downarrow$ 130

3.70. $\displaystyle\int \frac{\sqrt{c+dx}\,(A+Bx+Cx^2)}{(a+bx)^{3/2}\sqrt{e+fx}}\,dx$

$$\frac{(bc-ad)\left(\frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}\left(8a^2Cdf^2-abf(6Bdf+cCf+3Cde)+b^2(3df(Af+Be)-Ce(2de-cf))\right)E\left(\arcsin\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{ad-bc}}\right)|\frac{(bc-ad)f}{d(be-af)}\right)}{b\sqrt{d}f\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}+\frac{2\sqrt{ad-bc}(be-a\cdots}{\cdots}\right.}{3bf}}{b(\cdots}$$

$$\frac{2(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

input
```
Int[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^(3/2)*Sqrt[e + f*x]),x]
```

output
```
(-2*(A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*Sqrt[e + f*x])/(b*(b*c - a*d)*
(b*e - a*f)*Sqrt[a + b*x]) + ((2*((4*a^2*C*d*f)/b + b*(c*C*e + 3*A*d*f) -
a*(C*d*e + c*C*f + 3*B*d*f))*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])/(3
*f) + ((b*c - a*d)*((2*Sqrt[-(b*c) + a*d]*(8*a^2*C*d*f^2 - a*b*f*(3*C*d*e
+ c*C*f + 6*B*d*f) + b^2*(3*d*f*(B*e + A*f) - C*e*(2*d*e - c*f)))*Sqrt[(b*
(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b
*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(b*Sqrt[d]*f*S
qrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]) + (2*Sqrt[-(b*c) + a*d]*(b*e
 - a*f)*(d*e - c*f)*(2*b*C*e - 3*b*B*f + 4*a*C*f)*Sqrt[(b*(c + d*x))/(b*c
- a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a +
 b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(b*Sqrt[d]*f
*Sqrt[c + d*x]*Sqrt[e + f*x])))/(3*b*f))/(b*(b*c - a*d)*(b*e - a*f))
```

3.70.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

3.70. $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{3/2}\sqrt{e+fx}}\,dx$

rule 2117
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Si
mp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*
(e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1
) - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x]
, x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n, 2*p]
```

3.70.4 Maple [A] (verified)

Time = 3.02 (sec) , antiderivative size = 861, normalized size of antiderivative = 1.59

method	result
elliptic	$\dfrac{\sqrt{(bx+a)(dx+c)(fx+e)}\left(\frac{2\left(bdf\,x^2+bcfx+bdex+bce\right)\left(b^2A-abB+C\,a^2\right)}{b^3(af-be)\sqrt{\left(x+\frac{a}{b}\right)\left(bdf\,x^2+bcfx+bdex+bce\right)}}+\frac{2C\sqrt{bdf\,x^3+adf\,x^2+bcf\,x^2+bde\,x^2+acfx+adex+bcex+ace}}{3b^2f}+\frac{2\left(A\right.}{}\right.}{}$
default	Expression too large to display

input
```
int((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(3/2)/(f*x+e)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
((b*x+a)*(d*x+c)*(f*x+e))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*
(2*(b*d*f*x^2+b*c*f*x+b*d*e*x+b*c*e)*(A*b^2-B*a*b+C*a^2)/b^3/(a*f-b*e)/((x
+a/b)*(b*d*f*x^2+b*c*f*x+b*d*e*x+b*c*e))^(1/2)+2/3*C/b^2/f*(b*d*f*x^3+a*d*
f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)+2*((A*b^2*d
-B*a*b*d+B*b^2*c+C*a^2*d-C*a*b*c)/b^3-(A*b^2-B*a*b+C*a^2)/b^3*(a*d*f-b*c*f
-b*d*e)/(a*f-b*e)-(b*c*f+b*d*e)*(A*b^2-B*a*b+C*a^2)/b^3/(a*f-b*e)-2/3*C/b^
2/f*(1/2*a*c*f+1/2*a*d*e+1/2*b*c*e))*(e/f-c/d)*((x+e/f)/(e/f-c/d))^(1/2)*(
(x+a/b)/(-e/f+a/b))^(1/2)*((x+c/d)/(-e/f+c/d))^(1/2)/(b*d*f*x^3+a*d*f*x^2+
b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*EllipticF(((x+e/f
)/(e/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1/2))+2*(1/b^2*(B*b*d-C*a*d+C*
b*c)-(A*b^2-B*a*b+C*a^2)/b^2*d*f/(a*f-b*e)-2/3*C/b^2/f*(a*d*f+b*c*f+b*d*e)
)*(e/f-c/d)*((x+e/f)/(e/f-c/d))^(1/2)*((x+a/b)/(-e/f+a/b))^(1/2)*((x+c/d)/
(-e/f+c/d))^(1/2)/(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x
+b*c*e*x+a*c*e)^(1/2)*((-e/f+a/b)*EllipticE(((x+e/f)/(e/f-c/d))^(1/2),((-e
/f+c/d)/(-e/f+a/b))^(1/2))-a/b*EllipticF(((x+e/f)/(e/f-c/d))^(1/2),((-e/f+
c/d)/(-e/f+a/b))^(1/2))))
```

3.70.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 1336, normalized size of antiderivative = 2.47

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{3/2}\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(3/2)/(f*x+e)^(1/2),x, algor
ithm="fricas")
```

output
```
2/9*(3*(C*a*b^3*d^2*e*f^2 - (4*C*a^2*b^2 - 3*B*a*b^3 + 3*A*b^4)*d^2*f^3 +
(C*b^4*d^2*e*f^2 - C*a*b^3*d^2*f^3)*x)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*
x + e) + (2*C*a*b^3*d^2*e^3 - (2*C*a*b^3*c*d - (2*C*a^2*b^2 - 3*B*a*b^3)*d
^2)*e^2*f - (C*a*b^3*c^2 + 6*(C*a^2*b^2 - B*a*b^3)*c*d - (7*C*a^3*b - 6*B*
a^2*b^2 + 6*A*a*b^3)*d^2)*e*f^2 + (C*a^2*b^2*c^2 + (5*C*a^3*b - 3*B*a^2*b^
2 - 3*A*a*b^3)*c*d - (8*C*a^4 - 6*B*a^3*b + 3*A*a^2*b^2)*d^2)*f^3 + (2*C*b
^4*d^2*e^3 - (2*C*b^4*c*d - (2*C*a*b^3 - 3*B*b^4)*d^2)*e^2*f - (C*b^4*c^2
+ 6*(C*a*b^3 - B*b^4)*c*d - (7*C*a^2*b^2 - 6*B*a*b^3 + 6*A*b^4)*d^2)*e*f^2
 + (C*a*b^3*c^2 + (5*C*a^2*b^2 - 3*B*a*b^3 - 3*A*b^4)*c*d - (8*C*a^3*b - 6
*B*a^2*b^2 + 3*A*a*b^3)*d^2)*f^3)*x)*sqrt(b*d*f)*weierstrassPInverse(4/3*(
b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)
/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f - 3
*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 + (2*b^3*c^3 - 3*a*b^2*c^2*
d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3), 1/3*(3*b*d*f*x + b*d*e
+ (b*c + a*d)*f)/(b*d*f)) + 3*(2*C*a*b^3*d^2*e^2*f - (C*a*b^3*c*d - 3*(C*a
^2*b^2 - B*a*b^3)*d^2)*e*f^2 + (C*a^2*b^2*c*d - (8*C*a^3*b - 6*B*a^2*b^2 +
 3*A*a*b^3)*d^2)*f^3 + (2*C*b^4*d^2*e^2*f - (C*b^4*c*d - 3*(C*a*b^3 - B*b^
4)*d^2)*e*f^2 + (C*a*b^3*c*d - (8*C*a^2*b^2 - 6*B*a*b^3 + 3*A*b^4)*d^2)*f^
3)*x)*sqrt(b*d*f)*weierstrassZeta(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*e
*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3...
```

3.70.6 Sympy [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{3/2}\sqrt{e+fx}}\,dx = \int \frac{\sqrt{c+dx}\,(A+Bx+Cx^2)}{(a+bx)^{\frac{3}{2}}\sqrt{e+fx}}\,dx$$

input
```
integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)/(b*x+a)**(3/2)/(f*x+e)**(1/2),x)
```

output
```
Integral(sqrt(c + d*x)*(A + B*x + C*x**2)/((a + b*x)**(3/2)*sqrt(e + f*x))
, x)
```

3.70.7 Maxima [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{3/2}\sqrt{e+fx}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{dx+c}}{(bx+a)^{\frac{3}{2}}\sqrt{fx+e}}\,dx$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(3/2)/(f*x+e)^(1/2),x, algor
ithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(d*x + c)/((b*x + a)^(3/2)*sqrt(f*x + e)),
 x)
```

3.70.8 Giac [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{3/2}\sqrt{e+fx}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{dx+c}}{(bx+a)^{\frac{3}{2}}\sqrt{fx+e}}\,dx$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(3/2)/(f*x+e)^(1/2),x, algor
ithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(d*x + c)/((b*x + a)^(3/2)*sqrt(f*x + e)),
 x)
```

3.70.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{3/2}\sqrt{e+fx}}\,dx = \int \frac{\sqrt{c+d\,x}\,(C\,x^2+B\,x+A)}{\sqrt{e+f\,x}\,{(a+b\,x)}^{3/2}}\,dx$$

input
```
int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)^(3/2)),
x)
```

output
```
int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)^(3/2)),
 x)
```

3.71 $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{5/2}\sqrt{e+fx}}\,dx$

3.71.1 Optimal result

Integrand size = 38, antiderivative size = 597

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{5/2}\sqrt{e+fx}}\,dx =$$

$$-\frac{2(4a^2Cf+b^2(3Be-2Af)-ab(6Ce+Bf))\sqrt{c+dx}\sqrt{e+fx}}{3b^2(be-af)^2\sqrt{a+bx}}$$

$$-\frac{2(Ab^2-a(bB-aC))\,(c+dx)^{3/2}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^{3/2}}$$

$$+\frac{2\sqrt{d}(8a^3Cdf^2-a^2bf(13Cde+7cCf+2Bdf)+ab^2(3Ce(de+4cf)+f(4Bde+Bcf-Adf))-b^3(Adef}{3b^3\sqrt{-bc+ad}f(be-af)^2\sqrt{c+dx}}$$

$$+\frac{2(de-cf)\,(4a^2Cdf+b^2(3cCe+Adf)-ab(Bdf+3C(de+cf)))\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}}{\sqrt{-}}\right.\right.}{3b^3\sqrt{d}\sqrt{-bc+ad}f(be-af)\sqrt{c+dx}\sqrt{e+fx}}$$

output
```
-2/3*(A*b^2-a*(B*b-C*a))*(d*x+c)^(3/2)*(f*x+e)^(1/2)/b/(-a*d+b*c)/(-a*f+b*
e)/(b*x+a)^(3/2)-2/3*(4*a^2*C*f+b^2*(-2*A*f+3*B*e)-a*b*(B*f+6*C*e))*(d*x+c
)^(1/2)*(f*x+e)^(1/2)/b^2/(-a*f+b*e)^2/(b*x+a)^(1/2)+2/3*(8*a^3*C*d*f^2-a^
2*b*f*(2*B*d*f+7*C*c*f+13*C*d*e)+a*b^2*(3*C*e*(4*c*f+d*e)+f*(-A*d*f+B*c*f+
4*B*d*e))-b^3*(A*d*e*f+c*(-2*A*f^2+3*B*e*f+3*C*e^2)))*EllipticE(d^(1/2)*(b
*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*d^(1/2)*(b*
(d*x+c)/(-a*d+b*c))^(1/2)*(f*x+e)^(1/2)/b^3/f/(-a*f+b*e)^2/(a*d-b*c)^(1/2)
/(d*x+c)^(1/2)/(b*(f*x+e)/(-a*f+b*e))^(1/2)+2/3*(-c*f+d*e)*(4*a^2*C*d*f+b^
2*(A*d*f+3*C*c*e)-a*b*(B*d*f+3*C*(c*f+d*e)))*EllipticF(d^(1/2)*(b*x+a)^(1/
2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(b*(d*x+c)/(-a*d+b*c
))^(1/2)*(b*(f*x+e)/(-a*f+b*e))^(1/2)/b^3/f/(-a*f+b*e)/d^(1/2)/(a*d-b*c)^(
1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)
```

3.71.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 28.13 (sec) , antiderivative size = 724, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{5/2}\sqrt{e+fx}}\,dx =$$

$$-\frac{2\left(b^2\sqrt{-a+\frac{bc}{d}}f(c+dx)(e+fx)\left((Ab^2+a(-bB+aC))(bc-ad)(be-af)+(-5a^3Cdf+b^3(3Bce+A\right.\right.}{}$$

input
```
Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^(5/2)*Sqrt[e + f*x]
),x]
```

output
```
(-2*(b^2*Sqrt[-a + (b*c)/d]*f*(c + d*x)*(e + f*x)*((A*b^2 + a*(-(b*B) + a*
C))*(b*c - a*d)*(b*e - a*f) + (-5*a^3*C*d*f + b^3*(3*B*c*e + A*d*e - 2*A*c
*f) - a*b^2*(6*c*C*e + 4*B*d*e + B*c*f - A*d*f) + a^2*b*(7*C*d*e + 4*c*C*f
 + 2*B*d*f))*(a + b*x)) + (a + b*x)*(b^2*Sqrt[-a + (b*c)/d]*(8*a^3*C*d*f^2
 - a^2*b*f*(13*C*d*e + 7*c*C*f + 2*B*d*f) - b^3*(3*c*C*e^2 + A*d*e*f + c*f
*(3*B*e - 2*A*f)) + a*b^2*(3*C*e*(d*e + 4*c*f) + f*(4*B*d*e + B*c*f - A*d*
f)))*(c + d*x)*(e + f*x) + I*(b*c - a*d)*f*(8*a^3*C*d*f^2 - a^2*b*f*(13*C*
d*e + 7*c*C*f + 2*B*d*f) - b^3*(3*c*C*e^2 + A*d*e*f + c*f*(3*B*e - 2*A*f))
 + a*b^2*(3*C*e*(d*e + 4*c*f) + f*(4*B*d*e + B*c*f - A*d*f)))*(a + b*x)^(3
/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*El
lipticE[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*
f - a*d*f)] + I*b*(b*c - a*d)*f*(d*e - c*f)*(-4*a^2*C*f + b^2*(-3*B*e + 2*
A*f) + a*b*(6*C*e + B*f))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))
]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticF[I*ArcSinh[Sqrt[-a + (b*c)/d]
/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)])))/(3*b^4*Sqrt[-a + (b*c
)/d]*(b*c - a*d)*f*(b*e - a*f)^2*(a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*
x])
```

3.71.3 Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 630, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2117, 27, 167, 27, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{(a+bx)^{5/2}\sqrt{e+fx}}\,dx$$

$$\Big\downarrow \text{ 2117}$$

$$-\frac{2\int -\frac{\sqrt{c+dx}\left(C(3de+cf)a^2-b(3cCe+3Bde+Bcf-3Adf)a+b^2(3Bce-2Acf)-b\left(-\frac{4Cdfa^2}{b}+Bdfa+3C(de+cf)a-b(3cCe+Adf)\right)x\right)}{2b(a+bx)^{3/2}\sqrt{e+fx}}dx}{3(bc-ad)(be-af)}-\frac{2(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\sqrt{c+dx}\left(C(3de+cf)a^2-b(3cCe+3Bde+Bcf-3Adf)a+b^2c(3Be-2Af)-b\left(-\frac{4Cdfa^2}{b}+Bdfa+3C(de+cf)a-b(3cCe+Adf)\right)x\right)}{(a+bx)^{3/2}\sqrt{e+fx}}dx}{3b(bc-ad)(be-af)}-$$

$$\frac{2(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$\Big\downarrow$ 167

$$\frac{2\int -\frac{4Cdf(de+cf)a^3-b\left(Bdf(de+cf)+C\left(6d^2e^2+11cdfe+3c^2f^2\right)\right)a^2+b^2\left(6Cefc^2+d\left(9Ce^2+2Bfe+Af^2\right)c+d^2e(3Be-2Af)\right)a-b^3ce(3cCe+3Bde-Adf)+d(8Cdf}{2\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}}{b(be-af)}$$

$$\frac{2(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$\Big\downarrow$ 27

$$-\frac{\int \frac{4Cdf(de+cf)a^3-b\left(Bdf(de+cf)+C\left(6d^2e^2+11cdfe+3c^2f^2\right)\right)a^2+b^2\left(6Cefc^2+d\left(9Ce^2+2Bfe+Af^2\right)c+d^2e(3Be-2Af)\right)a-b^3ce(3cCe+3Bde-Adf)+d(8Cdf^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}}{b(be-af)}$$

3

$$\frac{2(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$\Big\downarrow$ 176

$$-\frac{\frac{(be-af)(de-cf)\left(4a^2Cdf-ab(Bdf+3C(cf+de))+b^2(Adf+3cCe)\right)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx}{f}+\frac{d\left(8a^3Cdf^2-a^2bf(2Bdf+7cCf+13Cde)+ab^2(f(-Adf+Bcf+4B\right.}{}}{b(be-af)}$$

$3b(bc-ad)(b$

$$\frac{2(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$\Big\downarrow$ 124

$$-\frac{\frac{(be-af)(de-cf)\left(4a^2Cdf-ab(Bdf+3C(cf+de))+b^2(Adf+3cCe)\right)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx}{f}+\frac{d\sqrt{e+fx}\sqrt{\frac{b(c+dx)}{bc-ad}}\left(8a^3Cdf^2-a^2bf(2Bdf+7cCf+13Cde)+ab^2(\right.}{}}{b(be-af)}$$

$3b($

$$\frac{2(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$\Big\downarrow$ 123

3.71. $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{5/2}\sqrt{e+fx}}\,dx$

$$-\frac{\frac{(be-af)(de-cf)\left(4a^2Cdf-ab(Bdf+3C(cf+de))+b^2(Adf+3cCe)\right)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx}{f}+\frac{2\sqrt{d}\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}\left(8a^3Cdf^2-a^2bf(2Bdf+7cCf+13\right.}{}}{b(be-af)}$$

$$\frac{2(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$\downarrow$ 131

$$-\frac{\frac{(be-af)(de-cf)\sqrt{\frac{b(c+dx)}{bc-ad}}\left(4a^2Cdf-ab(Bdf+3C(cf+de))+b^2(Adf+3cCe)\right)\int \frac{1}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}\sqrt{e+fx}}dx}{f\sqrt{c+dx}}+\frac{2\sqrt{d}\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}\left(8a^3Cdf^2-\right.}{}}{b(be-af)}$$

$$\frac{2(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$\downarrow$ 131

$$-\frac{\frac{(be-af)(de-cf)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\left(4a^2Cdf-ab(Bdf+3C(cf+de))+b^2(Adf+3cCe)\right)\int \frac{1}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}\sqrt{\frac{be}{be-af}+\frac{bfx}{be-af}}}dx}{f\sqrt{c+dx}\sqrt{e+fx}}+\frac{2\sqrt{d}\sqrt{e+fx}\sqrt{ad-bc}}{}}{b(}$$

$$\frac{2(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$\downarrow$ 130

$$-\frac{2\sqrt{c+dx}\sqrt{e+fx}(bc-ad)\left(4a^2Cf-ab(Bf+6Ce)+b^2(3Be-2Af)\right)}{b\sqrt{a+bx}(be-af)}-\frac{\frac{2\sqrt{ad-bc}(be-af)(de-cf)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\left(4a^2Cdf-ab(Bdf+3C(cf+de))+b^2\right.}{b\sqrt{d}f\sqrt{c+dx}\sqrt{e+fx}}}{}$$

$$\frac{2(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

input
```
Int[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^(5/2)*Sqrt[e + f*x]),x]
```

3.71. $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{5/2}\sqrt{e+fx}}\,dx$

output
```
(-2*(A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*Sqrt[e + f*x])/(3*b*(b*c - a*d
)*(b*e - a*f)*(a + b*x)^(3/2)) + ((-2*(b*c - a*d)*(4*a^2*C*f + b^2*(3*B*e
- 2*A*f) - a*b*(6*C*e + B*f))*Sqrt[c + d*x]*Sqrt[e + f*x])/(b*(b*e - a*f)*
Sqrt[a + b*x]) - ((2*Sqrt[d]*Sqrt[-(b*c) + a*d]*(8*a^3*C*d*f^2 - a^2*b*f*(
13*C*d*e + 7*c*C*f + 2*B*d*f) - b^3*(3*c*C*e^2 + A*d*e*f + c*f*(3*B*e - 2*
A*f)) + a*b^2*(3*C*e*(d*e + 4*c*f) + f*(4*B*d*e + B*c*f - A*d*f)))*Sqrt[(b
*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a +
b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))]])/(b*f*Sqrt[c +
 d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]) + (2*Sqrt[-(b*c) + a*d]*(b*e - a*f)
*(d*e - c*f)*(4*a^2*C*d*f + b^2*(3*c*C*e + A*d*f) - a*b*(B*d*f + 3*C*(d*e
+ c*f)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*E
llipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*
f)/(d*(b*e - a*f))]])/(b*Sqrt[d]*f*Sqrt[c + d*x]*Sqrt[e + f*x]))/(b*(b*e -
a*f)))/(3*b*(b*c - a*d)*(b*e - a*f))
```

3.71.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

3.71. $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{5/2}\sqrt{e+fx}}\,dx$

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 167
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2*p]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 2117
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Si
mp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*
(e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1
) - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x]
, x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n, 2*p]
```

3.71. $\int \frac{\sqrt{c+dx}\,(A+Bx+Cx^2)}{(a+bx)^{5/2}\sqrt{e+fx}}\,dx$

3.71.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1268 vs. 2(543) = 1086.

Time = 4.20 (sec) , antiderivative size = 1269, normalized size of antiderivative = 2.13

method	result	size
elliptic	Expression too large to display	1269
default	Expression too large to display	15367

input
```
int((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(5/2)/(f*x+e)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
((b*x+a)*(d*x+c)*(f*x+e))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*
(2/3*(A*b^2-B*a*b+C*a^2)/b^4/(a*f-b*e)*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*
e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)/(x+a/b)^2+2/3*(b*d*f*x^2+b*c*f*
x+b*d*e*x+b*c*e)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^3*(A*a*b^2*d*f-2*A*b^
3*c*f+A*b^3*d*e+2*B*a^2*b*d*f-B*a*b^2*c*f-4*B*a*b^2*d*e+3*B*b^3*c*e-5*C*a^
3*d*f+4*C*a^2*b*c*f+7*C*a^2*b*d*e-6*C*a*b^2*c*e)/(a*f-b*e)/((x+a/b)*(b*d*f
*x^2+b*c*f*x+b*d*e*x+b*c*e))^(1/2)+2*((B*b*d-2*C*a*d+C*b*c)/b^3+1/3*(A*b^2
-B*a*b+C*a^2)/b^3*d*f/(a*f-b*e)-1/3/b^3*(a*d*f-b*c*f-b*d*e)*(A*a*b^2*d*f-2
*A*b^3*c*f+A*b^3*d*e+2*B*a^2*b*d*f-B*a*b^2*c*f-4*B*a*b^2*d*e+3*B*b^3*c*e-5
*C*a^3*d*f+4*C*a^2*b*c*f+7*C*a^2*b*d*e-6*C*a*b^2*c*e)/(a^2*d*f-a*b*c*f-a*b
*d*e+b^2*c*e)/(a*f-b*e)-1/3*(b*c*f+b*d*e)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e
)/b^3*(A*a*b^2*d*f-2*A*b^3*c*f+A*b^3*d*e+2*B*a^2*b*d*f-B*a*b^2*c*f-4*B*a*b
^2*d*e+3*B*b^3*c*e-5*C*a^3*d*f+4*C*a^2*b*c*f+7*C*a^2*b*d*e-6*C*a*b^2*c*e)/
(a*f-b*e))*(e/f-c/d)*((x+e/f)/(e/f-c/d))^(1/2)*((x+a/b)/(-e/f+a/b))^(1/2)*
((x+c/d)/(-e/f+c/d))^(1/2)/(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*
x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*EllipticF(((x+e/f)/(e/f-c/d))^(1/2),((-e/f+
c/d)/(-e/f+a/b))^(1/2))+2*(C*d/b^2-1/3*d*f/b^2*(A*a*b^2*d*f-2*A*b^3*c*f+A*
b^3*d*e+2*B*a^2*b*d*f-B*a*b^2*c*f-4*B*a*b^2*d*e+3*B*b^3*c*e-5*C*a^3*d*f+4*
C*a^2*b*c*f+7*C*a^2*b*d*e-6*C*a*b^2*c*e)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)
/(a*f-b*e))*(e/f-c/d)*((x+e/f)/(e/f-c/d))^(1/2)*((x+a/b)/(-e/f+a/b))^(1...
```

3.71.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.31 (sec) , antiderivative size = 2429, normalized size of antiderivative = 4.07

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{5/2}\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(5/2)/(f*x+e)^(1/2),x, algor
ithm="fricas")
```

output
```
2/9*(3*(((5*C*a^2*b^4 - 2*B*a*b^5 - A*b^6)*c*d - 3*(2*C*a^3*b^3 - B*a^2*b^
4)*d^2)*e*f^2 - (3*(C*a^3*b^3 - A*a*b^5)*c*d - (4*C*a^4*b^2 - B*a^3*b^3 -
2*A*a^2*b^4)*d^2)*f^3 + ((3*(2*C*a*b^5 - B*b^6)*c*d - (7*C*a^2*b^4 - 4*B*a
*b^5 + A*b^6)*d^2)*e*f^2 - ((4*C*a^2*b^4 - B*a*b^5 - 2*A*b^6)*c*d - (5*C*a
^3*b^3 - 2*B*a^2*b^4 - A*a*b^5)*d^2)*f^3)*x)*sqrt(b*x + a)*sqrt(d*x + c)*s
qrt(f*x + e) - (3*(C*a^2*b^4*c*d - C*a^3*b^3*d^2)*e^3 - (6*C*a^2*b^4*c^2 -
 3*(5*C*a^3*b^3 - 2*B*a^2*b^4)*c*d + (8*C*a^4*b^2 - 5*B*a^3*b^3 - A*a^2*b^
4)*d^2)*e^2*f + (3*(2*C*a^3*b^3 + B*a^2*b^4)*c^2 - (25*C*a^4*b^2 - 4*B*a^3
*b^3 - 2*A*a^2*b^4)*c*d + (17*C*a^5*b - 5*B*a^4*b^2 - 4*A*a^3*b^3)*d^2)*e*
f^2 - ((2*C*a^4*b^2 + B*a^3*b^3 + 2*A*a^2*b^4)*c^2 - (11*C*a^5*b - 2*B*a^4
*b^2 + 2*A*a^3*b^3)*c*d + (8*C*a^6 - 2*B*a^5*b - A*a^4*b^2)*d^2)*f^3 + (3*
(C*b^6*c*d - C*a*b^5*d^2)*e^3 - (6*C*b^6*c^2 - 3*(5*C*a*b^5 - 2*B*b^6)*c*d
 + (8*C*a^2*b^4 - 5*B*a*b^5 - A*b^6)*d^2)*e^2*f + (3*(2*C*a*b^5 + B*b^6)*c
^2 - (25*C*a^2*b^4 - 4*B*a*b^5 - 2*A*b^6)*c*d + (17*C*a^3*b^3 - 5*B*a^2*b^
4 - 4*A*a*b^5)*d^2)*e*f^2 - ((2*C*a^2*b^4 + B*a*b^5 + 2*A*b^6)*c^2 - (11*C
*a^3*b^3 - 2*B*a^2*b^4 + 2*A*a*b^5)*c*d + (8*C*a^4*b^2 - 2*B*a^3*b^3 - A*a
^2*b^4)*d^2)*f^3)*x^2 + 2*(3*(C*a*b^5*c*d - C*a^2*b^4*d^2)*e^3 - (6*C*a*b^
5*c^2 - 3*(5*C*a^2*b^4 - 2*B*a*b^5)*c*d + (8*C*a^3*b^3 - 5*B*a^2*b^4 - A*a
*b^5)*d^2)*e^2*f + (3*(2*C*a^2*b^4 + B*a*b^5)*c^2 - (25*C*a^3*b^3 - 4*B*a^
2*b^4 - 2*A*a*b^5)*c*d + (17*C*a^4*b^2 - 5*B*a^3*b^3 - 4*A*a^2*b^4)*d^2...
```

3.71.6 Sympy [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{5/2}\sqrt{e+fx}}\,dx = \int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{\frac{5}{2}}\sqrt{e+fx}}\,dx$$

input
```
integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)/(b*x+a)**(5/2)/(f*x+e)**(1/2),x)
```

output
```
Integral(sqrt(c + d*x)*(A + B*x + C*x**2)/((a + b*x)**(5/2)*sqrt(e + f*x))
, x)
```

3.71.7 Maxima [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{5/2}\sqrt{e+fx}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{dx+c}}{(bx+a)^{\frac{5}{2}}\sqrt{fx+e}}\,dx$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(5/2)/(f*x+e)^(1/2),x, algor
ithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(d*x + c)/((b*x + a)^(5/2)*sqrt(f*x + e)),
 x)
```

3.71.8 Giac [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{5/2}\sqrt{e+fx}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{dx+c}}{(bx+a)^{\frac{5}{2}}\sqrt{fx+e}}\,dx$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(5/2)/(f*x+e)^(1/2),x, algor
ithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(d*x + c)/((b*x + a)^(5/2)*sqrt(f*x + e)),
 x)
```

3.71.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{5/2}\sqrt{e+fx}}\,dx = \int \frac{\sqrt{c+d\,x}\,(C\,x^2+B\,x+A)}{\sqrt{e+f\,x}\,(a+b\,x)^{5/2}}\,dx$$

input
```
int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)^(5/2)),
x)
```

output
```
int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)^(5/2)),
 x)
```

3.71. $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{5/2}\sqrt{e+fx}}\,dx$

3.72 $\int \frac{\sqrt{c+dx}\,(A+Bx+Cx^2)}{(a+bx)^{7/2}\sqrt{e+fx}}\,dx$

3.72.1 Optimal result

Integrand size = 38, antiderivative size = 1034

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{7/2}\sqrt{e+fx}}\,dx = \frac{2(4a^3Cdf - b^3(5Bce - 2Ade - 4Acf) + ab^2(10cCe + 3Bde + Bcf - 6A}{15b^2(bc-ad)(be-af)^2(a+}$$

$$-\frac{2(8a^4Cd^2f^2 - a^3bdf(23Cde + 13cCf - 2Bdf) - b^4(2Ad^2e^2 - cde(5Be - 3Af) - c^2(15Ce^2 - 10Bef + 8}{}$$

$$-\frac{2(Ab^2 - a(bB - aC))\,(c+dx)^{3/2}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$$+\frac{2\sqrt{d}(8a^4Cd^2f^2 - a^3bdf(23Cde + 13cCf - 2Bdf) - b^4(2Ad^2e^2 - cde(5Be - 3Af) - c^2(15Ce^2 - 10Bef +}{}$$

$$+\frac{2\sqrt{d}(de-cf)\,(4a^3Cdf - b^3(5Bce - 2Ade - 4Acf) + ab^2(10cCe + 3Bde + Bcf - 6Adf) - a^2b(8Cde + 6}{15b^3(-bc+ad)^{3/2}(be-af)^2\sqrt{c+dx}\sqrt{e}}$$

output
```
-2/5*(A*b^2-a*(B*b-C*a))*(d*x+c)^(3/2)*(f*x+e)^(1/2)/b/(-a*d+b*c)/(-a*f+b*
e)/(b*x+a)^(5/2)+2/15*(4*a^3*C*d*f-b^3*(-4*A*c*f-2*A*d*e+5*B*c*e)+a*b^2*(-
6*A*d*f+B*c*f+3*B*d*e+10*C*c*e)-a^2*b*(-B*d*f+6*C*c*f+8*C*d*e))*(d*x+c)^(1
/2)*(f*x+e)^(1/2)/b^2/(-a*d+b*c)/(-a*f+b*e)^2/(b*x+a)^(3/2)-2/15*(8*a^4*C*
d^2*f^2-a^3*b*d*f*(-2*B*d*f+13*C*c*f+23*C*d*e)-b^4*(2*A*d^2*e^2-c*d*e*(-3*
A*f+5*B*e)-c^2*(8*A*f^2-10*B*e*f+15*C*e^2))-a^2*b^2*(d*f*(-3*A*d*f+2*B*c*f
+7*B*d*e)-C*(3*c^2*f^2+37*c*d*e*f+23*d^2*e^2))-a*b^3*(d^2*e*(-7*A*f+3*B*e)
+2*c^2*f*(-B*f+5*C*e)+c*d*(40*C*e^2-13*f*(-A*f+B*e))))*(d*x+c)^(1/2)*(f*x+
e)^(1/2)/b^2/(-a*d+b*c)^2/(-a*f+b*e)^3/(b*x+a)^(1/2)+2/15*(8*a^4*C*d^2*f^2
-a^3*b*d*f*(-2*B*d*f+13*C*c*f+23*C*d*e)-b^4*(2*A*d^2*e^2-c*d*e*(-3*A*f+5*B
*e)-c^2*(8*A*f^2-10*B*e*f+15*C*e^2))-a^2*b^2*(d*f*(-3*A*d*f+2*B*c*f+7*B*d*
e)-C*(3*c^2*f^2+37*c*d*e*f+23*d^2*e^2))-a*b^3*(d^2*e*(-7*A*f+3*B*e)+2*c^2*
f*(-B*f+5*C*e)+c*d*(40*C*e^2-13*f*(-A*f+B*e))))*EllipticE(d^(1/2)*(b*x+a)^
(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*d^(1/2)*(b*(d*x+c
)/(-a*d+b*c))^(1/2)*(f*x+e)^(1/2)/b^3/(a*d-b*c)^(3/2)/(-a*f+b*e)^3/(d*x+c)
^(1/2)/(b*(f*x+e)/(-a*f+b*e))^(1/2)+2/15*(-c*f+d*e)*(4*a^3*C*d*f-b^3*(-4*A
*c*f-2*A*d*e+5*B*c*e)+a*b^2*(-6*A*d*f+B*c*f+3*B*d*e+10*C*c*e)-a^2*b*(-B*d*
f+6*C*c*f+8*C*d*e))*EllipticF(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d
+b*c)*f/d/(-a*f+b*e))^(1/2))*d^(1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(b*(f*x+
e)/(-a*f+b*e))^(1/2)/b^3/(a*d-b*c)^(3/2)/(-a*f+b*e)^2/(d*x+c)^(1/2)/(f*...
```

3.72.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 33.50 (sec) , antiderivative size = 1449, normalized size of antiderivative = 1.40

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{7/2}\sqrt{e+fx}}\,dx = \sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\Bigg(-\frac{2(Ab^2-abB+a^2C)}{5b^2(be-af)(a+bx)^3}$$
$$-\frac{2(5b^3Bce-10ab^2cCe+Ab^3de-6ab^2Bde+11a^2bCde-4Ab^3cf-ab^2Bcf+6a^2bcCf+3aAb^2df+2a^2}{15b^2(bc-ad)(be-af)^2(a+bx)^2}$$
$$-\frac{2(15b^4c^2Ce^2+5b^4Bcde^2-40ab^3cCde^2-2Ab^4d^2e^2-3ab^3Bd^2e^2+23a^2b^2Cd^2e^2-10b^4Bc^2ef-10ab^3c^2C}{}$$
$$+\frac{2(a+bx)^{3/2}\left(\sqrt{-a+\frac{bc}{d}}(8a^4Cd^2f^2+a^3bdf(-23Cde-13cCf+2Bdf)+b^4(-2Ad^2e^2+cde(5Be-3Af}{}$$

input
```
Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^(7/2)*Sqrt[e + f*x]
),x]
```

output
```
Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*((-2*(A*b^2 - a*b*B + a^2*C))/(5
*b^2*(b*e - a*f)*(a + b*x)^3) - (2*(5*b^3*B*c*e - 10*a*b^2*c*C*e + A*b^3*d
*e - 6*a*b^2*B*d*e + 11*a^2*b*C*d*e - 4*A*b^3*c*f - a*b^2*B*c*f + 6*a^2*b*
c*C*f + 3*a*A*b^2*d*f + 2*a^2*b*B*d*f - 7*a^3*C*d*f))/(15*b^2*(b*c - a*d)*
(b*e - a*f)^2*(a + b*x)^2) - (2*(15*b^4*c^2*C*e^2 + 5*b^4*B*c*d*e^2 - 40*a
*b^3*c*C*d*e^2 - 2*A*b^4*d^2*e^2 - 3*a*b^3*B*d^2*e^2 + 23*a^2*b^2*C*d^2*e^
2 - 10*b^4*B*c^2*e*f - 10*a*b^3*c^2*C*e*f - 3*A*b^4*c*d*e*f + 13*a*b^3*B*c
*d*e*f + 37*a^2*b^2*c*C*d*e*f + 7*a*A*b^3*d^2*e*f - 7*a^2*b^2*B*d^2*e*f -
23*a^3*b*C*d^2*e*f + 8*A*b^4*c^2*f^2 + 2*a*b^3*B*c^2*f^2 + 3*a^2*b^2*c^2*C
*f^2 - 13*a*A*b^3*c*d*f^2 - 2*a^2*b^2*B*c*d*f^2 - 13*a^3*b*c*C*d*f^2 + 3*a
^2*A*b^2*d^2*f^2 + 2*a^3*b*B*d^2*f^2 + 8*a^4*C*d^2*f^2))/(15*b^2*(b*c - a*
d)^2*(b*e - a*f)^3*(a + b*x))) + (2*(a + b*x)^(3/2)*(Sqrt[-a + (b*c)/d]*(8
*a^4*C*d^2*f^2 + a^3*b*d*f*(-23*C*d*e - 13*c*C*f + 2*B*d*f) + b^4*(-2*A*d^
2*e^2 + c*d*e*(5*B*e - 3*A*f) + c^2*(15*C*e^2 - 10*B*e*f + 8*A*f^2)) + a^2
*b^2*(d*f*(-7*B*d*e - 2*B*c*f + 3*A*d*f) + C*(23*d^2*e^2 + 37*c*d*e*f + 3*
c^2*f^2)) + a*b^3*(d^2*e*(-3*B*e + 7*A*f) + 2*c^2*f*(-5*C*e + B*f) + c*d*(
-40*C*e^2 + 13*f*(B*e - A*f))))*(d + (b*c)/(a + b*x) - (a*d)/(a + b*x))*(f
 + (b*e)/(a + b*x) - (a*f)/(a + b*x)) + (I*(-(b*c) + a*d)*f*(-8*a^4*C*d^2*
f^2 + a^3*b*d*f*(23*C*d*e + 13*c*C*f - 2*B*d*f) + b^4*(2*A*d^2*e^2 + c*d*e
*(-5*B*e + 3*A*f) + c^2*(-15*C*e^2 + 10*B*e*f - 8*A*f^2)) - a^2*b^2*(d*...
```

3.72.3 Rubi [A] (verified)

Time = 2.30 (sec) , antiderivative size = 1073, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2117, 27, 167, 27, 169, 27, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{7/2}\sqrt{e+fx}}\,dx$$

$$\Big\downarrow \; 2117$$

$$-\frac{2\int -\frac{\sqrt{c+dx}\left(C(3de+cf)a^2-b(5cCe+3Bde+Bcf-5Adf)a+b^2(5Bce-2Ade-4Acf)+b\left(\frac{4Cdfa^2}{b}-5Cdea-5cCfa+Bdfa+5bcCe-Abdf\right)x\right)}{2b(a+bx)^{5/2}\sqrt{e+fx}}d}{5(bc-ad)(be-af)}$$

$$\frac{2(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{5b(a+bx)^{5/2}(bc-ad)(be-af)}$$

$\big\downarrow$ 27

$$\frac{\int \frac{\sqrt{c+dx}\left(C(3de+cf)a^2-b(5cCe+3Bde+Bcf-5Adf)a+b^2(5Bce-2A(de+2cf))+b\left(\frac{4Cdfa^2}{b}+Bdfa-5C(de+cf)a+b(5cCe-Adf)\right)x\right)}{(a+bx)^{5/2}\sqrt{e+fx}}dx}{5b(bc-ad)(be-af)}-$$

$$\frac{2(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{5b(a+bx)^{5/2}(bc-ad)(be-af)}$$

$\big\downarrow$ 167

$$\frac{2\int -\frac{4Cdf(de+cf)a^3+b\left(Bdf(de+cf)-C\left(8d^2e^2+17cdfe+3c^2f^2\right)\right)a^2+b^2\left(2f(5Ce-Bf)c^2+d\left(25Ce^2-8Bfe+9Af^2\right)c+3d^2e(Be-2Af)\right)a+b^3\left(-\left(\left(15Ce^2-10Bfe\right.\right.\right.}{2(a+bx)^3}}{3b(}$$

$$\frac{2(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{5b(a+bx)^{5/2}(bc-ad)(be-af)}$$

$\big\downarrow$ 27

$$\frac{\frac{2\sqrt{c+dx}\sqrt{e+fx}(4a^3Cdf-a^2b(-Bdf+6cCf+8Cde)+ab^2(-6Adf+Bcf+3Bde+10cCe)-b^3(-4Acf-2Ade+5Bce))}{3b(a+bx)^{3/2}(be-af)}-\int\frac{4Cdf(de+cf)a^3+b\left(Bdf(d\right.}{}}{}$$

$$\frac{2(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{5b(a+bx)^{5/2}(bc-ad)(be-af)}$$

$\big\downarrow$ 169

$$\frac{\frac{2\left(4Cdfa^3-b(8Cde+6cCf-Bdf)a^2+b^2(10cCe+3Bde+Bcf-6Adf)a-b^3(5Bce-2Ade-4Acf)\right)\sqrt{c+dx}\sqrt{e+fx}}{3b(be-af)(a+bx)^{3/2}}-\frac{2\left(8Cd^2f^2a^4-bdf(23Cde+13cCf-\right.}{}}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$\big\downarrow$ 27

3.72. $\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{(a+bx)^{7/2}\sqrt{e+fx}}\,dx$

$$\frac{\frac{2\left(4Cdfa^3-b(8Cde+6cCf-Bdf)a^2+b^2(10cCe+3Bde+Bcf-6Adf)a-b^3(5Bce-2Ade-4Acf)\right)\sqrt{c+dx}\sqrt{e+fx}}{3b(be-af)(a+bx)^{3/2}}-\frac{2\left(8Cd^2f^2a^4-bdf(23Cde+13cCf-\right.}{}}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$\downarrow$ 176

$$\frac{\frac{2\left(4Cdfa^3-b(8Cde+6cCf-Bdf)a^2+b^2(10cCe+3Bde+Bcf-6Adf)a-b^3(5Bce-2Ade-4Acf)\right)\sqrt{c+dx}\sqrt{e+fx}}{3b(be-af)(a+bx)^{3/2}}-\frac{2\left(8Cd^2f^2a^4-bdf(23Cde+13cCf-\right.}{}}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$\downarrow$ 124

$$\frac{\frac{2\left(4Cdfa^3-b(8Cde+6cCf-Bdf)a^2+b^2(10cCe+3Bde+Bcf-6Adf)a-b^3(5Bce-2Ade-4Acf)\right)\sqrt{c+dx}\sqrt{e+fx}}{3b(be-af)(a+bx)^{3/2}}-\frac{2\left(8Cd^2f^2a^4-bdf(23Cde+13cCf-\right.}{}}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$\downarrow$ 123

$$\frac{\frac{2\left(4Cdfa^3-b(8Cde+6cCf-Bdf)a^2+b^2(10cCe+3Bde+Bcf-6Adf)a-b^3(5Bce-2Ade-4Acf)\right)\sqrt{c+dx}\sqrt{e+fx}}{3b(be-af)(a+bx)^{3/2}}-\frac{2\left(8Cd^2f^2a^4-bdf(23Cde+13cCf-\right.}{}}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$\downarrow$ 131

3.72. $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{7/2}\sqrt{e+fx}}\,dx$

$$\frac{\frac{2(4Cdfa^3-b(8Cde+6cCf-Bdf)a^2+b^2(10cCe+3Bde+Bcf-6Adf)a-b^3(5Bce-2Ade-4Acf))\sqrt{c+dx}\sqrt{e+fx}}{3b(be-af)(a+bx)^{3/2}}-\frac{2\left(8Cd^2f^2a^4-bdf(23Cde+13cCf-\right.}{}}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$\downarrow$ 131

$$\frac{\frac{2(4Cdfa^3-b(8Cde+6cCf-Bdf)a^2+b^2(10cCe+3Bde+Bcf-6Adf)a-b^3(5Bce-2Ade-4Acf))\sqrt{c+dx}\sqrt{e+fx}}{3b(be-af)(a+bx)^{3/2}}-\frac{2\left(8Cd^2f^2a^4-bdf(23Cde+13cCf-\right.}{}}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$\downarrow$ 130

$$\frac{\frac{2(4Cdfa^3-b(8Cde+6cCf-Bdf)a^2+b^2(10cCe+3Bde+Bcf-6Adf)a-b^3(5Bce-2Ade-4Acf))\sqrt{c+dx}\sqrt{e+fx}}{3b(be-af)(a+bx)^{3/2}}-\frac{2\left(8Cd^2f^2a^4-bdf(23Cde+13cCf-\right.}{}}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

input
```
Int[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^(7/2)*Sqrt[e + f*x]),x]
```

3.72. $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{7/2}\sqrt{e+fx}}\,dx$

output
```
(-2*(A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*Sqrt[e + f*x])/(5*b*(b*c - a*d
)*(b*e - a*f)*(a + b*x)^(5/2)) + ((2*(4*a^3*C*d*f - b^3*(5*B*c*e - 2*A*d*e
 - 4*A*c*f) + a*b^2*(10*c*C*e + 3*B*d*e + B*c*f - 6*A*d*f) - a^2*b*(8*C*d*
e + 6*c*C*f - B*d*f))*Sqrt[c + d*x]*Sqrt[e + f*x])/(3*b*(b*e - a*f)*(a + b
*x)^(3/2)) - ((2*(8*a^4*C*d^2*f^2 - a^3*b*d*f*(23*C*d*e + 13*c*C*f - 2*B*d
*f) - b^4*(2*A*d^2*e^2 - c*d*e*(5*B*e - 3*A*f) - c^2*(15*C*e^2 - 10*B*e*f
+ 8*A*f^2)) - a^2*b^2*(d*f*(7*B*d*e + 2*B*c*f - 3*A*d*f) - C*(23*d^2*e^2 +
 37*c*d*e*f + 3*c^2*f^2)) - a*b^3*(d^2*e*(3*B*e - 7*A*f) + 2*c^2*f*(5*C*e
- B*f) + c*d*(40*C*e^2 - 13*f*(B*e - A*f))))*Sqrt[c + d*x]*Sqrt[e + f*x])/
((b*c - a*d)*(b*e - a*f)*Sqrt[a + b*x]) - (d*((2*Sqrt[-(b*c) + a*d]*(8*a^4
*C*d^2*f^2 - a^3*b*d*f*(23*C*d*e + 13*c*C*f - 2*B*d*f) - b^4*(2*A*d^2*e^2
- c*d*e*(5*B*e - 3*A*f) - c^2*(15*C*e^2 - 10*B*e*f + 8*A*f^2)) - a^2*b^2*(
d*f*(7*B*d*e + 2*B*c*f - 3*A*d*f) - C*(23*d^2*e^2 + 37*c*d*e*f + 3*c^2*f^2
)) - a*b^3*(d^2*e*(3*B*e - 7*A*f) + 2*c^2*f*(5*C*e - B*f) + c*d*(40*C*e^2
- 13*f*(B*e - A*f))))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*Ellipt
icE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d
*(b*e - a*f))])/(b*Sqrt[d]*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)])
+ (2*Sqrt[-(b*c) + a*d]*(b*e - a*f)*(d*e - c*f)*(4*a^3*C*d*f - b^3*(5*B*c*
e - 2*A*d*e - 4*A*c*f) + a*b^2*(10*c*C*e + 3*B*d*e + B*c*f - 6*A*d*f) - a^
2*b*(8*C*d*e + 6*c*C*f - B*d*f))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[(...
```

3.72.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 167
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2*p]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

3.72. $\int \frac{\sqrt{c+dx}\,(A+Bx+Cx^2)}{(a+bx)^{7/2}\sqrt{e+fx}}\,dx$

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 2117
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Si
mp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*
(e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1
) - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x]
, x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n, 2*p]
```

3.72.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2329 vs. 2(972) = 1944.

Time = 5.68 (sec) , antiderivative size = 2330, normalized size of antiderivative = 2.25

method	result	size
elliptic	Expression too large to display	2330
default	Expression too large to display	36158

input
```
int((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(7/2)/(f*x+e)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
((b*x+a)*(d*x+c)*(f*x+e))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*
(2/5*(A*b^2-B*a*b+C*a^2)/b^5/(a*f-b*e)*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*
e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)/(x+a/b)^3+2/15*(3*A*a*b^2*d*f-4
*A*b^3*c*f+A*b^3*d*e+2*B*a^2*b*d*f-B*a*b^2*c*f-6*B*a*b^2*d*e+5*B*b^3*c*e-7
*C*a^3*d*f+6*C*a^2*b*c*f+11*C*a^2*b*d*e-10*C*a*b^2*c*e)/b^4/(a^2*d*f-a*b*c
*f-a*b*d*e+b^2*c*e)/(a*f-b*e)*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c
*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)/(x+a/b)^2+2/15*(b*d*f*x^2+b*c*f*x+b*d*e*
x+b*c*e)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)^2/b^3*(3*A*a^2*b^2*d^2*f^2-13*A
*a*b^3*c*d*f^2+7*A*a*b^3*d^2*e*f+8*A*b^4*c^2*f^2-3*A*b^4*c*d*e*f-2*A*b^4*d
^2*e^2+2*B*a^3*b*d^2*f^2-2*B*a^2*b^2*c*d*f^2-7*B*a^2*b^2*d^2*e*f+2*B*a*b^3
*c^2*f^2+13*B*a*b^3*c*d*e*f-3*B*a*b^3*d^2*e^2-10*B*b^4*c^2*e*f+5*B*b^4*c*d
*e^2+8*C*a^4*d^2*f^2-13*C*a^3*b*c*d*f^2-23*C*a^3*b*d^2*e*f+3*C*a^2*b^2*c^2
*f^2+37*C*a^2*b^2*c*d*e*f+23*C*a^2*b^2*d^2*e^2-10*C*a*b^3*c^2*e*f-40*C*a*b
^3*c*d*e^2+15*C*b^4*c^2*e^2)/(a*f-b*e)/((x+a/b)*(b*d*f*x^2+b*c*f*x+b*d*e*x
+b*c*e))^(1/2)+2*(C*d/b^3+1/15*d*f*(3*A*a*b^2*d*f-4*A*b^3*c*f+A*b^3*d*e+2*
B*a^2*b*d*f-B*a*b^2*c*f-6*B*a*b^2*d*e+5*B*b^3*c*e-7*C*a^3*d*f+6*C*a^2*b*c*
f+11*C*a^2*b*d*e-10*C*a*b^2*c*e)/b^3/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/(a*
f-b*e)-1/15/b^3*(a*d*f-b*c*f-b*d*e)*(3*A*a^2*b^2*d^2*f^2-13*A*a*b^3*c*d*f^
2+7*A*a*b^3*d^2*e*f+8*A*b^4*c^2*f^2-3*A*b^4*c*d*e*f-2*A*b^4*d^2*e^2+2*B*a^
3*b*d^2*f^2-2*B*a^2*b^2*c*d*f^2-7*B*a^2*b^2*d^2*e*f+2*B*a*b^3*c^2*f^2+1...
```

3.72.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.00 (sec) , antiderivative size = 4867, normalized size of antiderivative = 4.71

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{7/2}\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(7/2)/(f*x+e)^(1/2),x, algor
ithm="fricas")
```

output
```
-2/45*(3*((15*C*a^4*b^4*d^3 + (8*C*a^2*b^6 + 2*B*a*b^7 + 3*A*b^8)*c^2*d -
5*(5*C*a^3*b^5 + A*a*b^7)*c*d^2)*e^2*f - (10*(B*a^2*b^6 + A*a*b^7)*c^2*d -
 15*(C*a^4*b^4 + B*a^3*b^5 + A*a^2*b^6)*c*d^2 + (11*C*a^5*b^3 + 9*B*a^4*b^
4 + A*a^3*b^5)*d^3)*e*f^2 + (15*A*a^2*b^6*c^2*d - (6*C*a^5*b^3 - B*a^4*b^4
 + 26*A*a^3*b^5)*c*d^2 + (4*C*a^6*b^2 + B*a^5*b^3 + 9*A*a^4*b^4)*d^3)*f^3
+ ((15*C*b^8*c^2*d - 5*(8*C*a*b^7 - B*b^8)*c*d^2 + (23*C*a^2*b^6 - 3*B*a*b
^7 - 2*A*b^8)*d^3)*e^2*f - (10*(C*a*b^7 + B*b^8)*c^2*d - (37*C*a^2*b^6 + 1
3*B*a*b^7 - 3*A*b^8)*c*d^2 + (23*C*a^3*b^5 + 7*B*a^2*b^6 - 7*A*a*b^7)*d^3)
*e*f^2 + ((3*C*a^2*b^6 + 2*B*a*b^7 + 8*A*b^8)*c^2*d - (13*C*a^3*b^5 + 2*B*
a^2*b^6 + 13*A*a*b^7)*c*d^2 + (8*C*a^4*b^4 + 2*B*a^3*b^5 + 3*A*a^2*b^6)*d^
3)*f^3)*x^2 + ((5*(4*C*a*b^7 + B*b^8)*c^2*d - (59*C*a^2*b^6 + B*a*b^7 - A*
b^8)*c*d^2 + 5*(7*C*a^3*b^5 - A*a*b^7)*d^3)*e^2*f - 2*((2*C*a^2*b^6 + 13*B
*a*b^7 + 2*A*b^8)*c^2*d - 20*(C*a^3*b^5 + B*a^2*b^6)*c*d^2 + (14*C*a^4*b^4
 + 11*B*a^3*b^5 - 6*A*a^2*b^6)*d^3)*e*f^2 + (5*(B*a^2*b^6 + 4*A*a*b^7)*c^2
*d - (13*C*a^4*b^4 + 7*B*a^3*b^5 + 33*A*a^2*b^6)*c*d^2 + 3*(3*C*a^5*b^3 +
2*B*a^4*b^4 + 3*A*a^3*b^5)*d^3)*f^3)*x)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f
*x + e) - ((30*C*a^3*b^5*c^2*d - 5*(10*C*a^4*b^4 + B*a^3*b^5)*c*d^2 + (22*
C*a^5*b^3 + 3*B*a^4*b^4 + 2*A*a^3*b^5)*d^3)*e^3 - (15*C*a^3*b^5*c^3 + 5*(5
*C*a^4*b^4 + 2*B*a^3*b^5)*c^2*d - (67*C*a^5*b^3 + 33*B*a^4*b^4 + 2*A*a^3*b
^5)*c*d^2 + (33*C*a^6*b^2 + 17*B*a^5*b^3 + 8*A*a^4*b^4)*d^3)*e^2*f + (1...
```

3.72.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{7/2}\sqrt{e+fx}}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)/(b*x+a)**(7/2)/(f*x+e)**(1/2),x)
```

output
```
Timed out
```

3.72.7 Maxima [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{7/2}\sqrt{e+fx}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{dx+c}}{(bx+a)^{\frac{7}{2}}\sqrt{fx+e}}\,dx$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(7/2)/(f*x+e)^(1/2),x, algor
ithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(d*x + c)/((b*x + a)^(7/2)*sqrt(f*x + e)),
 x)
```

3.72.8 Giac [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{7/2}\sqrt{e+fx}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{dx+c}}{(bx+a)^{\frac{7}{2}}\sqrt{fx+e}}\,dx$$

input
```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(7/2)/(f*x+e)^(1/2),x, algor
ithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(d*x + c)/((b*x + a)^(7/2)*sqrt(f*x + e)),
 x)
```

3.72.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{7/2}\sqrt{e+fx}}\,dx = \int \frac{\sqrt{c+d\,x}\,(C\,x^2+B\,x+A)}{\sqrt{e+f\,x}\,{(a+b\,x)}^{7/2}}\,dx$$

input
```
int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)^(7/2)),
x)
```

output
```
int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)^(7/2)),
 x)
```

3.73 $\int \frac{(a+bx)^{3/2}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$

3.73.1 Optimal result

Integrand size = 38, antiderivative size = 838

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx =$$

$$-\frac{2(5bdf(5bcCe+aCde+acCf-7Abdf)+(3adf-4b(de+cf))(2aCdf-b(7Bdf-6C(de+cf))))\sqrt{a+}}{105bd^3f^3}$$

$$-\frac{2(2aCdf-b(7Bdf-6C(de+cf)))(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}{35bd^2f^2}$$

$$+\frac{2C(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}{7bdf}$$

$$-\frac{2\sqrt{-bc+ad}\left(3bdf(5adf(5bcCe+aCde+acCf-7Abdf)-(3bce+ade+acf)(2aCdf-b(7Bdf-6C(d\right.}{}$$

$$-\frac{2\sqrt{-bc+ad}(be-af)\left(3a^2Cd^2f^2(de-cf)-3abdf(7df(3Bde+2Bcf-5Adf)-C(16d^2e^2+8cdef+1\right.}{}$$

output
```
-2/35*(2*a*C*d*f-b*(7*B*d*f-6*C*(c*f+d*e)))*(b*x+a)^(3/2)*(d*x+c)^(1/2)*(f
*x+e)^(1/2)/b/d^2/f^2+2/7*C*(b*x+a)^(5/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/d/
f-2/105*(5*b*d*f*(-7*A*b*d*f+C*a*c*f+C*a*d*e+5*C*b*c*e)+(3*a*d*f-4*b*(c*f+
d*e))*(2*a*C*d*f-b*(7*B*d*f-6*C*(c*f+d*e))))*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(
f*x+e)^(1/2)/b/d^3/f^3-2/105*(3*b*d*f*(5*a*d*f*(-7*A*b*d*f+C*a*c*f+C*a*d*e
+5*C*b*c*e)-(a*c*f+a*d*e+3*b*c*e)*(2*a*C*d*f-b*(7*B*d*f-6*C*(c*f+d*e))))+2
*(1/2*a*d*f-b*(c*f+d*e))*(5*b*d*f*(-7*A*b*d*f+C*a*c*f+C*a*d*e+5*C*b*c*e)+(
3*a*d*f-4*b*(c*f+d*e))*(2*a*C*d*f-b*(7*B*d*f-6*C*(c*f+d*e)))))*EllipticE(d
^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(a
*d-b*c)^(1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(f*x+e)^(1/2)/b^2/d^(7/2)/f^4/(
d*x+c)^(1/2)/(b*(f*x+e)/(-a*f+b*e))^(1/2)-2/105*(-a*f+b*e)*(3*a^2*C*d^2*f^
2*(-c*f+d*e)-3*a*b*d*f*(7*d*f*(-5*A*d*f+2*B*c*f+3*B*d*e)-C*(11*c^2*f^2+8*c
*d*e*f+16*d^2*e^2))-b^2*(C*(24*c^3*f^3+17*c^2*d*e*f^2+16*c*d^2*e^2*f+48*d^
3*e^3)+7*d*f*(5*A*d*f*(c*f+2*d*e)-B*(4*c^2*f^2+3*c*d*e*f+8*d^2*e^2))))*Ell
ipticF(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(
1/2))*(a*d-b*c)^(1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(b*(f*x+e)/(-a*f+b*e))^
(1/2)/b^2/d^(7/2)/f^4/(d*x+c)^(1/2)/(f*x+e)^(1/2)
```

3.73.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 29.12 (sec) , antiderivative size = 1000, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \frac{2\left(-b^2\sqrt{-a+\frac{bc}{d}}\left(6a^3Cd^3f^3+3a^2bd^2f^2(-7Bdf+4C(de+cf))-ab^2\right.\right.}{}$$

input
```
Integrate[((a + b*x)^(3/2)*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]
),x]
```

output
```
(2*(-(b^2*Sqrt[-a + (b*c)/d]*(6*a^3*C*d^3*f^3 + 3*a^2*b*d^2*f^2*(-7*B*d*f
+ 4*C*(d*e + c*f)) - a*b^2*d*f*(C*(72*d^2*e^2 + 62*c*d*e*f + 72*c^2*f^2) +
 7*d*f*(20*A*d*f - 13*B*(d*e + c*f))) + b^3*(8*C*(6*d^3*e^3 + 5*c*d^2*e^2*
f + 5*c^2*d*e*f^2 + 6*c^3*f^3) + 7*d*f*(10*A*d*f*(d*e + c*f) - B*(8*d^2*e^
2 + 7*c*d*e*f + 8*c^2*f^2))))*(c + d*x)*(e + f*x)) + b^2*Sqrt[-a + (b*c)/d
]*d*f*(a + b*x)*(c + d*x)*(e + f*x)*(3*a^2*C*d^2*f^2 + 3*a*b*d*f*(14*B*d*f
 + C*(-11*d*e - 11*c*f + 8*d*f*x)) + b^2*(7*d*f*(5*A*d*f + B*(-4*d*e - 4*c
*f + 3*d*f*x)) + C*(24*c^2*f^2 + c*d*f*(23*e - 18*f*x) + 3*d^2*(8*e^2 - 6*
e*f*x + 5*f^2*x^2)))) - I*(b*c - a*d)*f*(6*a^3*C*d^3*f^3 + 3*a^2*b*d^2*f^2
*(-7*B*d*f + 4*C*(d*e + c*f)) - a*b^2*d*f*(C*(72*d^2*e^2 + 62*c*d*e*f + 72
*c^2*f^2) + 7*d*f*(20*A*d*f - 13*B*(d*e + c*f))) + b^3*(8*C*(6*d^3*e^3 + 5
*c*d^2*e^2*f + 5*c^2*d*e*f^2 + 6*c^3*f^3) + 7*d*f*(10*A*d*f*(d*e + c*f) -
B*(8*d^2*e^2 + 7*c*d*e*f + 8*c^2*f^2))))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x)
)/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticE[I*ArcSinh[Sqr
t[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)] + I*b*(b*
c - a*d)*f*(3*a^2*C*d^2*f^2*(d*e - c*f) - 3*a*b*d*f*(7*d*f*(-2*B*d*e - 3*B
*c*f + 5*A*d*f) + C*(11*d^2*e^2 + 8*c*d*e*f + 16*c^2*f^2)) + b^2*(C*(24*d^
3*e^3 + 17*c*d^2*e^2*f + 16*c^2*d*e*f^2 + 48*c^3*f^3) + 7*d*f*(5*A*d*f*(d*
e + 2*c*f) - B*(4*d^2*e^2 + 3*c*d*e*f + 8*c^2*f^2))))*(a + b*x)^(3/2)*Sqrt
[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*Ellipti...
```

3.73.3 Rubi [A] (verified)

Time = 1.59 (sec) , antiderivative size = 857, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2118, 27, 171, 27, 171, 27, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/2} (A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}} dx$$

$$\Big\downarrow \text{2118}$$

$$\frac{2\int -\frac{b(a+bx)^{3/2}(5bcCe+aCde+acCf-7Abdf-(7bBdf-2aCdf-6bC(de+cf))x)}{2\sqrt{c+dx}\sqrt{e+fx}}dx}{7b^2df}+\frac{2C(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}{7bdf}$$

$$\Big\downarrow \text{27}$$

$$\frac{2C(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}{7bdf}-\frac{\int\frac{(a+bx)^{3/2}(5bcCe+aCde+acCf-7Abdf-(7bBdf-2aCdf-6bC(de+cf))x)}{\sqrt{c+dx}\sqrt{e+fx}}dx}{7bdf}$$

$\downarrow$ 171

$$\frac{2C(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}{7bdf}-$$

$$2\int\frac{\sqrt{a+bx}(5adf(5bcCe+aCde+acCf-7Abdf)+(3bce+ade+acf)(7bBdf-2aCdf-6bC(de+cf))+(5bdf(5bcCe+aCde+acCf-7Abdf)-(3adf-4b(de+cf))(7bBdf-2a}{2\sqrt{c+dx}\sqrt{e+fx}} \quad 5df \quad 7bdf$$

$\downarrow$ 27

$$\frac{2C(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}{7bdf}-$$

$$\int\frac{\sqrt{a+bx}(5adf(5bcCe+aCde+acCf-7Abdf)+(3bce+ade+acf)(7bBdf-2aCdf-6bC(de+cf))+(5bdf(5bcCe+aCde+acCf-7Abdf)-(3adf-4b(de+cf))(7bBdf-2aC}{\sqrt{c+dx}\sqrt{e+fx}} \quad 5df \quad 7bdf$$

$\downarrow$ 171

$$\frac{2C(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}{7bdf}-$$

$$2\int 3adf(5adf(5bcCe+aCde+acCf-7Abdf)+(3bce+ade+acf)(7bBdf-2aCdf-6bC(de+cf)))-(bce+ade+acf)(5bdf(5bcCe+aCde+acCf-7Abdf)-(3adf-4b(de+c$$

$\downarrow$ 27

$$\frac{2C(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}{7bdf}-$$

$$\int 3adf(5adf(5bcCe+aCde+acCf-7Abdf)+(3bce+ade+acf)(7bBdf-2aCdf-6bC(de+cf)))-(bce+ade+acf)(5bdf(5bcCe+aCde+acCf-7Abdf)-(3adf-4b(de+cf)$$

$\downarrow$ 176

$$\frac{2C(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}{7bdf}-$$

$$\frac{(be-af)\left(3a^2Cd^2f^2(de-cf)-3abdf\left(7df(-5Adf+2Bcf+3Bde)-C\left(11c^2f^2+8cdef+16d^2e^2\right)\right)-\left(b^2\left(7df\left(5Adf(cf+2de)-B\left(4c^2f^2+3cdef+8d^2e^2\right)\right)+C\left(24c^3f^3+\right.\right.\right.\right.}{f}$$

$\downarrow$ 124

3.73. $\int\frac{(a+bx)^{3/2}\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$

$$\frac{2C(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}{7bdf}-$$

$$\frac{(be-af)\left(3a^2Cd^2f^2(de-cf)-3abdf\left(7df(-5Adf+2Bcf+3Bde)-C\left(11c^2f^2+8cdef+16d^2e^2\right)\right)-\left(b^2\left(7df\left(5Adf(cf+2de)-B\left(4c^2f^2+3cdef+8d^2e^2\right)\right)+C\left(24c^3f^3+\right.\right.\right.\right.}{f}$$

$$\Big\downarrow \; 123$$

$$\frac{2C(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}{7bdf}-$$

$$\frac{(be-af)\left(3a^2Cd^2f^2(de-cf)-3abdf\left(7df(-5Adf+2Bcf+3Bde)-C\left(11c^2f^2+8cdef+16d^2e^2\right)\right)-\left(b^2\left(7df\left(5Adf(cf+2de)-B\left(4c^2f^2+3cdef+8d^2e^2\right)\right)+C\left(24c^3f^3+\right.\right.\right.\right.}{f}$$

$$\Big\downarrow \; 131$$

$$\frac{2C(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}{7bdf}-$$

$$\frac{2\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}(5bdf(5bcCe+aCde+acCf-7Abdf)-(3adf-4b(de+cf))(7bBdf-2aCdf-6bC(de+cf)))}{3df}+\frac{2\sqrt{ad-bc}\left(3bdf(5adf(5bcCe+aCde+acCf-7Abdf)\right.}{}$$

$$\Big\downarrow \; 131$$

$$\frac{2C(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}{7bdf}-$$

$$\frac{2\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}(5bdf(5bcCe+aCde+acCf-7Abdf)-(3adf-4b(de+cf))(7bBdf-2aCdf-6bC(de+cf)))}{3df}+\frac{2\sqrt{ad-bc}\left(3bdf(5adf(5bcCe+aCde+acCf-7Abdf)\right.}{}$$

$$\Big\downarrow \; 130$$

$$\frac{2C(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}{7bdf}-$$

$$\frac{2\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}(5bdf(5bcCe+aCde+acCf-7Abdf)-(3adf-4b(de+cf))(7bBdf-2aCdf-6bC(de+cf)))}{3df}+\frac{2\sqrt{ad-bc}\left(3bdf(5adf(5bcCe+aCde+acCf-7Abdf)\right.}{}$$

3.73. $\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$

input
```
Int[((a + b*x)^(3/2)*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

output
```
(2*C*(a + b*x)^(5/2)*Sqrt[c + d*x]*Sqrt[e + f*x])/(7*b*d*f) - ((-2*(7*b*B*
d*f - 2*a*C*d*f - 6*b*C*(d*e + c*f))*(a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e
+ f*x])/(5*d*f) + ((2*(5*b*d*f*(5*b*c*C*e + a*C*d*e + a*c*C*f - 7*A*b*d*f)
 - (3*a*d*f - 4*b*(d*e + c*f))*(7*b*B*d*f - 2*a*C*d*f - 6*b*C*(d*e + c*f))
)*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])/(3*d*f) + ((2*Sqrt[-(b*c) + a
*d]*(3*b*d*f*(5*a*d*f*(5*b*c*C*e + a*C*d*e + a*c*C*f - 7*A*b*d*f) + (3*b*c
*e + a*d*e + a*c*f)*(7*b*B*d*f - 2*a*C*d*f - 6*b*C*(d*e + c*f))) + 2*((a*d
*f)/2 - b*(d*e + c*f))*(5*b*d*f*(5*b*c*C*e + a*C*d*e + a*c*C*f - 7*A*b*d*f
) - (3*a*d*f - 4*b*(d*e + c*f))*(7*b*B*d*f - 2*a*C*d*f - 6*b*C*(d*e + c*f)
)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE[ArcSin[(Sqrt[d
]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(b
*Sqrt[d]*f*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]) + (2*Sqrt[-(b*c)
 + a*d]*(b*e - a*f)*(3*a^2*C*d^2*f^2*(d*e - c*f) - 3*a*b*d*f*(7*d*f*(3*B*d
*e + 2*B*c*f - 5*A*d*f) - C*(16*d^2*e^2 + 8*c*d*e*f + 11*c^2*f^2)) - b^2*(
C*(48*d^3*e^3 + 16*c*d^2*e^2*f + 17*c^2*d*e*f^2 + 24*c^3*f^3) + 7*d*f*(5*A
*d*f*(2*d*e + c*f) - B*(8*d^2*e^2 + 3*c*d*e*f + 4*c^2*f^2))))*Sqrt[(b*(c +
 d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt
[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/
(b*Sqrt[d]*f*Sqrt[c + d*x]*Sqrt[e + f*x]))/(3*d*f))/(5*d*f))/(7*b*d*f)
```

3.73.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

3.73. $\int \frac{(a+bx)^{3/2}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

3.73.4 Maple [A] (verified)

Time = 2.92 (sec) , antiderivative size = 1233, normalized size of antiderivative = 1.47

method	result	size
elliptic	Expression too large to display	1233
default	Expression too large to display	9580

input
```
int((b*x+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
((b*x+a)*(d*x+c)*(f*x+e))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*
(2/7*b*C/d/f*x^2*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+
b*c*e*x+a*c*e)^(1/2)+2/5*(B*b^2+2*C*a*b-2/7*b*C/d/f*(3*a*d*f+3*b*c*f+3*b*d
*e))/b/d/f*x*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*
e*x+a*c*e)^(1/2)+2/3*(b^2*A+2*a*b*B+C*a^2-2/7*b*C/d/f*(5/2*a*c*f+5/2*a*d*e
+5/2*b*c*e)-2/5*(B*b^2+2*C*a*b-2/7*b*C/d/f*(3*a*d*f+3*b*c*f+3*b*d*e))/b/d/
f*(2*a*d*f+2*b*c*f+2*b*d*e))/b/d/f*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^
2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)+2*(a^2*A-2/5*(B*b^2+2*C*a*b-2/7*b*C
/d/f*(3*a*d*f+3*b*c*f+3*b*d*e))/b/d/f*a*c*e-2/3*(b^2*A+2*a*b*B+C*a^2-2/7*b
*C/d/f*(5/2*a*c*f+5/2*a*d*e+5/2*b*c*e)-2/5*(B*b^2+2*C*a*b-2/7*b*C/d/f*(3*a
*d*f+3*b*c*f+3*b*d*e))/b/d/f*(2*a*d*f+2*b*c*f+2*b*d*e))/b/d/f*(1/2*a*c*f+1
/2*a*d*e+1/2*b*c*e))*(e/f-c/d)*((x+e/f)/(e/f-c/d))^(1/2)*((x+a/b)/(-e/f+a/
b))^(1/2)*((x+c/d)/(-e/f+c/d))^(1/2)/(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*
x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*EllipticF(((x+e/f)/(e/f-c/d))^(1/
2),((-e/f+c/d)/(-e/f+a/b))^(1/2))+2*(2*a*b*A+a^2*B-4/7*b*C/d/f*a*c*e-2/5*(
B*b^2+2*C*a*b-2/7*b*C/d/f*(3*a*d*f+3*b*c*f+3*b*d*e))/b/d/f*(3/2*a*c*f+3/2*
a*d*e+3/2*b*c*e)-2/3*(b^2*A+2*a*b*B+C*a^2-2/7*b*C/d/f*(5/2*a*c*f+5/2*a*d*e
+5/2*b*c*e)-2/5*(B*b^2+2*C*a*b-2/7*b*C/d/f*(3*a*d*f+3*b*c*f+3*b*d*e))/b/d/
f*(2*a*d*f+2*b*c*f+2*b*d*e))/b/d/f*(a*d*f+b*c*f+b*d*e))*(e/f-c/d)*((x+e/f)
/(e/f-c/d))^(1/2)*((x+a/b)/(-e/f+a/b))^(1/2)*((x+c/d)/(-e/f+c/d))^(1/2)...
```

3.73.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 1388, normalized size of antiderivative = 1.66

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algor
ithm="fricas")
```

output
```
2/315*(3*(15*C*b^4*d^4*f^4*x^2 + 24*C*b^4*d^4*e^2*f^2 + (23*C*b^4*c*d^3 -
(33*C*a*b^3 + 28*B*b^4)*d^4)*e*f^3 + (24*C*b^4*c^2*d^2 - (33*C*a*b^3 + 28*
B*b^4)*c*d^3 + (3*C*a^2*b^2 + 42*B*a*b^3 + 35*A*b^4)*d^4)*f^4 - 3*(6*C*b^4
*d^4*e*f^3 + (6*C*b^4*c*d^3 - (8*C*a*b^3 + 7*B*b^4)*d^4)*f^4)*x)*sqrt(b*x
+ a)*sqrt(d*x + c)*sqrt(f*x + e) + (48*C*b^4*d^4*e^4 + 8*(2*C*b^4*c*d^3 -
(12*C*a*b^3 + 7*B*b^4)*d^4)*e^3*f + (11*C*b^4*c^2*d^2 - 7*(4*C*a*b^3 + 3*B
*b^4)*c*d^3 + (39*C*a^2*b^2 + 119*B*a*b^3 + 70*A*b^4)*d^4)*e^2*f^2 + (16*C
*b^4*c^3*d - 7*(4*C*a*b^3 + 3*B*b^4)*c^2*d^2 + 7*(C*a^2*b^2 + 7*B*a*b^3 +
5*A*b^4)*c*d^3 + (9*C*a^3*b - 56*B*a^2*b^2 - 175*A*a*b^3)*d^4)*e*f^3 + (48
*C*b^4*c^4 - 8*(12*C*a*b^3 + 7*B*b^4)*c^3*d + (39*C*a^2*b^2 + 119*B*a*b^3
+ 70*A*b^4)*c^2*d^2 + (9*C*a^3*b - 56*B*a^2*b^2 - 175*A*a*b^3)*c*d^3 + (6*
C*a^4 - 21*B*a^3*b + 175*A*a^2*b^2)*d^4)*f^4)*sqrt(b*d*f)*weierstrassPInve
rse(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 - a*b*c*d + a^2*
d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*
e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 + (2*b^3*c^3 - 3*a
*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3), 1/3*(3*b*d*f*x
 + b*d*e + (b*c + a*d)*f)/(b*d*f)) + 3*(48*C*b^4*d^4*e^3*f + 8*(5*C*b^4*c*
d^3 - (9*C*a*b^3 + 7*B*b^4)*d^4)*e^2*f^2 + (40*C*b^4*c^2*d^2 - (62*C*a*b^3
 + 49*B*b^4)*c*d^3 + (12*C*a^2*b^2 + 91*B*a*b^3 + 70*A*b^4)*d^4)*e*f^3 + (
48*C*b^4*c^3*d - 8*(9*C*a*b^3 + 7*B*b^4)*c^2*d^2 + (12*C*a^2*b^2 + 91*B...
```

3.73.6 Sympy [F]

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx=\int \frac{(a+bx)^{\frac{3}{2}}\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

input
```
integrate((b*x+a)**(3/2)*(C*x**2+B*x+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

output
```
Integral((a + b*x)**(3/2)*(A + B*x + C*x**2)/(sqrt(c + d*x)*sqrt(e + f*x))
, x)
```

3.73.7 Maxima [F]

$$\int \frac{(a+bx)^{3/2}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx=\int \frac{(Cx^2+Bx+A)(bx+a)^{\frac{3}{2}}}{\sqrt{dx+c}\sqrt{fx+e}}\,dx$$

input
```
integrate((b*x+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algor
ithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*(b*x + a)^(3/2)/(sqrt(d*x + c)*sqrt(f*x + e)),
 x)
```

3.73.8 Giac [F]

$$\int \frac{(a+bx)^{3/2}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx=\int \frac{(Cx^2+Bx+A)(bx+a)^{\frac{3}{2}}}{\sqrt{dx+c}\sqrt{fx+e}}\,dx$$

input
```
integrate((b*x+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algor
ithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*(b*x + a)^(3/2)/(sqrt(d*x + c)*sqrt(f*x + e)),
 x)
```

3.73.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx=\int \frac{(a+b\,x)^{3/2}\,(C\,x^2+B\,x+A)}{\sqrt{e+f\,x}\,\sqrt{c+d\,x}}\,dx$$

input
```
int(((a + b*x)^(3/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(c + d*x)^(1/2)),
x)
```

output
```
int(((a + b*x)^(3/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(c + d*x)^(1/2)),
 x)
```

3.74 $\int \frac{\sqrt{a+bx}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$

3.74.1 Optimal result

Integrand size = 38, antiderivative size = 528

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$$
$$= -\frac{2(2aCdf-b(5Bdf-4C(de+cf)))\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{15bd^2f^2}$$
$$+\frac{2C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}{5bdf}$$
$$-\frac{2\sqrt{-bc+ad}(3bdf(3bcCe+aCde+acCf-5Abdf)+(adf-2b(de+cf))(2aCdf-b(5Bdf-4C(de-}{15b^2d^{5/2}f^3\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}$$
$$-\frac{2\sqrt{-bc+ad}(be-af)\left(aCdf(de-cf)-b(5df(2Bde+Bcf-3Adf)-C(8d^2e^2+3cdef+4c^2f^2))\right)\sqrt{}}{15b^2d^{5/2}f^3\sqrt{c+dx}\sqrt{e+fx}}$$

output
```
2/5*C*(b*x+a)^(3/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/d/f-2/15*(2*a*C*d*f-b*(5
*B*d*f-4*C*(c*f+d*e)))*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/d^2/f^2
-2/15*(3*b*d*f*(-5*A*b*d*f+C*a*c*f+C*a*d*e+3*C*b*c*e)+(a*d*f-2*b*(c*f+d*e)
)*(2*a*C*d*f-b*(5*B*d*f-4*C*(c*f+d*e))))*EllipticE(d^(1/2)*(b*x+a)^(1/2)/(
a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(a*d-b*c)^(1/2)*(b*(d*x+
c)/(-a*d+b*c))^(1/2)*(f*x+e)^(1/2)/b^2/d^(5/2)/f^3/(d*x+c)^(1/2)/(b*(f*x+e
)/(-a*f+b*e))^(1/2)-2/15*(-a*f+b*e)*(a*C*d*f*(-c*f+d*e)-b*(5*d*f*(-3*A*d*f
+B*c*f+2*B*d*e)-C*(4*c^2*f^2+3*c*d*e*f+8*d^2*e^2)))*EllipticF(d^(1/2)*(b*x
+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(a*d-b*c)^(1/
2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(b*(f*x+e)/(-a*f+b*e))^(1/2)/b^2/d^(5/2)/f
^3/(d*x+c)^(1/2)/(f*x+e)^(1/2)
```

3.74.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.66 (sec) , antiderivative size = 615, normalized size of antiderivative = 1.16

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx =$$

$$-\frac{2\left(b^2\sqrt{-a+\frac{bc}{d}}(2a^2Cd^2f^2+abdf(-5Bdf+3C(de+cf))-b^2(C(8d^2e^2+7cdef+8c^2f^2)+5df(3Adf\right.}{}$$

input
```
Integrate[(Sqrt[a + b*x]*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]),
x]
```

output
```
(-2*(b^2*Sqrt[-a + (b*c)/d]*(2*a^2*C*d^2*f^2 + a*b*d*f*(-5*B*d*f + 3*C*(d*
e + c*f)) - b^2*(C*(8*d^2*e^2 + 7*c*d*e*f + 8*c^2*f^2) + 5*d*f*(3*A*d*f -
2*B*(d*e + c*f))))*(c + d*x)*(e + f*x) - b^2*Sqrt[-a + (b*c)/d]*d*f*(a + b
*x)*(c + d*x)*(e + f*x)*(5*b*B*d*f + a*C*d*f + b*C*(-4*d*e - 4*c*f + 3*d*f
*x)) + I*(b*c - a*d)*f*(2*a^2*C*d^2*f^2 + a*b*d*f*(-5*B*d*f + 3*C*(d*e + c
*f)) - b^2*(C*(8*d^2*e^2 + 7*c*d*e*f + 8*c^2*f^2) + 5*d*f*(3*A*d*f - 2*B*(
d*e + c*f))))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e
 + f*x))/(f*(a + b*x))]*EllipticE[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*
x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)] + I*b*(b*c - a*d)*f*(a*C*d*f*(-(d*e)
 + c*f) + b*C*(4*d^2*e^2 + 3*c*d*e*f + 8*c^2*f^2) + 5*b*d*f*(3*A*d*f - B*(
d*e + 2*c*f)))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(
e + f*x))/(f*(a + b*x))]*EllipticF[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b
*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)]))/(15*b^3*Sqrt[-a + (b*c)/d]*d^3*f^
3*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])
```

3.74.3 Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 542, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2118, 27, 171, 27, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}dx$$

$$\Big\downarrow \text{2118}$$

$$\frac{2\int -\frac{b\sqrt{a+bx}(3bcCe+aCde+acCf-5Abdf-(5bBdf-2aCdf-4bC(de+cf))x)}{2\sqrt{c+dx}\sqrt{e+fx}}dx}{5b^2df}+\frac{2C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}{5bdf}$$

$$\Big\downarrow \text{27}$$

$$\frac{2C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}{5bdf}-\frac{\int \frac{\sqrt{a+bx}(3bcCe+aCde+acCf-5Abdf-(5bBdf-2aCdf-4bC(de+cf))x)}{\sqrt{c+dx}\sqrt{e+fx}}dx}{5bdf}$$

$$\Big\downarrow \text{171}$$

$$\frac{2C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}{5bdf}-$$

$$\frac{\frac{2\int \frac{3adf(3bcCe+aCde+acCf-5Abdf)+(bce+ade+acf)(5bBdf-2aCdf-4bC(de+cf))+(3bdf(3bcCe+aCde+acCf-5Abdf)-(adf-2b(de+cf))(5bBdf-2aCdf-4bC}{2\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}}{3df}}{5bdf}$$

$$\Big\downarrow \; 27$$

$$\frac{2C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}{5bdf} - \frac{\frac{\int \frac{3adf(3bcCe+aCde+acCf-5Abdf)+(bce+ade+acf)(5bBdf-2aCdf-4bC(de+cf))+(3bdf(3bcCe+aCde+acCf-5Abdf)-(adf-2b(de+cf))(5bBdf-2aCdf-4bC(d}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}}{3df}}{5bdf}$$

$$\Big\downarrow \; 176$$

$$\frac{2C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}{5bdf} - \frac{\frac{\frac{(be-af)\left(aCdf(de-cf)+5bdf(3Adf-B(cf+2de))+bC\left(4c^2f^2+3cdef+8d^2e^2\right)\right)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx}{f} + \frac{(3bdf(acCf+aCde-5Abdf+3bcCe)-(adf-2b(cf+de))}{f}}{3df}}{5bdf}$$

$$\Big\downarrow \; 124$$

$$\frac{2C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}{5bdf} - \frac{\frac{\frac{(be-af)\left(aCdf(de-cf)+5bdf(3Adf-B(cf+2de))+bC\left(4c^2f^2+3cdef+8d^2e^2\right)\right)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx}{f} + \frac{\sqrt{e+fx}\sqrt{\frac{b(c+dx)}{bc-ad}}(3bdf(acCf+aCde-5Abdf+3bcCe)-}{}}{3df}}{5bdf}$$

$$\Big\downarrow \; 123$$

$$\frac{2C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}{5bdf} - \frac{\frac{\frac{(be-af)\left(aCdf(de-cf)+5bdf(3Adf-B(cf+2de))+bC\left(4c^2f^2+3cdef+8d^2e^2\right)\right)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx}{f} + \frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}(3bdf(acCf+aCde-5Abdf}{}}{3df}}{5b}$$

$$\Big\downarrow \; 131$$

$$\frac{2C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}{5bdf} - \frac{\frac{(be-af)\sqrt{\frac{b(c+dx)}{bc-ad}}\left(aCdf(de-cf)+5bdf(3Adf-B(cf+2de))+bC\left(4c^2f^2+3cdef+8d^2e^2\right)\right)\int \frac{1}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}\sqrt{e+fx}}dx}{f\sqrt{c+dx}} + \frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}(3bdf}{}}{3df}$$

$$\Big\downarrow \; 131$$

3.74. $\int \frac{\sqrt{a+bx}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$

$$\frac{2C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}{5bdf} - \frac{\frac{(be-af)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\left(aCdf(de-cf)+5bdf(3Adf-B(cf+2de))+bC\left(4c^2f^2+3cdef+8d^2e^2\right)\right)\int \frac{1}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}\sqrt{\frac{be}{be-af}+\frac{bfx}{be-af}}}dx}{f\sqrt{c+dx}\sqrt{e+fx}} + \frac{2\sqrt{e+fx}\sqrt{\cdots}}{\cdots}}{3df}$$

$$\Big\downarrow \ 130$$

$$\frac{2C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}{5bdf} - \frac{\frac{2\sqrt{ad-bc}(be-af)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\left(aCdf(de-cf)+5bdf(3Adf-B(cf+2de))+bC\left(4c^2f^2+3cdef+8d^2e^2\right)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{ad-bc}}\right),\frac{(bc-ad)f}{d(be-af)}\right)}{b\sqrt{d}f\sqrt{c+dx}\sqrt{e+fx}} + \frac{2\sqrt{e+f\cdots}}{\cdots}}{3df}$$

input

```
Int[(Sqrt[a + b*x]*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

output

```
(2*C*(a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x])/(5*b*d*f) - ((-2*(5*b*B*
d*f - 2*a*C*d*f - 4*b*C*(d*e + c*f))*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
f*x])/(3*d*f) + ((2*Sqrt[-(b*c) + a*d]*(3*b*d*f*(3*b*c*C*e + a*C*d*e + a*c
*C*f - 5*A*b*d*f) - (a*d*f - 2*b*(d*e + c*f))*(5*b*B*d*f - 2*a*C*d*f - 4*b
*C*(d*e + c*f)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE[A
rcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e
 - a*f))])/(b*Sqrt[d]*f*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]) + (
2*Sqrt[-(b*c) + a*d]*(b*e - a*f)*(a*C*d*f*(d*e - c*f) + b*C*(8*d^2*e^2 + 3
*c*d*e*f + 4*c^2*f^2) + 5*b*d*f*(3*A*d*f - B*(2*d*e + c*f)))*Sqrt[(b*(c +
d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[
d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(
b*Sqrt[d]*f*Sqrt[c + d*x]*Sqrt[e + f*x]))/(3*d*f))/(5*b*d*f)
```

3.74. $\int \frac{\sqrt{a+bx}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$

3.74.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

3.74.4 Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 812, normalized size of antiderivative = 1.54

method	result
elliptic	$\frac{\sqrt{(bx+a)(dx+c)(fx+e)}\left(\frac{2Cx\sqrt{bdf\,x^3+adf\,x^2+bcf\,x^2+bde\,x^2+acfx+adex+bcex+ace}}{5df}+\frac{2\left(Bb+Ca-\frac{2C(2adf+2bcf+2bde)}{5df}\right)\sqrt{bdf\,x^3+adf\,x^2\ldots}}{3bdf}\ldots\right.}{\ldots}$
default	Expression too large to display

3.74. $\int \frac{\sqrt{a+bx}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$

input
```
int((b*x+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
((b*x+a)*(d*x+c)*(f*x+e))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*
(2/5*C/d/f*x*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*
e*x+a*c*e)^(1/2)+2/3*(B*b+C*a-2/5*C/d/f*(2*a*d*f+2*b*c*f+2*b*d*e))/b/d/f*(
b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/
2)+2*(A*a-2/5*C/d/f*a*c*e-2/3*(B*b+C*a-2/5*C/d/f*(2*a*d*f+2*b*c*f+2*b*d*e)
)/b/d/f*(1/2*a*c*f+1/2*a*d*e+1/2*b*c*e))*(e/f-c/d)*((x+e/f)/(e/f-c/d))^(1/
2)*((x+a/b)/(-e/f+a/b))^(1/2)*((x+c/d)/(-e/f+c/d))^(1/2)/(b*d*f*x^3+a*d*f*
x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*EllipticF(((x
+e/f)/(e/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1/2))+2*(A*b+B*a-2/5*C/d/f
*(3/2*a*c*f+3/2*a*d*e+3/2*b*c*e)-2/3*(B*b+C*a-2/5*C/d/f*(2*a*d*f+2*b*c*f+2
*b*d*e))/b/d/f*(a*d*f+b*c*f+b*d*e))*(e/f-c/d)*((x+e/f)/(e/f-c/d))^(1/2)*((
x+a/b)/(-e/f+a/b))^(1/2)*((x+c/d)/(-e/f+c/d))^(1/2)/(b*d*f*x^3+a*d*f*x^2+b
*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*((-e/f+a/b)*Ellipt
icE(((x+e/f)/(e/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1/2))-a/b*EllipticF
(((x+e/f)/(e/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1/2))))
```

3.74.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 1036, normalized size of antiderivative = 1.96

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$$
$$= \frac{2\left(3\left(3\,Cb^3d^3f^3x-4\,Cb^3d^3ef^2-\left(4\,Cb^3cd^2-\left(Cab^2+5\,Bb^3\right)d^3\right)f^3\right)\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}-\left(8\,Cb^3d^3\right.\right.}{}$$

input
```
integrate((b*x+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algor
ithm="fricas")
```

output
```
2/45*(3*(3*C*b^3*d^3*f^3*x - 4*C*b^3*d^3*e*f^2 - (4*C*b^3*c*d^2 - (C*a*b^2
 + 5*B*b^3)*d^3)*f^3)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e) - (8*C*b^3
*d^3*e^3 + (3*C*b^3*c*d^2 - (7*C*a*b^2 + 10*B*b^3)*d^3)*e^2*f + (3*C*b^3*c
^2*d - (2*C*a*b^2 + 5*B*b^3)*c*d^2 - (2*C*a^2*b - 10*B*a*b^2 - 15*A*b^3)*d
^3)*e*f^2 + (8*C*b^3*c^3 - (7*C*a*b^2 + 10*B*b^3)*c^2*d - (2*C*a^2*b - 10*
B*a*b^2 - 15*A*b^3)*c*d^2 - (2*C*a^3 - 5*B*a^2*b + 30*A*a*b^2)*d^3)*f^3)*s
qrt(b*d*f)*weierstrassPInverse(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*e*f
+ (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 -
 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^
3)*e*f^2 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3)/(b
^3*d^3*f^3), 1/3*(3*b*d*f*x + b*d*e + (b*c + a*d)*f)/(b*d*f)) - 3*(8*C*b^3
*d^3*e^2*f + (7*C*b^3*c*d^2 - (3*C*a*b^2 + 10*B*b^3)*d^3)*e*f^2 + (8*C*b^3
*c^2*d - (3*C*a*b^2 + 10*B*b^3)*c*d^2 - (2*C*a^2*b - 5*B*a*b^2 - 15*A*b^3)
*d^3)*f^3)*sqrt(b*d*f)*weierstrassZeta(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d
^2)*e*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3*d
^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 +
a^2*b*d^3)*e*f^2 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)
*f^3)/(b^3*d^3*f^3), weierstrassPInverse(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b
*d^2)*e*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3
*d^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d...
```

3.74.6 Sympy [F]

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx=\int \frac{\sqrt{a+bx}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

input
```
integrate((b*x+a)**(1/2)*(C*x**2+B*x+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x)*(A + B*x + C*x**2)/(sqrt(c + d*x)*sqrt(e + f*x)), x
)
```

3.74.7 Maxima [F]

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{bx+a}}{\sqrt{dx+c}\sqrt{fx+e}}\,dx$$

input
```
integrate((b*x+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algor
ithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)), x
)
```

3.74.8 Giac [F]

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{bx+a}}{\sqrt{dx+c}\sqrt{fx+e}}\,dx$$

input
```
integrate((b*x+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algor
ithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)), x
)
```

3.74.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{\sqrt{a+b\,x}\,(C\,x^2+B\,x+A)}{\sqrt{e+f\,x}\,\sqrt{c+d\,x}}\,dx$$

input
```
int(((a + b*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(c + d*x)^(1/2)),
x)
```

output
```
int(((a + b*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(c + d*x)^(1/2)),
 x)
```

3.75 $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx$

3.75.1 Optimal result

Integrand size = 38, antiderivative size = 387

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx = \frac{2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3bdf} - \frac{2\sqrt{-bc+ad}(2aCdf-b(3Bdf-2C(de+cf)))\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{e+fx}E\left(\arcsin\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{-bc+ad}}\right)|\frac{(bc-ad)f}{d(be-af)}\right)}{3b^2d^{3/2}f^2\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}} + \frac{2\sqrt{-bc+ad}(aCf(de-cf)-b(3df(Be-Af)-Ce(2de+cf)))\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\,\mathrm{EllipticF}\left(\arcsin\left(\right.\right.}{3b^2d^{3/2}f^2\sqrt{c+dx}\sqrt{e+fx}}$$

output
```
2/3*C*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/d/f-2/3*(2*a*C*d*f-b*(3*
B*d*f-2*C*(c*f+d*e)))*EllipticE(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a
*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(a*d-b*c)^(1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2
)*(f*x+e)^(1/2)/b^2/d^(3/2)/f^2/(d*x+c)^(1/2)/(b*(f*x+e)/(-a*f+b*e))^(1/2)
+2/3*(a*C*f*(-c*f+d*e)-b*(3*d*f*(-A*f+B*e)-C*e*(c*f+2*d*e)))*EllipticF(d^(
1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(a*d
-b*c)^(1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(b*(f*x+e)/(-a*f+b*e))^(1/2)/b^2/
d^(3/2)/f^2/(d*x+c)^(1/2)/(f*x+e)^(1/2)
```

3.75.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.49 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.08

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx$$
$$= \frac{\sqrt{a+bx}\left(2b^2Cdf(c+dx)(e+fx)-\frac{2b^2(-3bBdf+2aCdf+2bC(de+cf))(c+dx)(e+fx)}{a+bx}+2i\sqrt{-a+\frac{bc}{d}}df(3bBdf-2a\right.}{}$$

input
```
Integrate[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

output
```
(Sqrt[a + b*x]*(2*b^2*C*d*f*(c + d*x)*(e + f*x) - (2*b^2*(-3*b*B*d*f + 2*a
*C*d*f + 2*b*C*(d*e + c*f))*(c + d*x)*(e + f*x))/(a + b*x) + (2*I)*Sqrt[-a
 + (b*c)/d]*d*f*(3*b*B*d*f - 2*a*C*d*f - 2*b*C*(d*e + c*f))*Sqrt[a + b*x]*
Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*Ellipt
icE[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f -
a*d*f)] + ((2*I)*b*f*(a*C*d*(-(d*e) + c*f) + b*(2*c^2*C*f + 3*A*d^2*f + c*
d*(C*e - 3*B*f)))*Sqrt[a + b*x]*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*
(e + f*x))/(f*(a + b*x))]*EllipticF[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a +
b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)])/Sqrt[-a + (b*c)/d]))/(3*b^3*d^2*f
^2*Sqrt[c + d*x]*Sqrt[e + f*x])
```

3.75.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2118, 27, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx$$
$$\Big\downarrow \ 2118$$

$$\frac{2\int -\frac{b(bcCe+aCde+acCf-3Abdf-(3bBdf-2aCdf-2bC(de+cf))x)}{2\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx}{3b^2df}+\frac{2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3bdf}$$

$\downarrow$ 27

$$\frac{2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3bdf}-\frac{\int \frac{bcCe+aCde+acCf-3Abdf-(3bBdf-2aCdf-2bC(de+cf))x}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx}{3bdf}$$

$\downarrow$ 176

$$\frac{2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3bdf}-\frac{\frac{(-aCf(de-cf)+3bdf(Be-Af)-bCe(cf+2de))\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx}{f}-\frac{(-2aCdf+3bBdf-2bC(cf+de))\int \frac{\sqrt{e+fx}}{\sqrt{a+bx}\sqrt{c+dx}}dx}{f}}{3bdf}$$

$\downarrow$ 124

$$\frac{2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3bdf}-\frac{\frac{(-aCf(de-cf)+3bdf(Be-Af)-bCe(cf+2de))\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx}{f}-\frac{\sqrt{e+fx}\sqrt{\frac{b(c+dx)}{bc-ad}}(-2aCdf+3bBdf-2bC(cf+de))\int \frac{\sqrt{\frac{be}{be-af}+\frac{b\ldots}{be-\ldots}}}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}+\ldots}}}{f\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}}{3bdf}$$

$\downarrow$ 123

$$\frac{2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3bdf}-\frac{\frac{(-aCf(de-cf)+3bdf(Be-Af)-bCe(cf+2de))\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx}{f}-\frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}(-2aCdf+3bBdf-2bC(cf+de))E\left(\arcsin\ldots\right.}{b\sqrt{d}f\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}}{3bdf}$$

$\downarrow$ 131

$$\frac{2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3bdf}-\frac{\frac{\sqrt{\frac{b(c+dx)}{bc-ad}}(-aCf(de-cf)+3bdf(Be-Af)-bCe(cf+2de))\int \frac{1}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}\sqrt{e+fx}}dx}{f\sqrt{c+dx}}-\frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}(-2aCdf+3bBdf-2b\ldots}{b\sqrt{d}f\sqrt{c+dx}\ldots}}{3bdf}$$

$\downarrow$ 131

$$\frac{2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3bdf}-\frac{\frac{\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}(-aCf(de-cf)+3bdf(Be-Af)-bCe(cf+2de))\int \frac{1}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}\sqrt{\frac{be}{be-af}+\frac{bfx}{be-af}}}dx}{f\sqrt{c+dx}\sqrt{e+fx}}-\frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}(\ldots}{\ldots}}{3bdf}$$

3.75. $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx$

$\downarrow$ 130

$$\frac{2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3bdf}-$$

$$\frac{\frac{2\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}(-aCf(de-cf)+3bdf(Be-Af)-bCe(cf+2de))\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{ad-bc}}\right),\frac{(bc-ad)f}{d(be-af)}\right)}{b\sqrt{d}f\sqrt{c+dx}\sqrt{e+fx}}-\frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c-}{bc-}}}{}}{3bdf}$$

input
```
Int[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

output
```
(2*C*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])/(3*b*d*f) - ((-2*Sqrt[-(b*
c) + a*d]*(3*b*B*d*f - 2*a*C*d*f - 2*b*C*(d*e + c*f))*Sqrt[(b*(c + d*x))/(
b*c - a*d)]*Sqrt[e + f*x]*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(
b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(b*Sqrt[d]*f*Sqrt[c + d*x]
*Sqrt[(b*(e + f*x))/(b*e - a*f)]) + (2*Sqrt[-(b*c) + a*d]*(3*b*d*f*(B*e -
A*f) - a*C*f*(d*e - c*f) - b*C*e*(2*d*e + c*f))*Sqrt[(b*(c + d*x))/(b*c -
a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b
*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(b*Sqrt[d]*f*S
qrt[c + d*x]*Sqrt[e + f*x]))/(3*b*d*f)
```

3.75.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

3.75. $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx$

3.75.4 Maple [A] (verified)

Time = 2.60 (sec) , antiderivative size = 615, normalized size of antiderivative = 1.59

method	result
elliptic	$\sqrt{(bx+a)(dx+c)(fx+e)}\left(\frac{2C\sqrt{bdf\,x^{3}+adf\,x^{2}+bcf\,x^{2}+bde\,x^{2}+acfx+adex+bcex+ace}}{3bdf}+\frac{2\left(A-\frac{2C\left(\frac{1}{2}acf+\frac{1}{2}ade+\frac{1}{2}bce\right)}{3bdf}\right)\left(\frac{e}{f}-\frac{c}{d}\right)\sqrt{\frac{x+\frac{e}{f}}{\frac{e}{f}-\frac{c}{d}}}\sqrt{}}{\sqrt{bdf\,x^{3}+adf\,x^{2}+bcf\,x^{2}+bde\,x^{2}+}}\right.$
default	Expression too large to display

input
```
int((C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
((b*x+a)*(d*x+c)*(f*x+e))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*
(2/3*C/b/d/f*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*
e*x+a*c*e)^(1/2)+2*(A-2/3*C/b/d/f*(1/2*a*c*f+1/2*a*d*e+1/2*b*c*e))*(e/f-c/
d)*((x+e/f)/(e/f-c/d))^(1/2)*((x+a/b)/(-e/f+a/b))^(1/2)*((x+c/d)/(-e/f+c/d
))^(1/2)/(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+
a*c*e)^(1/2)*EllipticF(((x+e/f)/(e/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(
1/2))+2*(B-2/3*C/b/d/f*(a*d*f+b*c*f+b*d*e))*(e/f-c/d)*((x+e/f)/(e/f-c/d))^
(1/2)*((x+a/b)/(-e/f+a/b))^(1/2)*((x+c/d)/(-e/f+c/d))^(1/2)/(b*d*f*x^3+a*d
*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*((-e/f+a/b
)*EllipticE(((x+e/f)/(e/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1/2))-a/b*E
llipticF(((x+e/f)/(e/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1/2))))
```

3.75.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 807, normalized size of antiderivative = 2.09

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

$$=\frac{2\left(3\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}Cb^2d^2f^2+(2\,Cb^2d^2e^2+(Cb^2cd+(Cab-3\,Bb^2)d^2)ef+(2\,Cb^2c^2+(Cab\right.}{}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algor
ithm="fricas")
```

output
```
2/9*(3*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*C*b^2*d^2*f^2 + (2*C*b^2*
d^2*e^2 + (C*b^2*c*d + (C*a*b - 3*B*b^2)*d^2)*e*f + (2*C*b^2*c^2 + (C*a*b
- 3*B*b^2)*c*d + (2*C*a^2 - 3*B*a*b + 9*A*b^2)*d^2)*f^2)*sqrt(b*d*f)*weier
strassPInverse(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 - a*b
*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 +
a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 + (2*b^
3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3), 1/3
*(3*b*d*f*x + b*d*e + (b*c + a*d)*f)/(b*d*f)) + 3*(2*C*b^2*d^2*e*f + (2*C*
b^2*c*d + (2*C*a*b - 3*B*b^2)*d^2)*f^2)*sqrt(b*d*f)*weierstrassZeta(4/3*(b
^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)/
(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f - 3*
(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 + (2*b^3*c^3 - 3*a*b^2*c^2*d
 - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3), weierstrassPInverse(4/3*
(b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2
)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f -
3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 + (2*b^3*c^3 - 3*a*b^2*c^2
*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3), 1/3*(3*b*d*f*x + b*d*e
 + (b*c + a*d)*f)/(b*d*f))))/(b^3*d^3*f^3)
```

3.75.6 Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx=\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(b*x+a)**(1/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(sqrt(a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)), x
)
```

3.75.7 Maxima [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algor
ithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)), x
)
```

3.75.8 Giac [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algor
ithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)), x
)
```

3.75.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx=\int \frac{C\,x^2+B\,x+A}{\sqrt{e+f\,x}\,\sqrt{a+b\,x}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)^(1/2)*(c + d*x)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)^(1/2)*(c + d*x)^(1/2)), x
)
```

3.76 $\int \frac{A+Bx+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx$

3.76.1 Optimal result

Integrand size = 38, antiderivative size = 422

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx = -\frac{2(Ab^2-a(bB-aC))\sqrt{c+dx}\sqrt{e+fx}}{b(bc-ad)(be-af)\sqrt{a+bx}}$$
$$-\frac{2(2a^2Cdf+b^2(cCe+Adf)-ab(Cde+cCf+Bdf))\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{e+fx}E\left(\arcsin\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{-bc+ad}}\right)|\frac{(bc-ad)f}{d(be-af)}\right)}{b^2\sqrt{d}\sqrt{-bc+ad}f(be-af)\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}$$
$$-\frac{2(aC(de-cf)-b(cCe-Bcf+Adf))\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{-bc+ad}}\right),\frac{(bc-ad)f}{d(be-af)}\right)}{b^2\sqrt{d}\sqrt{-bc+ad}f\sqrt{c+dx}\sqrt{e+fx}}$$

output
```
-2*(A*b^2-a*(B*b-C*a))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/(-a*d+b*c)/(-a*f+b*e)
/(b*x+a)^(1/2)-2*(2*a^2*C*d*f+b^2*(A*d*f+C*c*e)-a*b*(B*d*f+C*c*f+C*d*e))*E
llipticE(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))
^(1/2))*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(f*x+e)^(1/2)/b^2/f/(-a*f+b*e)/d^(1/2
)/(a*d-b*c)^(1/2)/(d*x+c)^(1/2)/(b*(f*x+e)/(-a*f+b*e))^(1/2)-2*(a*C*(-c*f+
d*e)-b*(A*d*f-B*c*f+C*c*e))*EllipticF(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2
),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(b*(f*x+
e)/(-a*f+b*e))^(1/2)/b^2/f/d^(1/2)/(a*d-b*c)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(
1/2)
```

3.76.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.27 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.13

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx = \frac{2\left(-b^2(Ab^2+a(-bB+aC))\,(c+dx)(e+fx)+\frac{b^2(2a^2Cdf+b^2(cCe+Adf)-}{}\right.}{}$$

input
```
Integrate[(A + B*x + C*x^2)/((a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x]),
x]
```

output
```
(2*(-(b^2*(A*b^2 + a*(-(b*B) + a*C))*(c + d*x)*(e + f*x)) + (b^2*(2*a^2*C*
d*f + b^2*(c*C*e + A*d*f) - a*b*(C*d*e + c*C*f + B*d*f))*(c + d*x)*(e + f*
x))/(d*f) + (I*(b*c - a*d)*(2*a^2*C*d*f + b^2*(c*C*e + A*d*f) - a*b*(C*d*e
 + c*C*f + B*d*f))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[
(b*(e + f*x))/(f*(a + b*x))]*EllipticE[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a
 + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)])/(Sqrt[-a + (b*c)/d]*d) + (I*b*
(-(b*c) + a*d)*(a*C*(d*e - c*f) + b*(c*C*e - B*d*e + A*d*f))*(a + b*x)^(3/
2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*Ell
ipticF[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f
 - a*d*f)])/(Sqrt[-a + (b*c)/d]*d)))/(b^3*(b*c - a*d)*(b*e - a*f)*Sqrt[a +
 b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])
```

3.76.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2117, 27, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}dx$$

$$\Big\downarrow \; 2117$$

$$-\frac{2\int -\frac{C(de+cf)a^2-b(cCe+Bde+Bcf-Adf)a+b^2Bce+b\left(\frac{2Cdfa^2}{b}-(Cde+cCf+Bdf)a+b(cCe+Adf)\right)x}{2b\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx}{(bc-ad)(be-af)}-\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$\downarrow$ 27

$$\frac{\int \frac{C(de+cf)a^2-b(cCe+Bde+Bcf-Adf)a+b^2Bce+b\left(\frac{2Cdfa^2}{b}-(Cde+cCf+Bdf)a+b(cCe+Adf)\right)x}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx}{b(bc-ad)(be-af)}-\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$\downarrow$ 176

$$\frac{\frac{\left(2a^2Cdf-ab(Bdf+cCf+Cde)+b^2(Adf+cCe)\right)\int \frac{\sqrt{e+fx}}{\sqrt{a+bx}\sqrt{c+dx}}dx}{f}+\frac{(be-af)(aC(de-cf)-b(Adf-Bcf+cCe))\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx}{f}}{b(bc-ad)(be-af)}-\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$\downarrow$ 124

$$\frac{\frac{\sqrt{e+fx}\sqrt{\frac{b(c+dx)}{bc-ad}}\left(2a^2Cdf-ab(Bdf+cCf+Cde)+b^2(Adf+cCe)\right)\int \frac{\sqrt{\frac{be}{be-af}+\frac{bfx}{be-af}}}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}}dx}{f\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}+\frac{(be-af)(aC(de-cf)-b(Adf-Bcf+cCe))\int \frac{\cdots}{\sqrt{a\cdots}}\cdots}{f}}{b(bc-ad)(be-af)}$$
$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$\downarrow$ 123

$$\frac{\frac{(be-af)(aC(de-cf)-b(Adf-Bcf+cCe))\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx}{f}+\frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}\left(2a^2Cdf-ab(Bdf+cCf+Cde)+b^2(Adf+cCe)\right)\cdots}{b\sqrt{d}f\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}\cdots}}{b(bc-ad)(be-af)}$$
$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$\downarrow$ 131

3.76. $\int \frac{A+Bx+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx$

$$\frac{\frac{(be-af)\sqrt{\frac{b(c+dx)}{bc-ad}}(aC(de-cf)-b(Adf-Bcf+cCe))\int \frac{1}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}\sqrt{e+fx}}dx}{f\sqrt{c+dx}}+\frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}(2a^2Cdf-ab(Bdf+cCf+C\cdots}{b\sqrt{d}f\sqrt{c+dx}\cdots}}{b(bc-ad)(be-af)}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$\big\downarrow$ 131

$$\frac{\frac{(be-af)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}(aC(de-cf)-b(Adf-Bcf+cCe))\int \frac{1}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}\sqrt{\frac{be}{be-af}+\frac{bfx}{be-af}}}dx}{f\sqrt{c+dx}\sqrt{e+fx}}+\frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}(2a^2C\cdots}{\cdots}}{b(bc-ad)(be-af)}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$\big\downarrow$ 130

$$\frac{\frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}(2a^2Cdf-ab(Bdf+cCf+Cde)+b^2(Adf+cCe))E\left(\arcsin\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{ad-bc}}\right)|\frac{(bc-ad)f}{d(be-af)}\right)}{b\sqrt{d}f\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}+\frac{2\sqrt{ad-bc}(be-af)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e\cdots}{be\cdots}}\cdots}{\cdots}}{b(bc-ad)(be-af)}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

input
```
Int[(A + B*x + C*x^2)/((a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

output
```
(-2*(A*b^2 - a*(b*B - a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/(b*(b*c - a*d)*(b
*e - a*f)*Sqrt[a + b*x]) + ((2*Sqrt[-(b*c) + a*d]*(2*a^2*C*d*f + b^2*(c*C*
e + A*d*f) - a*b*(C*d*e + c*C*f + B*d*f))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*
Sqrt[e + f*x]*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]]
, ((b*c - a*d)*f)/(d*(b*e - a*f))])/(b*Sqrt[d]*f*Sqrt[c + d*x]*Sqrt[(b*(e
 + f*x))/(b*e - a*f)]) + (2*Sqrt[-(b*c) + a*d]*(b*e - a*f)*(a*C*(d*e - c*f)
 - b*(c*C*e - B*c*f + A*d*f))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[(b*(e +
 f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) +
 a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(b*Sqrt[d]*f*Sqrt[c + d*x]*Sqrt[
e + f*x]))/(b*(b*c - a*d)*(b*e - a*f))
```

3.76. $\int \frac{A+Bx+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx$

3.76.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 2117
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Si
mp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*
(e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1
) - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x]
, x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n, 2*p]
```

3.76.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 783 vs. $2(382) = 764$.

Time = 3.34 (sec) , antiderivative size = 784, normalized size of antiderivative = 1.86

method	result
elliptic	$\frac{\sqrt{(bx+a)(dx+c)(fx+e)}\left(-\frac{2\left(bdf\,x^{2}+bcfx+bdex+bce\right)\left(b^{2}A-abB+C\,a^{2}\right)}{\left(a^{2}df-acfb-abde+b^{2}ce\right)b^{2}\sqrt{\left(x+\frac{a}{b}\right)\left(bdf\,x^{2}+bcfx+bdex+bce\right)}}+\frac{2\left(\frac{Bb-Ca}{b^{2}}+\frac{(adf-bcf-bde)\left(b^{2}A-abB+C\,a^{2}\right)}{b^{2}\left(a^{2}df-acfb-abde+b^{2}ce\right)}\right.}{\sqrt{b}}\right.}{}$
default	Expression too large to display

input
```
int((C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
((b*x+a)*(d*x+c)*(f*x+e))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*
(-2*(b*d*f*x^2+b*c*f*x+b*d*e*x+b*c*e)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^
2*(A*b^2-B*a*b+C*a^2)/((x+a/b)*(b*d*f*x^2+b*c*f*x+b*d*e*x+b*c*e))^(1/2)+2*
((B*b-C*a)/b^2+1/b^2*(a*d*f-b*c*f-b*d*e)*(A*b^2-B*a*b+C*a^2)/(a^2*d*f-a*b*
c*f-a*b*d*e+b^2*c*e)+(b*c*f+b*d*e)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2*(
A*b^2-B*a*b+C*a^2))*(e/f-c/d)*((x+e/f)/(e/f-c/d))^(1/2)*((x+a/b)/(-e/f+a/b
))^(1/2)*((x+c/d)/(-e/f+c/d))^(1/2)/(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x
^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*EllipticF(((x+e/f)/(e/f-c/d))^(1/2
),((-e/f+c/d)/(-e/f+a/b))^(1/2))+2*(C/b+1/b*d*f*(A*b^2-B*a*b+C*a^2)/(a^2*d
*f-a*b*c*f-a*b*d*e+b^2*c*e))*(e/f-c/d)*((x+e/f)/(e/f-c/d))^(1/2)*((x+a/b)/
(-e/f+a/b))^(1/2)*((x+c/d)/(-e/f+c/d))^(1/2)/(b*d*f*x^3+a*d*f*x^2+b*c*f*x^
2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*((-e/f+a/b)*EllipticE(((x
+e/f)/(e/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1/2))-a/b*EllipticF(((x+e/
f)/(e/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1/2))))
```

3.76.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 1240, normalized size of antiderivative = 2.94

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algor
ithm="fricas")
```

output
```
-2/3*(3*(C*a^2*b^2 - B*a*b^3 + A*b^4)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x
 + e)*d^2*f^2 + ((C*a*b^3*c*d - C*a^2*b^2*d^2)*e^2 + (C*a*b^3*c^2 + (2*C*a
^2*b^2 - 3*B*a*b^3)*c*d - (2*C*a^3*b - 2*B*a^2*b^2 - A*a*b^3)*d^2)*e*f - (
C*a^2*b^2*c^2 + (2*C*a^3*b - 2*B*a^2*b^2 - A*a*b^3)*c*d - (2*C*a^4 - B*a^3
*b - 2*A*a^2*b^2)*d^2)*f^2 + ((C*b^4*c*d - C*a*b^3*d^2)*e^2 + (C*b^4*c^2 +
 (2*C*a*b^3 - 3*B*b^4)*c*d - (2*C*a^2*b^2 - 2*B*a*b^3 - A*b^4)*d^2)*e*f -
(C*a*b^3*c^2 + (2*C*a^2*b^2 - 2*B*a*b^3 - A*b^4)*c*d - (2*C*a^3*b - B*a^2*
b^2 - 2*A*a*b^3)*d^2)*f^2)*x)*sqrt(b*d*f)*weierstrassPInverse(4/3*(b^2*d^2
*e^2 - (b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d
^2*f^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c
^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a
^2*b*c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3), 1/3*(3*b*d*f*x + b*d*e + (b*c
+ a*d)*f)/(b*d*f)) + 3*sqrt(b*d*f)*((C*a*b^3*c*d - C*a^2*b^2*d^2)*e*f - (C
*a^2*b^2*c*d - (2*C*a^3*b - B*a^2*b^2 + A*a*b^3)*d^2)*f^2 + ((C*b^4*c*d -
C*a*b^3*d^2)*e*f - (C*a*b^3*c*d - (2*C*a^2*b^2 - B*a*b^3 + A*b^4)*d^2)*f^2
)*x)*weierstrassZeta(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*e*f + (b^2*c^2
 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*
d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 +
 (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3
), weierstrassPInverse(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*e*f + (b^...
```

3.76.6 Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx=\int \frac{A+Bx+Cx^2}{(a+bx)^{\frac{3}{2}}\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(b*x+a)**(3/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/((a + b*x)**(3/2)*sqrt(c + d*x)*sqrt(e + f*x))
, x)
```

3.76.7 Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{Cx^2+Bx+A}{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\sqrt{fx+e}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algor
ithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/((b*x + a)^(3/2)*sqrt(d*x + c)*sqrt(f*x + e)),
 x)
```

3.76.8 Giac [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{Cx^2+Bx+A}{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\sqrt{fx+e}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algor
ithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)/((b*x + a)^(3/2)*sqrt(d*x + c)*sqrt(f*x + e)),
 x)
```

3.76.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{C\,x^2+B\,x+A}{\sqrt{e+f\,x}\,(a+b\,x)^{3/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)^(3/2)*(c + d*x)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)^(3/2)*(c + d*x)^(1/2)), x
)
```

3.77 $\int \frac{A+Bx+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx$

3.77.1 Optimal result

Integrand size = 38, antiderivative size = 642

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx = -\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^{3/2}}$$
$$+\frac{2\left(2a^3Cdf+ab^2(6cCe+Bde+Bcf-4Adf)-b^3(3Bce-2A(de+cf))+a^2b(Bdf-4C(de+cf))\right)\sqrt{c-\cdots}}{3b(bc-ad)^2(be-af)^2\sqrt{a+bx}}$$
$$-\frac{2\sqrt{d}\left(2a^3Cdf+ab^2(6cCe+Bde+Bcf-4Adf)-b^3(3Bce-2A(de+cf))+a^2b(Bdf-4C(de+cf))\right)\cdots}{3b^2(-bc+ad)^{3/2}(be-af)^2\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}$$
$$-\frac{2\left(a^2Cd(de-cf)-b^2(3c^2Ce-3Bcde+2Ad^2e+Acdf)+ab(3(c^2C+Ad^2)f-Bd(de+2cf))\right)\sqrt{\frac{b(c+dx)}{bc-ad}}\cdots}{3b^2\sqrt{d}(-bc+ad)^{3/2}(be-af)\sqrt{c+dx}\sqrt{e+fx}}$$

output
```
-2/3*(A*b^2-a*(B*b-C*a))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/(-a*d+b*c)/(-a*f+b*
e)/(b*x+a)^(3/2)+2/3*(2*a^3*C*d*f+a*b^2*(-4*A*d*f+B*c*f+B*d*e+6*C*c*e)-b^3
*(3*B*c*e-2*A*(c*f+d*e))+a^2*b*(B*d*f-4*C*(c*f+d*e)))*(d*x+c)^(1/2)*(f*x+e
)^(1/2)/b/(-a*d+b*c)^2/(-a*f+b*e)^2/(b*x+a)^(1/2)-2/3*(2*a^3*C*d*f+a*b^2*(
-4*A*d*f+B*c*f+B*d*e+6*C*c*e)-b^3*(3*B*c*e-2*A*(c*f+d*e))+a^2*b*(B*d*f-4*C
*(c*f+d*e)))*EllipticE(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f
/d/(-a*f+b*e))^(1/2))*d^(1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(f*x+e)^(1/2)/b
^2/(a*d-b*c)^(3/2)/(-a*f+b*e)^2/(d*x+c)^(1/2)/(b*(f*x+e)/(-a*f+b*e))^(1/2)
-2/3*(a^2*C*d*(-c*f+d*e)-b^2*(A*c*d*f+2*A*d^2*e-3*B*c*d*e+3*C*c^2*e)+a*b*(
3*(A*d^2+C*c^2)*f-B*d*(2*c*f+d*e)))*EllipticF(d^(1/2)*(b*x+a)^(1/2)/(a*d-b
*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(b*(d*x+c)/(-a*d+b*c))^(1/2)*
(b*(f*x+e)/(-a*f+b*e))^(1/2)/b^2/(a*d-b*c)^(3/2)/(-a*f+b*e)/d^(1/2)/(d*x+c
)^(1/2)/(f*x+e)^(1/2)
```

3.77.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 27.39 (sec) , antiderivative size = 699, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx =$$

$$-\frac{2\left(b^2\sqrt{-a+\frac{bc}{d}}(c+dx)(e+fx)\left(\left(Ab^2+a(-bB+aC)\right)(bc-ad)(be-af)+\left(-2a^3Cdf-ab^2(6cCe+B\right.\right.\right.}{}$$

input
```
Integrate[(A + B*x + C*x^2)/((a + b*x)^(5/2)*Sqrt[c + d*x]*Sqrt[e + f*x]),
x]
```

output
```
(-2*(b^2*Sqrt[-a + (b*c)/d]*(c + d*x)*(e + f*x)*((A*b^2 + a*(-(b*B) + a*C)
)*(b*c - a*d)*(b*e - a*f) + (-2*a^3*C*d*f - a*b^2*(6*c*C*e + B*d*e + B*c*f
 - 4*A*d*f) + b^3*(3*B*c*e - 2*A*(d*e + c*f)) + a^2*b*(-(B*d*f) + 4*C*(d*e
 + c*f)))*(a + b*x)) + (a + b*x)*(b^2*Sqrt[-a + (b*c)/d]*(2*a^3*C*d*f + a*
b^2*(6*c*C*e + B*d*e + B*c*f - 4*A*d*f) + b^3*(-3*B*c*e + 2*A*(d*e + c*f))
 + a^2*b*(B*d*f - 4*C*(d*e + c*f)))*(c + d*x)*(e + f*x) + I*(b*c - a*d)*f*
(2*a^3*C*d*f + a*b^2*(6*c*C*e + B*d*e + B*c*f - 4*A*d*f) + b^3*(-3*B*c*e +
 2*A*(d*e + c*f)) + a^2*b*(B*d*f - 4*C*(d*e + c*f)))*(a + b*x)^(3/2)*Sqrt[
(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticE[I
*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f
)] - I*b*(b*c - a*d)*(a^2*C*f*(d*e - c*f) + b^2*(3*c*C*e^2 + A*d*e*f + c*f
*(-3*B*e + 2*A*f)) + a*b*(-3*C*d*e^2 + f*(2*B*d*e + B*c*f - 3*A*d*f)))*(a
+ b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a +
b*x))]*EllipticF[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d
*f)/(b*c*f - a*d*f)])))/(3*b^3*Sqrt[-a + (b*c)/d]*(b*c - a*d)^2*(b*e - a*f
)^2*(a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x])
```

3.77.3 Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 672, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2117, 27, 169, 27, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

$$\Big\downarrow \text{2117}$$

$$-\frac{2\int -\frac{C(de+cf)a^2-b(3cCe+Bde+Bcf-3Adf)a+b^2(3Bce-2A(de+cf))+b\left(\frac{2Cdfa^2}{b}-3Cdea-3cCfa+Bdfa+3bcCe-Abdf\right)x}{2b(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}dx}{3(bc-ad)(be-af)}-\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{C(de+cf)a^2-b(3cCe+Bde+Bcf-3Adf)a+b^2(3Bce-2A(de+cf))+b\left(\frac{2Cdfa^2}{b}+Bdfa-3C(de+cf)a+b(3cCe-Adf)\right)x}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}dx}{3b(bc-ad)(be-af)} - \frac{2\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$\Big\downarrow$ 169

$$\frac{\frac{2\sqrt{c+dx}\sqrt{e+fx}(2a^3Cdf+a^2b(Bdf-4C(cf+de))+ab^2(-4Adf+Bcf+Bde+6cCe)-b^3(3Bce-2A(cf+de)))}{\sqrt{a+bx}(bc-ad)(be-af)} - 2\int \frac{Cdf(de+cf)a^3-b\left(C\left(3d^2e^2+5cdf\right.\right.}{}}{} \frac{2\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$\Big\downarrow$ 27

$$\frac{\frac{2\sqrt{c+dx}\sqrt{e+fx}(2a^3Cdf+a^2b(Bdf-4C(cf+de))+ab^2(-4Adf+Bcf+Bde+6cCe)-b^3(3Bce-2A(cf+de)))}{\sqrt{a+bx}(bc-ad)(be-af)} - \int \frac{Cdf(de+cf)a^3-b\left(C\left(3d^2e^2+5cdfe\right.\right.}{}}{} \frac{2\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$\Big\downarrow$ 176

$$\frac{\frac{2\sqrt{c+dx}\sqrt{e+fx}(2a^3Cdf+a^2b(Bdf-4C(cf+de))+ab^2(-4Adf+Bcf+Bde+6cCe)-b^3(3Bce-2A(cf+de)))}{\sqrt{a+bx}(bc-ad)(be-af)} - \frac{(be-af)\left(a^2Cd(de-cf)+ab(3f(A\right.}{}}{} \frac{2\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$\Big\downarrow$ 124

$$\frac{\frac{2\sqrt{c+dx}\sqrt{e+fx}(2a^3Cdf+a^2b(Bdf-4C(cf+de))+ab^2(-4Adf+Bcf+Bde+6cCe)-b^3(3Bce-2A(cf+de)))}{\sqrt{a+bx}(bc-ad)(be-af)} - \frac{(be-af)\left(a^2Cd(de-cf)+ab(3f(A\right.}{}}{} \frac{2\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$\Big\downarrow$ 123

3.77. $\displaystyle\int \frac{A+Bx+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx$

$$\frac{\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(2a^3Cdf+a^2b(Bdf-4C(cf+de))+ab^2(-4Adf+Bcf+Bde+6cCe)-b^3(3Bce-2A(cf+de))\right)}{\sqrt{a+bx}(bc-ad)(be-af)}-\frac{(be-af)\left(a^2Cd(de-cf)+ab(3f(A\ldots\right.}{\ldots}}{\ldots}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$\big\downarrow$ 131

$$\frac{\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(2a^3Cdf+a^2b(Bdf-4C(cf+de))+ab^2(-4Adf+Bcf+Bde+6cCe)-b^3(3Bce-2A(cf+de))\right)}{\sqrt{a+bx}(bc-ad)(be-af)}-\frac{\frac{(be-af)\sqrt{\frac{b(c+dx)}{bc-ad}}\left(a^2Cd(de-cf)+a\ldots\right.}{\ldots}}{\ldots}}{\ldots}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$\big\downarrow$ 131

$$\frac{\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(2a^3Cdf+a^2b(Bdf-4C(cf+de))+ab^2(-4Adf+Bcf+Bde+6cCe)-b^3(3Bce-2A(cf+de))\right)}{\sqrt{a+bx}(bc-ad)(be-af)}-\frac{\frac{(be-af)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\left(a^2Cd(\ldots\right.}{\ldots}}{\ldots}}{\ldots}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$\big\downarrow$ 130

$$\frac{\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(2a^3Cdf+a^2b(Bdf-4C(cf+de))+ab^2(-4Adf+Bcf+Bde+6cCe)-b^3(3Bce-2A(cf+de))\right)}{\sqrt{a+bx}(bc-ad)(be-af)}-\frac{\frac{2\sqrt{ad-bc}(be-af)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+f\ldots}{be-a\ldots}}}{\ldots}}{\ldots}}{\ldots}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

input
```
Int[(A + B*x + C*x^2)/((a + b*x)^(5/2)*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

3.77. $\int \frac{A+Bx+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx$

output
```
(-2*(A*b^2 - a*(b*B - a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/(3*b*(b*c - a*d)*
(b*e - a*f)*(a + b*x)^(3/2)) + ((2*(2*a^3*C*d*f + a*b^2*(6*c*C*e + B*d*e +
 B*c*f - 4*A*d*f) - b^3*(3*B*c*e - 2*A*(d*e + c*f)) + a^2*b*(B*d*f - 4*C*(
d*e + c*f)))*Sqrt[c + d*x]*Sqrt[e + f*x])/((b*c - a*d)*(b*e - a*f)*Sqrt[a
+ b*x]) - ((2*Sqrt[d]*Sqrt[-(b*c) + a*d]*(2*a^3*C*d*f + a*b^2*(6*c*C*e + B
*d*e + B*c*f - 4*A*d*f) - b^3*(3*B*c*e - 2*A*(d*e + c*f)) + a^2*b*(B*d*f -
 4*C*(d*e + c*f)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE
[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b
*e - a*f))])/(b*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]) + (2*Sqrt[-
(b*c) + a*d]*(b*e - a*f)*(a^2*C*d*(d*e - c*f) - b^2*(3*c^2*C*e - 3*B*c*d*e
 + 2*A*d^2*e + A*c*d*f) + a*b*(3*(c^2*C + A*d^2)*f - B*d*(d*e + 2*c*f)))*S
qrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[A
rcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e
 - a*f))])/(b*Sqrt[d]*Sqrt[c + d*x]*Sqrt[e + f*x]))/((b*c - a*d)*(b*e - a*
f)))/(3*b*(b*c - a*d)*(b*e - a*f))
```

3.77.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 2117
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Si
mp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*
(e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1
) - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x]
, x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n, 2*p]
```

3.77. $\int \frac{A+Bx+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx$

3.77.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1248 vs. $2(588) = 1176$.

Time = 4.56 (sec) , antiderivative size = 1249, normalized size of antiderivative = 1.95

method	result	size
elliptic	Expression too large to display	1249
default	Expression too large to display	13099

input
```
int((C*x^2+B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
((b*x+a)*(d*x+c)*(f*x+e))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*
(-2/3/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^3*(A*b^2-B*a*b+C*a^2)*(b*d*f*x^3
+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)/(x+a/b
)^2-2/3*(b*d*f*x^2+b*c*f*x+b*d*e*x+b*c*e)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e
)^2/b^2*(4*A*a*b^2*d*f-2*A*b^3*c*f-2*A*b^3*d*e-B*a^2*b*d*f-B*a*b^2*c*f-B*a
*b^2*d*e+3*B*b^3*c*e-2*C*a^3*d*f+4*C*a^2*b*c*f+4*C*a^2*b*d*e-6*C*a*b^2*c*e
)/((x+a/b)*(b*d*f*x^2+b*c*f*x+b*d*e*x+b*c*e))^(1/2)+2*(C/b^2-1/3*d*f/b^2*(
A*b^2-B*a*b+C*a^2)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)+1/3/b^2*(a*d*f-b*c*f-
b*d*e)*(4*A*a*b^2*d*f-2*A*b^3*c*f-2*A*b^3*d*e-B*a^2*b*d*f-B*a*b^2*c*f-B*a*
b^2*d*e+3*B*b^3*c*e-2*C*a^3*d*f+4*C*a^2*b*c*f+4*C*a^2*b*d*e-6*C*a*b^2*c*e)
/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)^2+1/3*(b*c*f+b*d*e)/(a^2*d*f-a*b*c*f-a*
b*d*e+b^2*c*e)^2/b^2*(4*A*a*b^2*d*f-2*A*b^3*c*f-2*A*b^3*d*e-B*a^2*b*d*f-B*
a*b^2*c*f-B*a*b^2*d*e+3*B*b^3*c*e-2*C*a^3*d*f+4*C*a^2*b*c*f+4*C*a^2*b*d*e-
6*C*a*b^2*c*e))*(e/f-c/d)*((x+e/f)/(e/f-c/d))^(1/2)*((x+a/b)/(-e/f+a/b))^(
1/2)*((x+c/d)/(-e/f+c/d))^(1/2)/(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a
*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*EllipticF(((x+e/f)/(e/f-c/d))^(1/2),((
-e/f+c/d)/(-e/f+a/b))^(1/2))+2/3/b*d*f*(4*A*a*b^2*d*f-2*A*b^3*c*f-2*A*b^3*
d*e-B*a^2*b*d*f-B*a*b^2*c*f-B*a*b^2*d*e+3*B*b^3*c*e-2*C*a^3*d*f+4*C*a^2*b*
c*f+4*C*a^2*b*d*e-6*C*a*b^2*c*e)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)^2*(e/f-
c/d)*((x+e/f)/(e/f-c/d))^(1/2)*((x+a/b)/(-e/f+a/b))^(1/2)*((x+c/d)/(-e/...
```

3.77.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.30 (sec) , antiderivative size = 2344, normalized size of antiderivative = 3.65

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algor
ithm="fricas")
```

output
```
2/9*(3*(((5*C*a^2*b^4 - 2*B*a*b^5 - A*b^6)*c*d - 3*(C*a^3*b^3 - A*a*b^5)*d
^2)*e*f - (3*(C*a^3*b^3 - A*a*b^5)*c*d - (C*a^4*b^2 + 2*B*a^3*b^3 - 5*A*a^
2*b^4)*d^2)*f^2 + ((3*(2*C*a*b^5 - B*b^6)*c*d - (4*C*a^2*b^4 - B*a*b^5 - 2
*A*b^6)*d^2)*e*f - ((4*C*a^2*b^4 - B*a*b^5 - 2*A*b^6)*c*d - (2*C*a^3*b^3 +
 B*a^2*b^4 - 4*A*a*b^5)*d^2)*f^2)*x)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x
+ e) + ((9*C*a^2*b^4*c^2 - 3*(4*C*a^3*b^3 + B*a^2*b^4)*c*d + (5*C*a^4*b^2
+ B*a^3*b^3 + 2*A*a^2*b^4)*d^2)*e^2 - (3*(4*C*a^3*b^3 + B*a^2*b^4)*c^2 - (
13*C*a^4*b^2 + 11*B*a^3*b^3 + A*a^2*b^4)*c*d + (5*C*a^5*b + 4*B*a^4*b^2 +
5*A*a^3*b^3)*d^2)*e*f + ((5*C*a^4*b^2 + B*a^3*b^3 + 2*A*a^2*b^4)*c^2 - (5*
C*a^5*b + 4*B*a^4*b^2 + 5*A*a^3*b^3)*c*d + (2*C*a^6 + B*a^5*b + 5*A*a^4*b^
2)*d^2)*f^2 + ((9*C*b^6*c^2 - 3*(4*C*a*b^5 + B*b^6)*c*d + (5*C*a^2*b^4 + B
*a*b^5 + 2*A*b^6)*d^2)*e^2 - (3*(4*C*a*b^5 + B*b^6)*c^2 - (13*C*a^2*b^4 +
11*B*a*b^5 + A*b^6)*c*d + (5*C*a^3*b^3 + 4*B*a^2*b^4 + 5*A*a*b^5)*d^2)*e*f
 + ((5*C*a^2*b^4 + B*a*b^5 + 2*A*b^6)*c^2 - (5*C*a^3*b^3 + 4*B*a^2*b^4 + 5
*A*a*b^5)*c*d + (2*C*a^4*b^2 + B*a^3*b^3 + 5*A*a^2*b^4)*d^2)*f^2)*x^2 + 2*
((9*C*a*b^5*c^2 - 3*(4*C*a^2*b^4 + B*a*b^5)*c*d + (5*C*a^3*b^3 + B*a^2*b^4
 + 2*A*a*b^5)*d^2)*e^2 - (3*(4*C*a^2*b^4 + B*a*b^5)*c^2 - (13*C*a^3*b^3 +
11*B*a^2*b^4 + A*a*b^5)*c*d + (5*C*a^4*b^2 + 4*B*a^3*b^3 + 5*A*a^2*b^4)*d^
2)*e*f + ((5*C*a^3*b^3 + B*a^2*b^4 + 2*A*a*b^5)*c^2 - (5*C*a^4*b^2 + 4*B*a
^3*b^3 + 5*A*a^2*b^4)*c*d + (2*C*a^5*b + B*a^4*b^2 + 5*A*a^3*b^3)*d^2)*...
```

3.77.6 Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{A+Bx+Cx^2}{(a+bx)^{\frac{5}{2}}\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(b*x+a)**(5/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/((a + b*x)**(5/2)*sqrt(c + d*x)*sqrt(e + f*x))
, x)
```

3.77.7 Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{Cx^2+Bx+A}{(bx+a)^{\frac{5}{2}}\sqrt{dx+c}\sqrt{fx+e}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algor
ithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/((b*x + a)^(5/2)*sqrt(d*x + c)*sqrt(f*x + e)),
 x)
```

3.77.8 Giac [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{Cx^2+Bx+A}{(bx+a)^{\frac{5}{2}}\sqrt{dx+c}\sqrt{fx+e}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algor
ithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)/((b*x + a)^(5/2)*sqrt(d*x + c)*sqrt(f*x + e)),
 x)
```

3.77.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{C\,x^2+B\,x+A}{\sqrt{e+f\,x}\,{\left(a+b\,x\right)}^{5/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)^(5/2)*(c + d*x)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)^(5/2)*(c + d*x)^(1/2)), x
)
```

3.77. $\int \frac{A+Bx+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx$

3.78 $\int \frac{A+Bx+Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx$

3.78.1 Optimal result

Integrand size = 38, antiderivative size = 1116

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx = -\frac{2(Ab^2-a(bB-aC))\sqrt{c+dx}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$$+\frac{2(2a^3Cdf+ab^2(10cCe+Bde+Bcf-8Adf)-b^3(5Bce-4A(de+cf))+3a^2b(Bdf-2C(de+cf)))\sqrt{}}{15b(bc-ad)^2(be-af)^2(a+bx)^{3/2}}$$

$$+\frac{2(2a^4Cd^2f^2+a^3bdf(3Bdf-7C(de+cf))-b^4(8Ad^2e^2-cde(10Be-7Af)+c^2(15Ce^2-10Bef+8Af}{}$$

$$+\frac{2\sqrt{d}(2a^4Cd^2f^2+a^3bdf(3Bdf-7C(de+cf))-b^4(8Ad^2e^2-cde(10Be-7Af)+c^2(15Ce^2-10Bef+8}{}$$

$$+\frac{2\sqrt{d}(a^3Cdf(de-cf)+b^3(8Ad^2e^2-cde(10Be-3Af)+c^2(15Ce^2-5Bef+4Af^2))+ab^2(d^2e(2Be-19}{}$$

output
```
-2/5*(A*b^2-a*(B*b-C*a))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/(-a*d+b*c)/(-a*f+b*
e)/(b*x+a)^(5/2)+2/15*(2*a^3*C*d*f+a*b^2*(-8*A*d*f+B*c*f+B*d*e+10*C*c*e)-b
^3*(5*B*c*e-4*A*(c*f+d*e))+3*a^2*b*(B*d*f-2*C*(c*f+d*e)))*(d*x+c)^(1/2)*(f
*x+e)^(1/2)/b/(-a*d+b*c)^2/(-a*f+b*e)^2/(b*x+a)^(3/2)+2/15*(2*a^4*C*d^2*f^
2+a^3*b*d*f*(3*B*d*f-7*C*(c*f+d*e))-b^4*(8*A*d^2*e^2-c*d*e*(-7*A*f+10*B*e)
+c^2*(8*A*f^2-10*B*e*f+15*C*e^2))-a*b^3*(d^2*e*(-23*A*f+2*B*e)-2*c^2*f*(-B
*f+5*C*e)-c*d*(23*A*f^2-33*B*e*f+10*C*e^2))-a^2*b^2*(C*(3*c^2*f^2-13*c*d*e
*f+3*d^2*e^2)+d*f*(23*A*d*f-7*B*(c*f+d*e))))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b
/(-a*d+b*c)^3/(-a*f+b*e)^3/(b*x+a)^(1/2)+2/15*(2*a^4*C*d^2*f^2+a^3*b*d*f*(
3*B*d*f-7*C*(c*f+d*e))-b^4*(8*A*d^2*e^2-c*d*e*(-7*A*f+10*B*e)+c^2*(8*A*f^2
-10*B*e*f+15*C*e^2))-a*b^3*(d^2*e*(-23*A*f+2*B*e)-2*c^2*f*(-B*f+5*C*e)-c*d
*(23*A*f^2-33*B*e*f+10*C*e^2))-a^2*b^2*(C*(3*c^2*f^2-13*c*d*e*f+3*d^2*e^2)
+d*f*(23*A*d*f-7*B*(c*f+d*e))))*EllipticE(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^
(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*d^(1/2)*(b*(d*x+c)/(-a*d+b*c))^(1
/2)*(f*x+e)^(1/2)/b^2/(a*d-b*c)^(5/2)/(-a*f+b*e)^3/(d*x+c)^(1/2)/(b*(f*x+e
)/(-a*f+b*e))^(1/2)+2/15*(a^3*C*d*f*(-c*f+d*e)+b^3*(8*A*d^2*e^2-c*d*e*(-3*
A*f+10*B*e)+c^2*(4*A*f^2-5*B*e*f+15*C*e^2))+a*b^2*(d^2*e*(-19*A*f+2*B*e)-c
^2*f*(-B*f+20*C*e)-c*d*(11*A*f^2-27*B*e*f+10*C*e^2))-3*a^2*b*(d*f*(-5*A*d*
f+3*B*c*f+2*B*d*e)-C*(3*c^2*f^2+c*d*e*f+d^2*e^2)))*EllipticF(d^(1/2)*(b*x+
a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*d^(1/2)*(b*...
```

3.78.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 33.54 (sec) , antiderivative size = 1258, normalized size of antiderivative = 1.13

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx =$$

$$-\frac{2\left(b^2\sqrt{-a+\frac{bc}{d}}(2a^4Cd^2f^2+a^3bdf(3Bdf-7C(de+cf))-b^4(8Ad^2e^2+cde(-10Be+7Af)+c^2(15Ce^2\right.}{}$$

input
```
Integrate[(A + B*x + C*x^2)/((a + b*x)^(7/2)*Sqrt[c + d*x]*Sqrt[e + f*x]),
x]
```

output
```
(-2*(b^2*Sqrt[-a + (b*c)/d]*(2*a^4*C*d^2*f^2 + a^3*b*d*f*(3*B*d*f - 7*C*(d
*e + c*f)) - b^4*(8*A*d^2*e^2 + c*d*e*(-10*B*e + 7*A*f) + c^2*(15*C*e^2 -
10*B*e*f + 8*A*f^2)) + a*b^3*(d^2*e*(-2*B*e + 23*A*f) - 2*c^2*f*(-5*C*e +
B*f) + c*d*(10*C*e^2 - 33*B*e*f + 23*A*f^2)) + a^2*b^2*(d*f*(7*B*d*e + 7*B
*c*f - 23*A*d*f) + C*(-3*d^2*e^2 + 13*c*d*e*f - 3*c^2*f^2)))*(a + b*x)^2*(
c + d*x)*(e + f*x) + b^2*Sqrt[-a + (b*c)/d]*(c + d*x)*(e + f*x)*(3*(A*b^2
+ a*(-(b*B) + a*C))*(b*c - a*d)^2*(b*e - a*f)^2 + (b*c - a*d)*(b*e - a*f)*
(-2*a^3*C*d*f - a*b^2*(10*c*C*e + B*d*e + B*c*f - 8*A*d*f) + b^3*(5*B*c*e
- 4*A*(d*e + c*f)) + 3*a^2*b*(-(B*d*f) + 2*C*(d*e + c*f)))*(a + b*x) + (-2
*a^4*C*d^2*f^2 + a^3*b*d*f*(-3*B*d*f + 7*C*(d*e + c*f)) + a*b^3*(d^2*e*(2*
B*e - 23*A*f) + 2*c^2*f*(-5*C*e + B*f) + c*d*(-10*C*e^2 + 33*B*e*f - 23*A*
f^2)) + b^4*(8*A*d^2*e^2 + c*d*e*(-10*B*e + 7*A*f) + c^2*(15*C*e^2 - 10*B*
e*f + 8*A*f^2)) + a^2*b^2*(C*(3*d^2*e^2 - 13*c*d*e*f + 3*c^2*f^2) + d*f*(2
3*A*d*f - 7*B*(d*e + c*f))))*(a + b*x)^2) + I*(b*c - a*d)*f*(2*a^4*C*d^2*f
^2 + a^3*b*d*f*(3*B*d*f - 7*C*(d*e + c*f)) - b^4*(8*A*d^2*e^2 + c*d*e*(-10
*B*e + 7*A*f) + c^2*(15*C*e^2 - 10*B*e*f + 8*A*f^2)) + a*b^3*(d^2*e*(-2*B*
e + 23*A*f) - 2*c^2*f*(-5*C*e + B*f) + c*d*(10*C*e^2 - 33*B*e*f + 23*A*f^2
)) + a^2*b^2*(d*f*(7*B*d*e + 7*B*c*f - 23*A*d*f) + C*(-3*d^2*e^2 + 13*c*d*
e*f - 3*c^2*f^2)))*(a + b*x)^(7/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[
(b*(e + f*x))/(f*(a + b*x))]*EllipticE[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqr...
```

3.78.3 Rubi [A] (verified)

Time = 2.41 (sec) , antiderivative size = 1176, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2117, 27, 169, 27, 169, 27, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}}dx$$

$$\Big\downarrow \text{2117}$$

$$-\frac{2\int -\frac{C(de+cf)a^2-b(5cCe+Bde+Bcf-5Adf)a+b^2(5Bce-4A(de+cf))+b\left(\frac{2Cdfa^2}{b}-5Cdea-5cCfa+3Bdfa+5bcCe-3Abdf\right)x}{2b(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}dx}{5(bc-ad)(be-af)}-\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{5b(a+bx)^{5/2}(bc-ad)(be-af)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{C(de+cf)a^2-b(5cCe+Bde+Bcf-5Adf)a+b^2(5Bce-4A(de+cf))+b\left(\frac{2Cdfa^2}{b}+3Bdfa-5C(de+cf)a+b(5cCe-3Adf)\right)x}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}dx}{5b(bc-ad)(be-af)}-$$
$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{5b(a+bx)^{5/2}(bc-ad)(be-af)}$$

$\downarrow$ 169

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}(2a^3Cdf+3a^2b(Bdf-2C(cf+de))+ab^2(-8Adf+Bcf+Bde+10cCe)-b^3(5Bce-4A(cf+de)))}{3(a+bx)^{3/2}(bc-ad)(be-af)}-\frac{2\int -\frac{Cdf(de+cf)a^3+3b\left(C\left(d^2e^2-\right.\right.}{}}{}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{5b(a+bx)^{5/2}(bc-ad)(be-af)}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{Cdf(de+cf)a^3+3b\left(C\left(d^2e^2-cdfe+c^2f^2\right)+df(5Adf-2B(de+cf))\right)a^2+b^2\left(-2f(5Ce-Bf)c^2-d\left(10Ce^2-28Bfe+19Af^2\right)c+d^2e(2Be-19Af)\right)a+b^3\left(\left(15Ce^2-10B\right.\right.}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+f}}}{3(bc-ad)(be-af)}}{}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{5b(a+bx)^{5/2}(bc-ad)(be-af)}$$

$\downarrow$ 169

$$\frac{\frac{2\sqrt{c+dx}\sqrt{e+fx}(2Cdfa^3+3b(Bdf-2C(de+cf))a^2+b^2(10cCe+Bde+Bcf-8Adf)a-b^3(5Bce-4A(de+cf)))}{3(bc-ad)(be-af)(a+bx)^{3/2}}+\frac{2\left(2Cd^2f^2a^4+bdf(3Bdf-7C(de+cf)\right.}{}}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$\downarrow$ 27

$$\frac{\frac{2\sqrt{c+dx}\sqrt{e+fx}(2Cdfa^3+3b(Bdf-2C(de+cf))a^2+b^2(10cCe+Bde+Bcf-8Adf)a-b^3(5Bce-4A(de+cf)))}{3(bc-ad)(be-af)(a+bx)^{3/2}}+\frac{2\left(2Cd^2f^2a^4+bdf(3Bdf-7C(de+cf)\right.}{}}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$\downarrow$ 176

3.78. $\int \frac{A+Bx+Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(2Cdfa^3+3b(Bdf-2C(de+cf))a^2+b^2(10cCe+Bde+Bcf-8Adf)a-b^3(5Bce-4A(de+cf))\right)}{3(bc-ad)(be-af)(a+bx)^{3/2}}+\frac{2\left(2Cd^2f^2a^4+bdf(3Bdf-7C(de+cf)\right)}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$$\Big\downarrow \; 124$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(2Cdfa^3+3b(Bdf-2C(de+cf))a^2+b^2(10cCe+Bde+Bcf-8Adf)a-b^3(5Bce-4A(de+cf))\right)}{3(bc-ad)(be-af)(a+bx)^{3/2}}+\frac{2\left(2Cd^2f^2a^4+bdf(3Bdf-7C(de+cf)\right)}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$$\Big\downarrow \; 123$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(2Cdfa^3+3b(Bdf-2C(de+cf))a^2+b^2(10cCe+Bde+Bcf-8Adf)a-b^3(5Bce-4A(de+cf))\right)}{3(bc-ad)(be-af)(a+bx)^{3/2}}+\frac{2\left(2Cd^2f^2a^4+bdf(3Bdf-7C(de+cf)\right)}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$$\Big\downarrow \; 131$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(2Cdfa^3+3b(Bdf-2C(de+cf))a^2+b^2(10cCe+Bde+Bcf-8Adf)a-b^3(5Bce-4A(de+cf))\right)}{3(bc-ad)(be-af)(a+bx)^{3/2}}+\frac{2\left(2Cd^2f^2a^4+bdf(3Bdf-7C(de+cf)\right)}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$$\Big\downarrow \; 131$$

3.78. $\int \frac{A+Bx+Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx$

$$\frac{\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(2Cdfa^3+3b(Bdf-2C(de+cf))a^2+b^2(10cCe+Bde+Bcf-8Adf)a-b^3(5Bce-4A(de+cf))\right)}{3(bc-ad)(be-af)(a+bx)^{3/2}}+\frac{2\left(2Cd^2f^2a^4+bdf(3Bdf-7C(de+cf)\right.}{}}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$$\Big\downarrow \; 130$$

$$\frac{\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(2Cdfa^3+3b(Bdf-2C(de+cf))a^2+b^2(10cCe+Bde+Bcf-8Adf)a-b^3(5Bce-4A(de+cf))\right)}{3(bc-ad)(be-af)(a+bx)^{3/2}}+\frac{2\left(2Cd^2f^2a^4+bdf(3Bdf-7C(de+cf)\right.}{}}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

input

```
Int[(A + B*x + C*x^2)/((a + b*x)^(7/2)*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

output

```
(-2*(A*b^2 - a*(b*B - a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/(5*b*(b*c - a*d)*
(b*e - a*f)*(a + b*x)^(5/2)) + ((2*(2*a^3*C*d*f + a*b^2*(10*c*C*e + B*d*e
+ B*c*f - 8*A*d*f) - b^3*(5*B*c*e - 4*A*(d*e + c*f)) + 3*a^2*b*(B*d*f - 2*
C*(d*e + c*f)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(3*(b*c - a*d)*(b*e - a*f)*(a
 + b*x)^(3/2)) + ((2*(2*a^4*C*d^2*f^2 + a^3*b*d*f*(3*B*d*f - 7*C*(d*e + c*
f)) - b^4*(8*A*d^2*e^2 - c*d*e*(10*B*e - 7*A*f) + c^2*(15*C*e^2 - 10*B*e*f
 + 8*A*f^2)) - a*b^3*(d^2*e*(2*B*e - 23*A*f) - 2*c^2*f*(5*C*e - B*f) - c*d
*(10*C*e^2 - 33*B*e*f + 23*A*f^2)) - a^2*b^2*(C*(3*d^2*e^2 - 13*c*d*e*f +
3*c^2*f^2) + d*f*(23*A*d*f - 7*B*(d*e + c*f))))*Sqrt[c + d*x]*Sqrt[e + f*x
])/((b*c - a*d)*(b*e - a*f)*Sqrt[a + b*x]) - (d*f*((2*Sqrt[-(b*c) + a*d]*(
2*a^4*C*d^2*f^2 + a^3*b*d*f*(3*B*d*f - 7*C*(d*e + c*f)) - b^4*(8*A*d^2*e^2
 - c*d*e*(10*B*e - 7*A*f) + c^2*(15*C*e^2 - 10*B*e*f + 8*A*f^2)) - a*b^3*(
d^2*e*(2*B*e - 23*A*f) - 2*c^2*f*(5*C*e - B*f) - c*d*(10*C*e^2 - 33*B*e*f
+ 23*A*f^2)) + a^2*b^2*(d*f*(7*B*d*e + 7*B*c*f - 23*A*d*f) - C*(3*d^2*e^2
- 13*c*d*e*f + 3*c^2*f^2)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*
EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)
*f)/(d*(b*e - a*f))])/(b*Sqrt[d]*f*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e -
 a*f)]) + (2*Sqrt[-(b*c) + a*d]*(b*e - a*f)*(a^3*C*d*f*(d*e - c*f) + b^3*(
8*A*d^2*e^2 - c*d*e*(10*B*e - 3*A*f) + c^2*(15*C*e^2 - 5*B*e*f + 4*A*f^2))
 + a*b^2*(d^2*e*(2*B*e - 19*A*f) - c^2*f*(20*C*e - B*f) - c*d*(10*C*e^2...
```

3.78. $\int \frac{A+Bx+Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx$

3.78.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 2117
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Si
mp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*
(e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1
) - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x]
, x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n, 2*p]
```

3.78.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2282 vs. 2(1054) = 2108.

Time = 6.00 (sec) , antiderivative size = 2283, normalized size of antiderivative = 2.05

method	result	size
elliptic	Expression too large to display	2283
default	Expression too large to display	32154

input
```
int((C*x^2+B*x+A)/(b*x+a)^(7/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
((b*x+a)*(d*x+c)*(f*x+e))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*
(-2/5/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^4*(A*b^2-B*a*b+C*a^2)*(b*d*f*x^3
+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)/(x+a/b
)^3-2/15*(8*A*a*b^2*d*f-4*A*b^3*c*f-4*A*b^3*d*e-3*B*a^2*b*d*f-B*a*b^2*c*f-
B*a*b^2*d*e+5*B*b^3*c*e-2*C*a^3*d*f+6*C*a^2*b*c*f+6*C*a^2*b*d*e-10*C*a*b^2
*c*e)/b^3/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)^2*(b*d*f*x^3+a*d*f*x^2+b*c*f*x
^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)/(x+a/b)^2-2/15*(b*d*f*x^
2+b*c*f*x+b*d*e*x+b*c*e)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)^3/b^2*(23*A*a^2
*b^2*d^2*f^2-23*A*a*b^3*c*d*f^2-23*A*a*b^3*d^2*e*f+8*A*b^4*c^2*f^2+7*A*b^4
*c*d*e*f+8*A*b^4*d^2*e^2-3*B*a^3*b*d^2*f^2-7*B*a^2*b^2*c*d*f^2-7*B*a^2*b^2
*d^2*e*f+2*B*a*b^3*c^2*f^2+33*B*a*b^3*c*d*e*f+2*B*a*b^3*d^2*e^2-10*B*b^4*c
^2*e*f-10*B*b^4*c*d*e^2-2*C*a^4*d^2*f^2+7*C*a^3*b*c*d*f^2+7*C*a^3*b*d^2*e*
f+3*C*a^2*b^2*c^2*f^2-13*C*a^2*b^2*c*d*e*f+3*C*a^2*b^2*d^2*e^2-10*C*a*b^3*
c^2*e*f-10*C*a*b^3*c*d*e^2+15*C*b^4*c^2*e^2)/((x+a/b)*(b*d*f*x^2+b*c*f*x+b
*d*e*x+b*c*e))^(1/2)+2*(-1/15*d*f*(8*A*a*b^2*d*f-4*A*b^3*c*f-4*A*b^3*d*e-3
*B*a^2*b*d*f-B*a*b^2*c*f-B*a*b^2*d*e+5*B*b^3*c*e-2*C*a^3*d*f+6*C*a^2*b*c*f
+6*C*a^2*b*d*e-10*C*a*b^2*c*e)/b^2/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)^2+1/1
5/b^2*(a*d*f-b*c*f-b*d*e)*(23*A*a^2*b^2*d^2*f^2-23*A*a*b^3*c*d*f^2-23*A*a*
b^3*d^2*e*f+8*A*b^4*c^2*f^2+7*A*b^4*c*d*e*f+8*A*b^4*d^2*e^2-3*B*a^3*b*d^2*
f^2-7*B*a^2*b^2*c*d*f^2-7*B*a^2*b^2*d^2*e*f+2*B*a*b^3*c^2*f^2+33*B*a*b^...
```

3.78.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.05 (sec) , antiderivative size = 5108, normalized size of antiderivative = 4.58

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^(7/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algor
ithm="fricas")
```

output
```
Too large to include
```

3.78.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx=\text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/(b*x+a)**(7/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

output
```
Timed out
```

3.78.7 Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx=\int \frac{Cx^2+Bx+A}{(bx+a)^{\frac{7}{2}}\sqrt{dx+c}\sqrt{fx+e}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^(7/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algor
ithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/((b*x + a)^(7/2)*sqrt(d*x + c)*sqrt(f*x + e)),
 x)
```

3.78.8 Giac [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx=\int \frac{Cx^2+Bx+A}{(bx+a)^{\frac{7}{2}}\sqrt{dx+c}\sqrt{fx+e}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^(7/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algor
ithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)/((b*x + a)^(7/2)*sqrt(d*x + c)*sqrt(f*x + e)),
 x)
```

3.78.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx=\int \frac{C\,x^2+B\,x+A}{\sqrt{e+f\,x}\,{(a+b\,x)}^{7/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)^(7/2)*(c + d*x)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)^(7/2)*(c + d*x)^(1/2)), x
)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)



(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#            Port of original Maple grading function by
#            Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#            added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
        return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinsta
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                  #is checked before calling the grading function that is passed.
                  #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =" "
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


    print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
return grade, grade_annotation
```